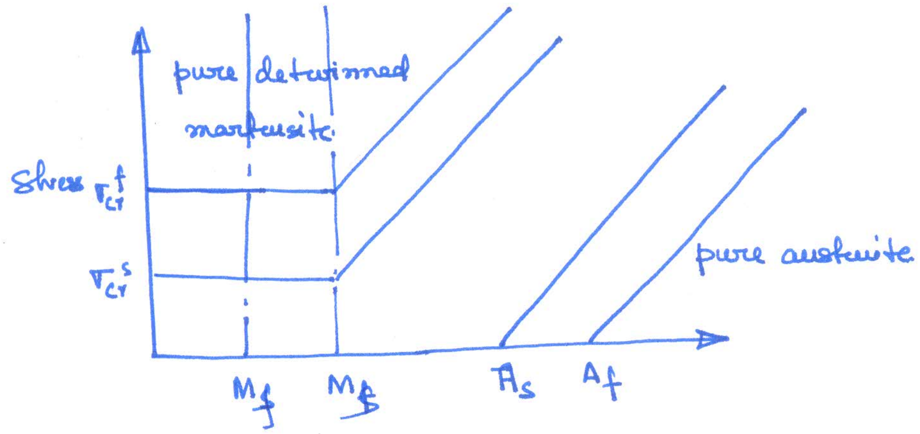
Brinson model



For A → M transformation

$T > M_s$ $\quad \sigma_{cr}^{s} + C_M(T - M_s) < \sigma < \sigma_{cr}^{f} + C_M(T - M_s)$

$$\xi_s = \frac{1-\xi_{s0}}{2} \cos\left[\frac{\pi}{\sigma_s^{cr} - \sigma_f^{cr}}\left\{\sigma - \sigma_{cr}^{f} - C_M(T - M_s)\right\}\right] + \frac{1+\xi_{s0}}{2}$$

$$\xi_T = \xi_{T0} - \frac{\xi_{T0}}{1-\xi_{s0}}(\xi_s - \xi_{s0})$$

For $T < M_s$ $\quad \sigma_{cr}^{s} < \sigma < \sigma_{cr}^{f}$

$$\xi_s = \frac{1-\xi_{s0}}{2} \cos\left[\frac{\pi}{\sigma_{cr}^{s} - \sigma_{cr}^{f}}(\sigma - \sigma_{cr}^{f})\right] + \frac{1+\xi_{0}}{2}$$

$$\xi_T = \xi_{T0} = \frac{\xi_{T0}}{1-\xi_{s0}}(\xi_s - \xi_{s0}) + \Delta_{T\xi}$$

$$\Delta_{T\xi} = \frac{1-\xi_{T0}}{2}\left\{\cos\left[\alpha_M (T - M_f)\right] + 1\right\} \quad M_f < T < M_s$$

~~otherw~~ otherwise $\Delta_{T\xi} = 0$

For M → A conversion

$T > A_s$ $\quad C_A(T - A_f) < \sigma < C_A(T - A_s)$

$$\xi = \frac{\xi_0}{2}\left[\cos \alpha_A\left(T - A_s - \frac{\sigma}{C_A}\right) + 1\right]$$

$$\xi_s = \xi_{s0} - \frac{\xi_{s0}}{\xi_0}(\xi_0 - \xi) \qquad \xi_T = \xi_{T0} - \frac{\xi_{T0}}{\xi_0}(\xi_0 - \xi)$$

Ex

$Y_A = 67$ GPa $M_f = 9°C$ $C_M = 8$ MPa/°C

$Y_M = 26$ GPa $M_s = 18°C$ $C_A = 14$ MPa/°C

$\epsilon_L = 7\%$ $A_s = 35°C$ $\sigma_{cr}^s = 100$ MPa

$A_f = 49°C$ $\sigma_{cr}^f = 170$ MPa.

An SMA wire with material properties listed above is initially at a temp of 5°C with $\xi_{T0} = 1$ and $\xi_{S0} = 0$
The material is assumed initially to be at a state of zero stress and strain.

(a) Compute the strain when the applied stress is 90 MPa

(b) Compute the stress induced and temp. induced martensitic fractions when the applied stress is 120 MPa

(c) Compute the strain when the applied stress is 120 MPa.

(a) $\sigma_{applied} = 90 \text{ MPa} < \sigma_{cr}^s \longrightarrow$ no conversion of temp. induced martensite to stress induced martensite.

Constitutive relation

$$\sigma = Y(\xi)\epsilon - Y(\xi)\epsilon_L \xi_S$$

$$\xi_S = 0 \Rightarrow \sigma = Y(\xi = 1)\epsilon$$

$$\Rightarrow \epsilon = \frac{\sigma}{Y_M} = \frac{90}{26 \times 10^3} = 0.38\%$$

(b) $T < M_f$

$$\sigma = Y(\xi)\epsilon - Y(\xi)\epsilon_L \xi_S$$

$$\xi_S = \frac{1 - \xi_{S0}}{2} \cos\left[\frac{\pi}{\sigma_{cr}^s - \sigma_{cr}^f}\left(\sigma - \sigma_{cr}^f\right)\right] + \frac{1 + \xi_{S0}}{2}$$

$$= \frac{1}{2}\cos\left[\frac{\pi}{-70}(-50)\right] + \frac{1}{2} = 0.188$$

$$\xi_T = 1 - 0.188 = 0.812$$

(c) $\sigma = Y_M\epsilon - Y_M\epsilon_L \xi_S^{\,0.188}$

$$\Rightarrow \epsilon = \sigma/Y_M + \epsilon_L \times 0.188 = 0.0038 + 0.07 \times 0.188 = 1.78\%$$

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परिशिष्ट/Supplement - 4

रोल नं./Roll No.
पाठ्यक्रम नाम/Course Name
शाखा/प्रभाग/Branch/Div.
शिक्षण बैच/Tutorial Batch
अनुभाग/Section
पाठ्यक्रम सं./Course No.
तिथि/Date

Electrical actuation of SMA

Joule heating is commonly used for raising the temp. of SMA for phase transformation.

A common model for heat transfer associated with electrical heating is

$$\underset{\text{mass density}}{\rho} \underset{\text{c/s area.}}{A} \overset{\text{specific heat}}{C_p} \frac{dT}{dt} = \underset{\text{current}}{i}^2 \overset{\text{resistance per unit length}}{R} - \underset{\text{heat transfer coeff}}{h_c} \underset{\text{circumferential area/length}}{A_c} (T(t) - \overset{\text{ambient temp.}}{T_\infty}) \quad \text{——— (1)}$$

$\Rightarrow$ Eqn. (1) can be rewritten as,

$$\frac{dT}{dt} + \frac{h_c A_c}{\rho A C_p} T(t) = \frac{i^2 R}{\rho A C_p} + \frac{h_c A_c}{\rho A C_p} T_\infty \qquad \frac{\rho A C_p}{h_c A_c} = t_h$$

$$\Rightarrow \frac{dT}{dt} + \frac{1}{t_h} T(t) = \frac{i^2 R}{\cancel{\rho A C_p}\; t_h h_c A_c} + \frac{T_\infty}{t_h} \quad \text{——— (2)}$$

Solving eqn. (2), we get,

$$T(t) - T_\infty = \frac{R}{h_c A_c}\left(1 - e^{-t/t_h}\right) i^2 + (T_0 - T_\infty)\, e^{-t/t_h}$$

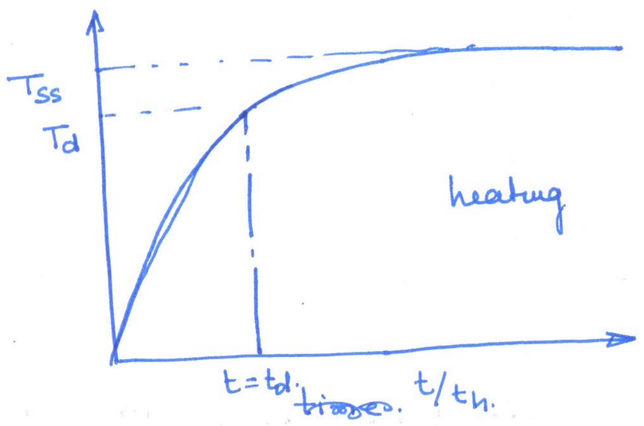
Usually $T_0 = T_\infty$

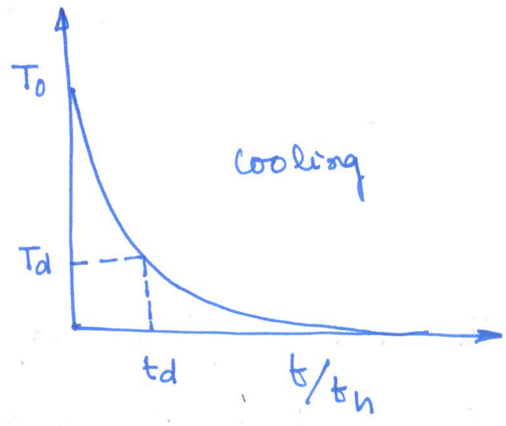
$$\therefore \quad T(t) - T_\infty = \frac{R}{h_c A_c}\left(1 - e^{-t/t_h}\right) i^2$$

The steady-state temp. is obtained by letting the exponential term going to zero

$$T_{ss} = T_\infty + \frac{i^2 R}{h_c A_c}$$

The temp will reach 95% of the steady-state at $t = 3t_h$.





$$t_d = -t_h \ln\left[1 - \frac{T_d - T_\infty}{i^2}\,\frac{h_c A_c}{R}\right]$$

$$\Rightarrow \quad t_d = -t_h \ln\left[\frac{T_{ss} - T_d}{T_{ss} - T_\infty}\right] \qquad \text{heating}$$

A model for the cooling of SMA wire can be derived setting $i = 0$

$$\Rightarrow \quad T(t) = T_\infty + (T_0 - T_\infty)\, e^{-t/t_h}$$

$$\Rightarrow \quad t_d = -t_h \ln \frac{T_d - T_\infty}{T_0 - T_\infty} \qquad \text{cooling}$$

The heating and cooling temp. desired will be related to the start and finish temps. of the material