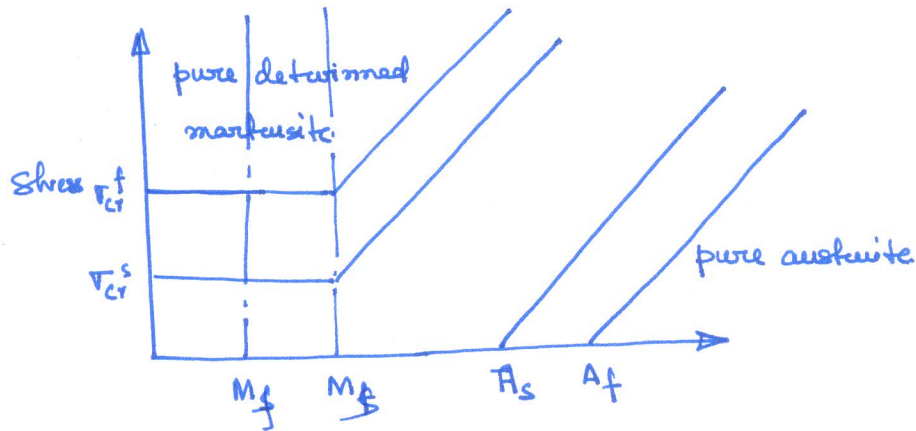


# Brinson model



For A  $\rightarrow$  M transformation

$$T > M_s \quad \sigma_{cr}^s + C_M(T - M_s) < \sigma < \sigma_{cr}^f + C_M(T - M_s)$$

$$\epsilon_s = \frac{1 - \epsilon_{s0}}{2} \ln \left[ \frac{\pi}{\sigma_{cr}^s - \sigma_{cr}^f} \left\{ \sigma - \sigma_{cr}^f - C_M(T - M_s) \right\} \right] + \frac{1 + \epsilon_{s0}}{2}$$

$$\epsilon_T = \epsilon_{s0} - \frac{\epsilon_{s0}}{1 - \epsilon_{s0}} (\epsilon_s - \epsilon_{s0})$$

For  $T < M_s$   $\sigma_{cr}^s < \sigma < \sigma_{cr}^f$

$$\epsilon_s = \frac{1 - \epsilon_{s0}}{2} \ln \left[ \frac{\pi}{\sigma_{cr}^s - \sigma_{cr}^f} (\sigma - \sigma_{cr}^f) \right] + \frac{1 + \epsilon_{s0}}{2}$$

$$\epsilon_T = \epsilon_{s0} - \frac{\epsilon_{s0}}{1 - \epsilon_{s0}} (\epsilon_s - \epsilon_{s0}) + \Delta T \epsilon_T$$

$$\Delta T \epsilon_T = \frac{1 - \epsilon_{s0}}{2} \left\{ \ln [\alpha_M (T - M_f)] + 1 \right\} \quad M_f < T < M_s$$

otherwise  $\Delta T \epsilon_T = 0$

For M  $\rightarrow$  A conversion

$$T > A_s \quad C_A(T - A_f) < \sigma < C_A(T - A_s)$$

$$\epsilon_f = \frac{\epsilon_{s0}}{2} \left[ \ln \left[ \frac{C_A(T - A_s - \frac{\sigma}{C_A})}{C_A(T - A_f)} \right] + 1 \right]$$

$$\epsilon_s = \epsilon_{s0} - \frac{\epsilon_{s0}}{\epsilon_{s0}} (\epsilon_{s0} - \epsilon_f) \quad \epsilon_T = \epsilon_{s0} - \frac{\epsilon_{s0}}{\epsilon_{s0}} (\epsilon_{s0} - \epsilon_f)$$

Ex

$$Y_A = 67 \text{ GPa}$$

$$Y_M = 26 \text{ GPa}$$

$$\epsilon_L = 7\%$$

$$M_f = 9^\circ\text{C}$$

$$M_s = 18^\circ\text{C}$$

$$A_s = 35^\circ\text{C}$$

$$A_f = 49^\circ\text{C}$$

$$C_M = 8 \text{ MPa}/^\circ\text{C}$$

$$C_A = 14 \text{ MPa}/^\circ\text{C}$$

$$\sigma_{cr}^S = 100 \text{ MPa}$$

$$\sigma_{cr}^T = 170 \text{ MPa}$$

An SMA wire with material properties listed above is initially at a temp of  $5^\circ\text{C}$  with  $\epsilon_{ST} = 1$  and  $\epsilon_{SS} = 0$

The material is assumed initially to be at a state of zero stress and strain.

- Compute the strain when the applied stress is 90 MPa
- Compute the stress induced and temp. induced martensitic fractions when the applied stress is 120 MPa
- Compute the strain when the applied stress is 120 MPa.

- (a)  $\sigma_{\text{applied}} = 90 \text{ MPa} < \sigma_{cr}^S \rightarrow$  no conversion of temp. induced martensite to stress induced martensite.

Constitutive relation

$$\sigma = Y(\epsilon_T) \epsilon - Y(\epsilon_T) \epsilon_L \epsilon_{SS}$$

$$\epsilon_{SS} = 0 \Rightarrow \sigma = Y(\epsilon_T) \epsilon$$

$$\Rightarrow \epsilon = \frac{\sigma}{Y_M} = \frac{90}{26 \times 10^3} = 0.38\%$$

(b)  $T < M_f$   $\sigma = Y(\epsilon_T) \epsilon - Y(\epsilon_T) \epsilon_L \epsilon_{SS}$

$$\epsilon_{SS} = \frac{1 - \epsilon_{SS0}}{2} \cos \left[ \frac{\pi}{\sigma_{cr}^S - \sigma_{cr}^T} (\sigma - \sigma_{cr}^T) \right] + \frac{1 + \epsilon_{SS0}}{2}$$

$$= \frac{1}{2} \cos \left[ \frac{\pi}{-70} (-50) \right] + \frac{1}{2} = 0.188$$

$$\epsilon_{ST} = 1 - 0.188 = 0.812$$

(c)  $\sigma = Y_M \epsilon - Y_M \epsilon_L \epsilon_{SS}''^{0.188}$

$$\Rightarrow \epsilon = \frac{\sigma}{Y_M} + \epsilon_L \times 0.188 = 0.0038 + 0.07 \times 0.188 = 1.78\%$$

