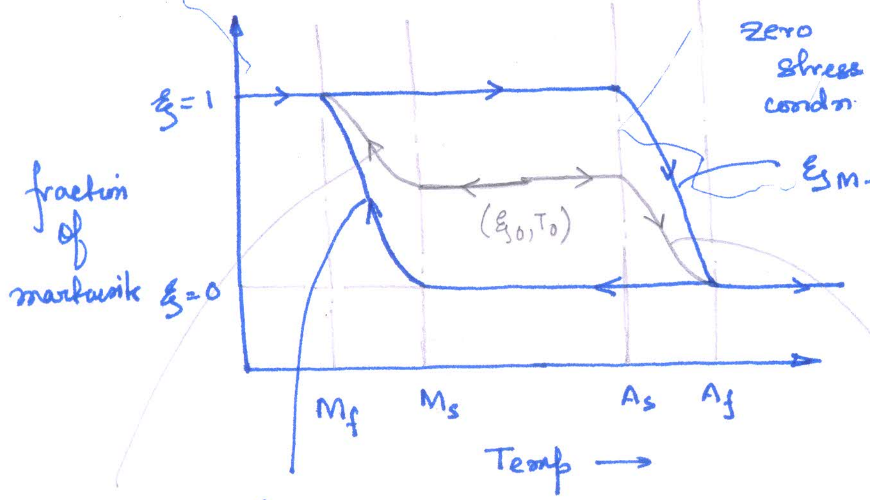
# Phase transformation for SMA




$$\xi_{M\to A} = \frac{1}{2}\left\{\cos \alpha_A (T - A_s) + 1\right\}$$

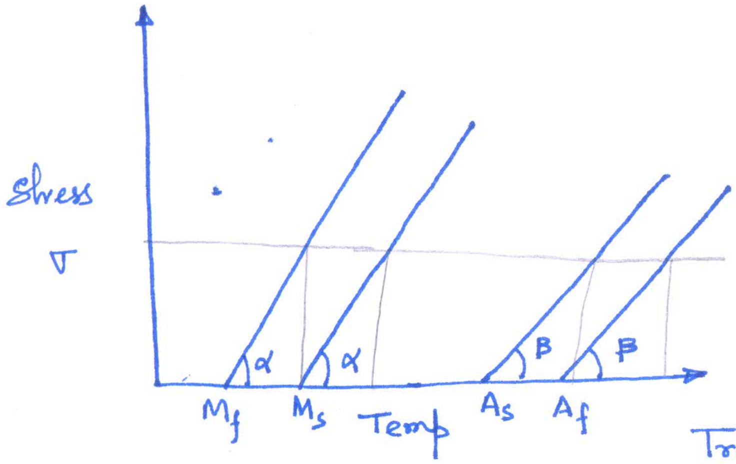
$$\alpha_A = \frac{\pi}{A_f - A_s}$$

$$\xi'_{M\to A} = \frac{\xi_0}{2}\left[\cos \alpha_A (T - A_s) + 1\right]$$

$$\xi_{A\to M} = \frac{1}{2}\left\{\cos \alpha_M (T - M_f) + 1\right\} \qquad \alpha_M = \frac{\pi}{M_s - M_f}$$

$$\xi'_{A\to M} = \frac{1-\xi_0}{2}\cos \alpha_M (T - M_f) + \frac{1+\xi_0}{2}$$

## Effect of stress on transition temperatures : ~~Brinson Model~~




$$M_f^* = M_f + \frac{\sigma}{\tan\alpha} = M_f + \frac{\sigma}{C_M}$$

$$M_s^* = M_s + \frac{\sigma}{\tan\alpha} = M_s + \frac{\sigma}{C_M}$$

$$A_s^* = A_s + \frac{\sigma}{\tan\beta} = A_s + \frac{\sigma}{C_A}$$

$$A_f^* = A_f + \frac{\sigma}{\tan\beta} = A_f + \frac{\sigma}{C_A}$$

Transition temps. are elevated in the presence of stress

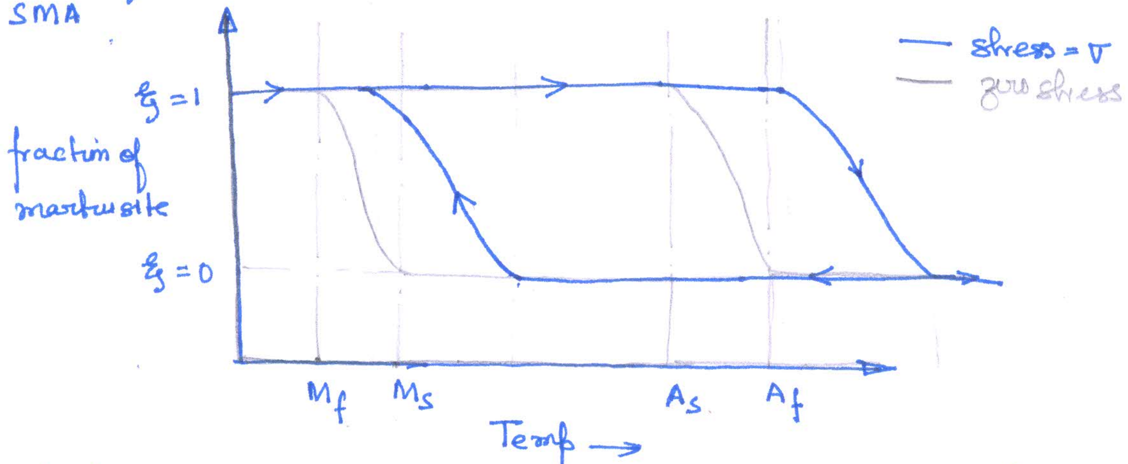
$$\xi_{M\to A} = \frac{1}{2}\left\{\cos \alpha_A \left(T - A_s - \frac{\sigma}{C_A}\right) + 1\right\}$$

$$\xi_{A\to M} = \frac{1}{2}\left\{\cos \alpha_M \left(T - M_f - \frac{\sigma}{C_M}\right) + 1\right\}$$

$$\xi'_{M\to A} = \frac{\xi_0}{2}\left[\cos \alpha_A \left(T - A_s - \frac{\sigma}{C_A}\right) + 1\right]$$

$$\xi'_{A\to M} = \frac{1-\xi_0}{2}\cos \alpha_M \left(T - M_f - \frac{\sigma}{C_M}\right) + \frac{1+\xi_0}{2}$$

Effect of stress on phase transformation of SMA




**Example** A NiTiNOL shape memory alloy wire with $M_s = 23°C$ and $M_f = 5°C$, $A_s = 29°C$ and $A_f = 51°C$, $C_A = 4.5\ MPa/°C$, $C_M = 11.3\ MPa/°C$ is in zero-stress state at a temp of $23°C$. (a) compute the $\xi$ if the material is cooled to a temp. of $15°C$ in stress free state.

$$\xi_{A \to M} = \frac{1}{2}\left\{\cos\left\{\alpha_M (T - M_f)\right\} + 1\right\} \qquad \alpha_M = \frac{\pi}{M_s - M_f} = \frac{\pi}{23-5} = \frac{\pi}{18}$$

$$= \frac{1}{2}\left\{\cos \frac{\pi}{18}(15 - 5) + 1\right\}$$

$$= \frac{1}{2}\left\{\cos \frac{5\pi}{9} + 1\right\} = 0.411$$

(b) Assuming the temp. is held constant at $15°C$, compute $\xi$ if $\sigma = 90\ MPa$ is applied.

$$\xi'_{A \to M} = \frac{1 - \xi_0}{2}\left\{\cos \alpha_M \left(T - M_f - \frac{\sigma}{C_M}\right)\right\} + \frac{1 + \xi_0}{2}$$

$$= \frac{1 - 0.411}{2}\left\{\cos \frac{\pi}{18}\left(15 - 5 - \frac{90}{11.3}\right)\right\} + \frac{1 + 0.411}{2}$$

$$= 0.982$$

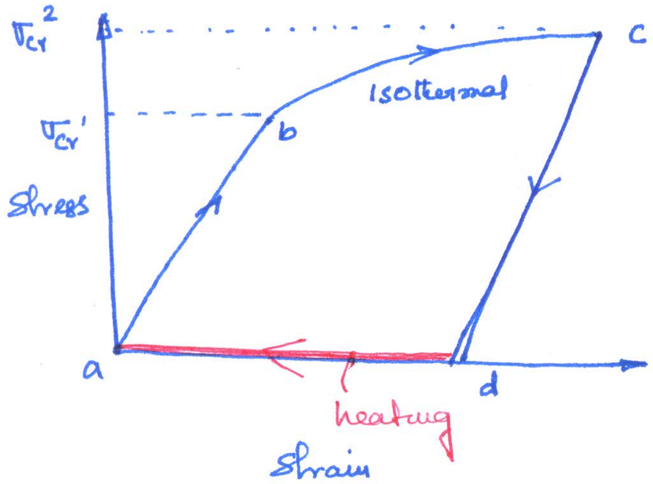
## 1-D constitutive model of SMA : Tanaka Model ~~Brinson model~~ (phenomenological model)

$$\sigma - \sigma_0 = Y(\epsilon - \epsilon_0) + \Omega(\xi - \xi_0)$$

Thermal expansion neglected

$\Omega \rightarrow$ transformation coefficient $= -Y\epsilon_L$

$\epsilon_L \rightarrow$ ~~recoverable~~ residual strain




### Assumptions

(i) zero initial stress and strain

(ii) Initially pure austenite $\xi_0 = 0$

(iii) Isothermal $M_s < T < A_s$

### a → b

$\sigma_0 = 0$

$\epsilon_0 = 0$

$\xi_0 = 0$

$$\boxed{\sigma = Y\epsilon}$$

The SMA wire is ~~loa~~ loaded isothermally till a critical stress ($\sigma_{cr}^1$) is reached which induces start of martensitic phase

$$\boxed{\begin{aligned} \sigma^b &= \sigma_{cr}^1 = C_M(T - M_s) \\ \epsilon^b &= \frac{C_M}{Y}(T - M_s) \end{aligned}}$$

$$\left(T = M_s^* = M_s + \frac{\sigma_{cr}^1}{C_M}\right)$$

### b → c

$\sigma_0 = \sigma^b$

$\epsilon_0 = \epsilon^b$

$\xi_0 = 0$

$$\sigma - \cancel{\sigma^b} = Y(\epsilon - \cancel{\epsilon^b}) - Y\epsilon_L\xi$$

$$\Rightarrow \boxed{\sigma = Y\epsilon - Y\epsilon_L\xi}$$

The SMA ~~wir~~ wire is loaded isothermally till the martensitic conversion is complete at $\sigma = \sigma_{cr}^2$

$$\boxed{\begin{aligned} \sigma^c &= \sigma_{cr}^2 = C_M(T - M_f) \\ \epsilon^c &= \frac{C_M}{Y}(T - M_f) + \epsilon_L \end{aligned}}$$

$$\left(T = M_f^* = M_f + \frac{\sigma_{cr}^2}{C_M}\right)$$

$\underline{\underline{c \to d}}$ The SMA wire is unloaded to zero stress isothermally. No phase transformation happens.

$\sigma_0 = \sigma^c$
$\epsilon_0 = \epsilon^c$
$\xi_0 = 1$

$$\sigma - \sigma^c = Y(\epsilon - \epsilon^c) - Y\epsilon_L(1-1)$$

$$\Rightarrow \sigma - Y\epsilon^c + Y\epsilon_L = Y(\epsilon - \epsilon^c)$$

$$\Rightarrow \boxed{\sigma = Y(\epsilon - \epsilon_L)}$$

$$\boxed{\begin{aligned} \sigma^d &= 0 \\ \epsilon^d &= \epsilon_L \end{aligned}}$$

$\underline{\underline{d \to a}}$ The SMA wire should be heated above $A_f$ for complete martensite to austenite conversion (no longer isothermal) and complete recovery of residual strain $\epsilon_L$