Example

A 10 cm long SMA with circular c/s is preloaded with a weight to induce austenite to martensite phase transformation. The wire is initially at complete austenite phase. The wire is at an ambient temp. of 25°c. The diameter of the wire is 0.5 mm

$Y_A = 67$ GPa	$M_f = 9°c$	$C_M = 8$ MPa/°c	Resistivity 70 μΩ.cm
$Y_M = 26$ GPa	$M_s = 18°c$	$C_A = 14$ MPa/°c	$h_c = 150\ J/m^2.°c.s$
$\epsilon_L = 7\%$	$A_s = 35°c$	$\sigma_{cr}^s = 100$ MPa	$\rho = 6450\ kg/m^3$
	$A_f = 49°c$	$\sigma_{cr}^f = 170$ MPa	$C_p = 0.2\ kCal/kg.°c$

(a) Determine the stress required to induce full austenite to martensite phase transformation

$$T_\infty > 25°c \Rightarrow \sigma_{applied} = \sigma_{cr}^f + C_M(T - M_s)$$
$$= 170 + 8(25-18)$$
$$= 226 \text{ MPa}.$$

(b) Compute the current reqd. to achieve steady-state temp. of 75°c

$$T_{ss} = T_\infty + \frac{i^2 R}{h_c A_c}$$

R = resistance/length

$$= \frac{76 \times 10^{-6} \times 10^{-2}}{\frac{\pi}{4}(0.5)^2 \times 10^{-6}} = 3.87\ \Omega/m$$

$$i^2 = \frac{(T_{ss} - T_\infty) h_c A_c}{R}$$

$$= \frac{(75-25) \times 150 \times \pi \times 0.5 \times 10^{-3}}{3.87}$$

$\Rightarrow i = 1.74$ A

(c) Find the time taken to start and complete martensite to austenite transformation

$$A_s^* = A_s + \frac{\sigma}{C_A} = 35 + \frac{226}{14} = 51.1°c$$

$$A_f^* = A_f + \frac{\sigma}{C_A} = 49 + \frac{226}{14} = 65.1°c.$$

$$t_n = \frac{\rho A C_p}{h_c A_c}$$
$$= \frac{6450 \times \frac{\pi}{4} \times (0.5)^2 \times 10^{-6} \times 0.2 \times 4185}{150 \times \pi \times 0.5 \times 10^{-3}} = 4.61\text{ s}$$

$$t_1 = t_{M\to A}^s = -t_n \ln\left[\frac{T_{ss} - T_d}{T_{ss} - T_\infty}\right] = 3.40\text{ s}$$

$$t_2 = t_{M\to A}^f = -t_n \ln\left[\frac{75 - 65.1}{75 - 25}\right] = 7.47\text{ s}$$

(d) Assuming that the current is set to zero when the material completes $M \rightarrow A$ phase transformation, plot

(i) Temp vs. time (ii) ξ_s vs. time & (iii) strain vs. time for t 0 to 30s

(i) Temp vs. time

heating

$$T(t) = \frac{R}{h_c A_c} i^2 \left(1 - e^{-t/t_H}\right) + T_\infty$$

$$= \frac{3.87 \times (1.74)^2}{150 \times \pi \times 0.5 \times 10^{-3}} \left(1 - e^{-t/4.61}\right) + 25$$

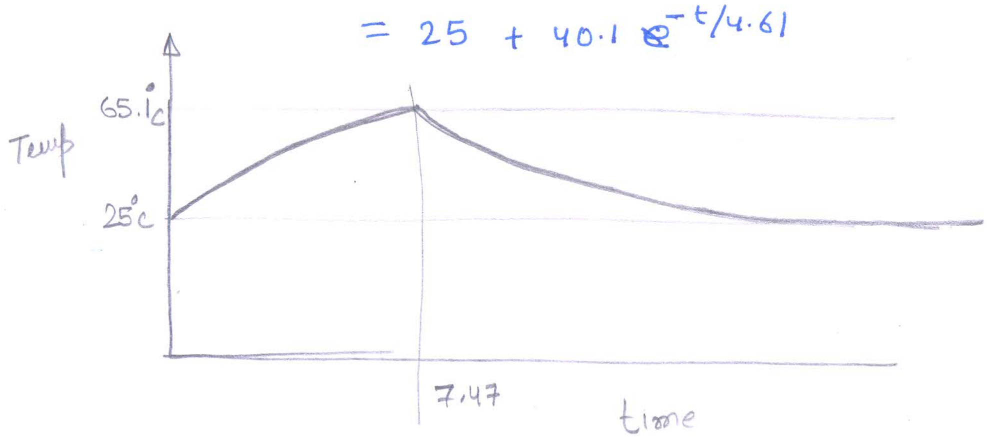
$$= 25 + 50\left(1 - e^{-t/4.61}\right) \quad 0 \le t \le 7.47$$

cooling

$$T(t) = T_\infty + (T_0 - T_\infty)\, e^{-t/t_H}$$

$$= 25 + (65.1 - 25)\, e^{-t/4.61}$$

$$= 25 + 40.1\, e^{-t/4.61}$$



(ii) ξ_s vs. time

$$\xi_s = 1 \qquad 0 \le t \le 3.40\,s$$

$$\xi_{s\,M\to A} = \frac{1}{2}\left[\cos\left\{\frac{\pi}{A_f - A_s}\left(T - A_s^*\right)\right\} + 1\right] \qquad 3.40 \le t \le 7.47$$

$$\xi_s = 0 \qquad 7.47 \le t \le t_3$$

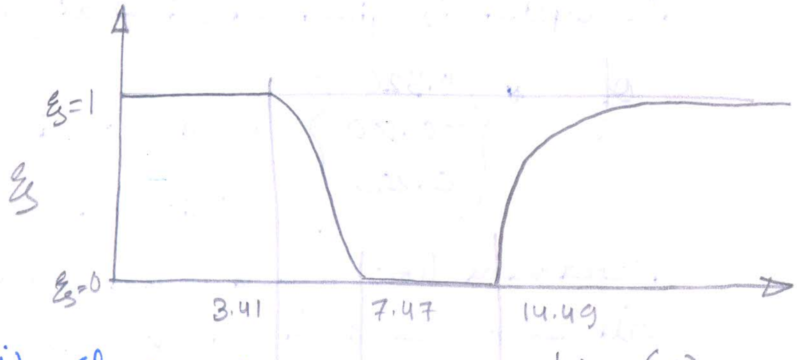
$$\xi_{s\,A\to M} = \frac{1}{2}\left\{\cos\frac{\pi}{\sigma_{cr}^s - \sigma_{cr}^f}\left[\sigma - \sigma_{cr}^f - C_M(T - M_s)\right]\right\} + \frac{1}{2} \qquad t \ge t_3$$

$$M_s^* = \frac{\sigma - \sigma_{cr}^s}{C_M} + M_s = \frac{226 - 100}{8} + 18 = 33.75°C$$

$$M_f^* = \frac{\sigma - \sigma_{cr}^f}{C_M} + M_f = 25°C \ (T_\infty)$$

$$t_3 = -t_n \ln\left[\frac{M_s^* - T_\infty}{T_0 - T_\infty}\right] = -4.61 \ln\left[\frac{33.75 - 25}{65.1 - 25}\right] = 7.02 \ (+7.47)$$

$$= 14.49\,s.$$



(iii) Strain vs. time