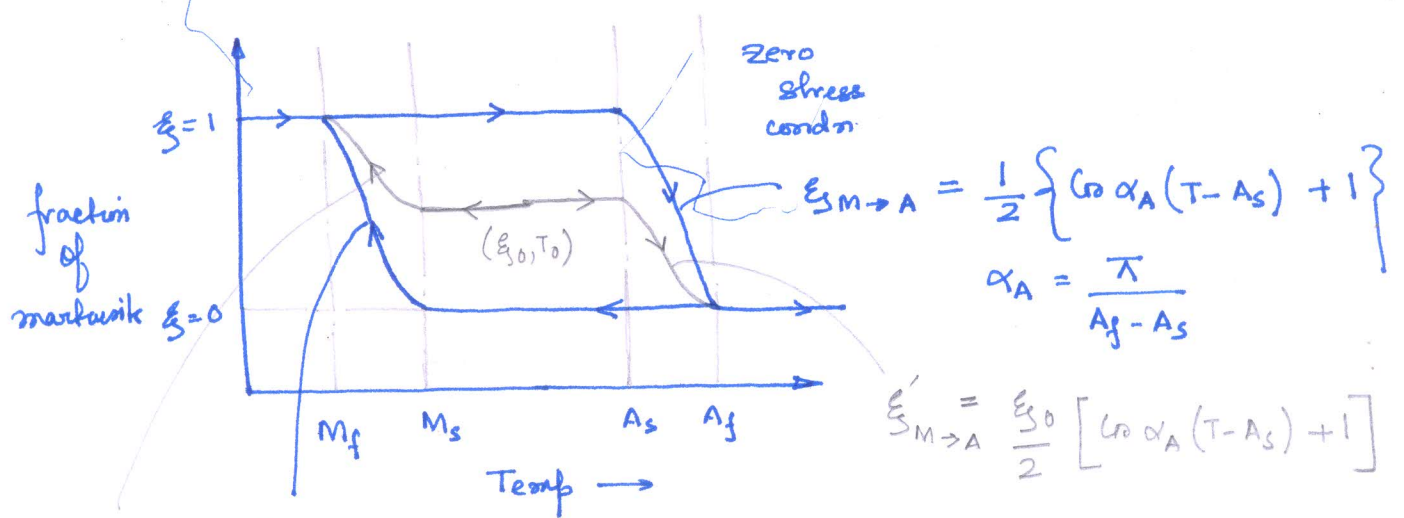


Phase transformation for SMA



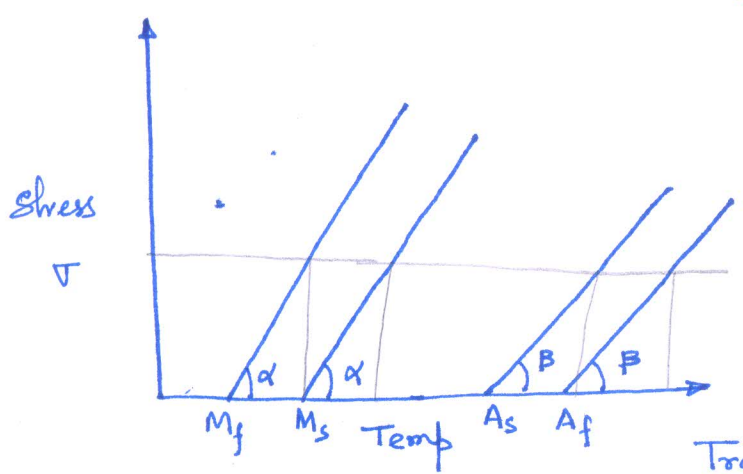
$$\xi'_{M \rightarrow A} = \frac{\xi_0}{2} \left[\cos \alpha_A (T - A_s) + 1 \right]$$

$$\xi_{A \rightarrow M} = \frac{1}{2} \left\{ \cos \alpha_M (T - M_f) + 1 \right\}$$

$$= \frac{1 - \xi_0}{2} \cos \alpha_M (T - M_f) + \frac{1 + \xi_0}{2}$$

$$\alpha_M = \frac{\pi}{M_s - M_f}$$

Effect of stress on transition temperatures: ~~Brinson~~ Model



$$M_f^* = M_f + \frac{\sigma}{\tan \alpha} = M_f + \frac{\sigma}{C_M}$$

$$M_s^* = M_s + \frac{\sigma}{\tan \alpha} = M_s + \frac{\sigma}{C_M}$$

$$A_s^* = A_s + \frac{\sigma}{\tan \beta} = A_s + \frac{\sigma}{C_A}$$

$$A_f^* = A_f + \frac{\sigma}{\tan \beta} = A_f + \frac{\sigma}{C_A}$$

Transition temps. are elevated in the presence of stress

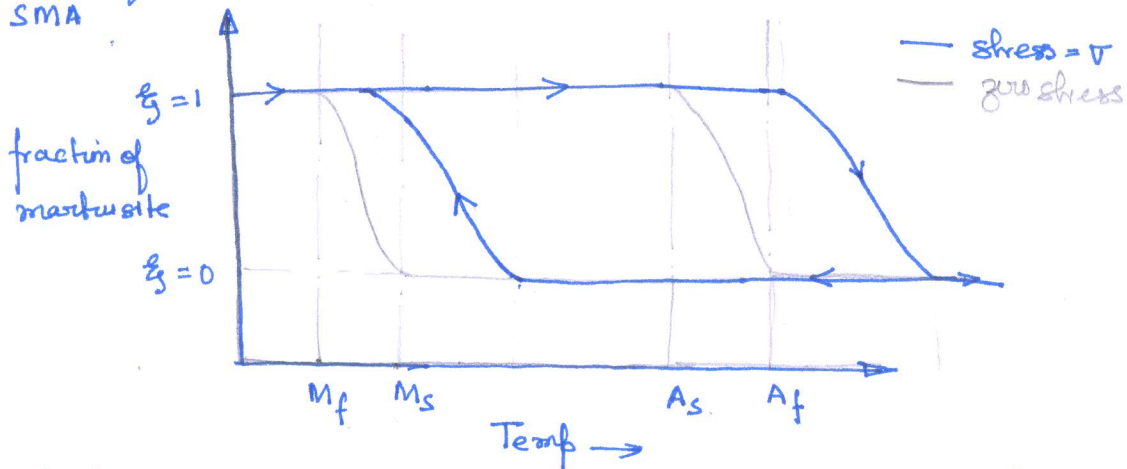
$$\xi_{M \rightarrow A} = \frac{1}{2} \left\{ \cos \alpha_A \left(T - A_s - \frac{\sigma}{C_A} \right) + 1 \right\}$$

$$\xi_{M \rightarrow M} = \frac{1}{2} \left\{ \cos \alpha_M \left(T - M_f - \frac{\sigma}{C_M} \right) + 1 \right\}$$

$$\xi'_{M \rightarrow A} = \frac{\xi_0}{2} \left[\cos \alpha_A \left(T - A_s - \frac{\sigma}{C_A} \right) + 1 \right]$$

$$\xi'_{A \rightarrow M} = \frac{1 - \xi_0}{2} \cos \alpha_M \left(T - M_f - \frac{\sigma}{C_M} \right) + \frac{1 + \xi_0}{2}$$

Effect of stress on phase transformation of SMA



Example

A NiTiNOI shape memory alloy wire with $M_s = 23^\circ\text{C}$ and $M_f = 5^\circ\text{C}$, $A_s = 29^\circ\text{C}$ and $A_f = 51^\circ\text{C}$, $C_A = 4.5 \text{ MPa}/^\circ\text{C}$, $C_M = 11.3 \text{ MPa}/^\circ\text{C}$ is in zero-stress state at a temp of 23°C . (a) compute the ϵ_M if the material is cooled to a temp. of 15°C in stress free state.

$$\epsilon_{A \rightarrow M} = \frac{1}{2} \left\{ \ln \left[\alpha_M (T - M_f) + 1 \right] \right\} \quad \alpha_M = \frac{\pi}{M_s - M_f} = \frac{\pi}{23 - 5} = \frac{\pi}{18}$$

$$= \frac{1}{2} \left\{ \ln \frac{\pi}{18} (15 - 5) + 1 \right\}$$

$$= \frac{1}{2} \left\{ \ln \frac{5\pi}{9} + 1 \right\} = 0.411$$

(b) Assuming the temp. is held constant at 15°C , compute ϵ_M if $\sigma = 90 \text{ MPa}$ is applied.

$$\epsilon'_{A \rightarrow M} = \frac{1 - \epsilon_{\sigma 0}}{2} \left\{ \ln \alpha_M \left(T - M_f - \frac{\sigma}{C_M} \right) \right\} + \frac{1 + \epsilon_{\sigma 0}}{2}$$

$$= \frac{1 - 0.411}{2} \left\{ \ln \frac{\pi}{18} \left(15 - 5 - \frac{90}{11.3} \right) \right\} + \frac{1 + 0.411}{2}$$

$$= 0.982$$

