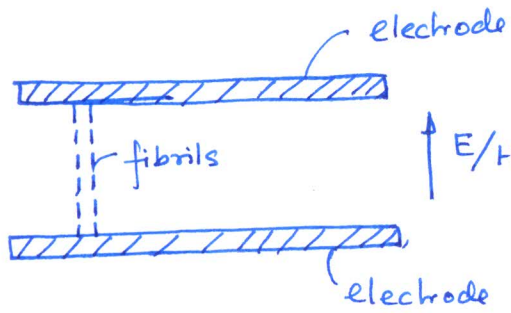


Modeling of ER/MR fluids

Bingham Plastic Model: Phenomenological model



Suspended particles align and form fibril/chain like structure which prevents fluid flow

Effect of fibrils on the viscosity of the fluid is negligible \rightarrow

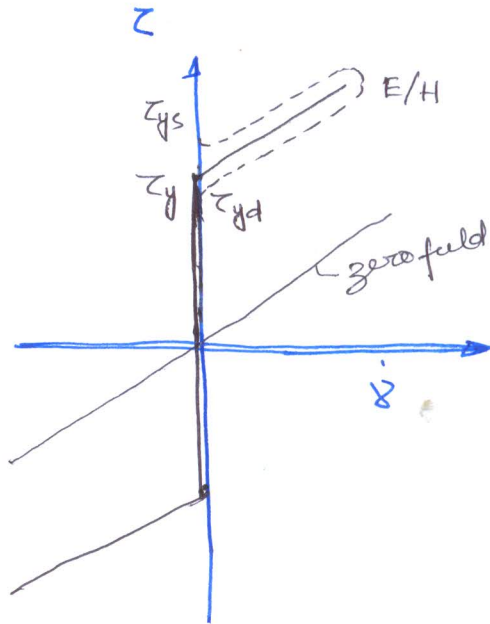
produces a shear stress largely independent on the strain rate referred to yield stress and denoted by τ_y .

Adding this yield stress to Newtonian mode results in Bingham plastic model

most common model or slight extension of the model

$$\tau = \tau_y (E/H) + \mu \dot{\gamma}$$

τ_y \rightarrow strength of electric field / magnetic field
 μ \rightarrow zero field viscosity



$\tau_{ys} > \tau_{y,d} \rightarrow$

Reasons for this behavior may include reattachment to the walls of the field induced fibrils are broken near their ends by the bulk shear of the fluid accompanying flow

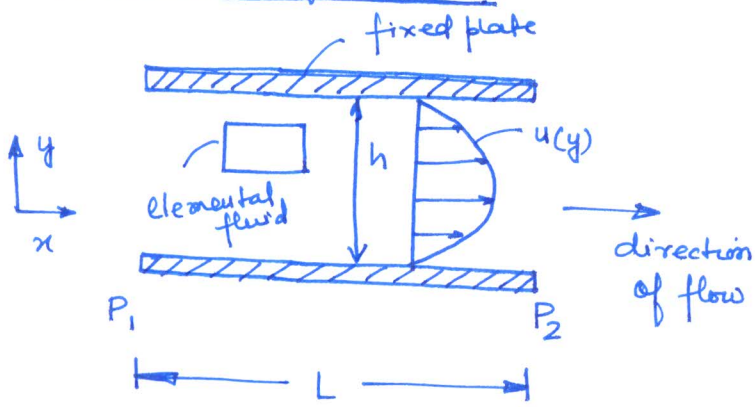
slight variation from ideal behavior can be neglected as the transient response is unimportant

Pre-yield response

$\tau < \tau_y$ no fluid flow occurs (however in reality there is a fluid flow) small.

Modelling of ER/MR fluid in flow mode / valve mode

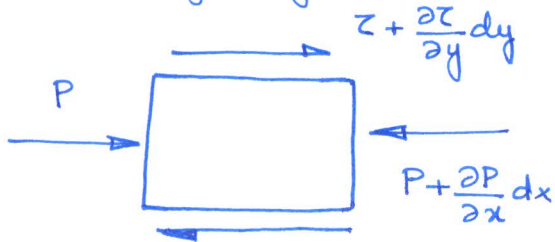
In absence of fluid flow



$$\Delta P = P_1 - P_2$$

$$\frac{\partial P}{\partial x} = -\frac{\Delta P}{L} = \text{constant}$$

Free body diagram



From eqn. (3), τ can be related to $\frac{\Delta P}{L}$ as,

$$\begin{aligned} \tau &= \mu \frac{\partial u}{\partial y} = \frac{\Delta P}{2L} (h - 2y) \\ &= \frac{\Delta P}{L} \left(\frac{h}{2} - y \right) \end{aligned}$$

rate of flow / m

$$\begin{aligned} \frac{Q}{b} &= 2 \int_0^{h/2} u(y) dy \\ &= \frac{\Delta P}{\mu L} \int_0^{h/2} (hy - y^2) dy \\ &= \frac{\Delta P h^3}{12 \mu b} \end{aligned}$$

Balancing force in x-dir, we get,

$$\frac{\partial \tau}{\partial y} dy dx b = \frac{\partial P}{\partial x} dx dy b$$

$$\Rightarrow \frac{\partial \tau}{\partial y} = \frac{\partial P}{\partial x} = -\frac{\Delta P}{L} \quad (1)$$

For Newtonian fluid,

$$\tau = \mu \frac{\partial u}{\partial y} \quad (2)$$

Substituting (2) in (1), we get,

$$\mu \frac{\partial^2 u}{\partial y^2} = -\frac{\Delta P}{L}$$

$$\Rightarrow u(y) = -\frac{\Delta P}{2\mu L} y^2 + Cy + D$$

Boundary condns on $u(y)$

$$u(y=0) = 0 \Rightarrow D = 0$$

$$u(y=h) = 0 \Rightarrow -\frac{\Delta P h^2}{2\mu L} + Ch = 0$$

$$\Rightarrow C = \frac{\Delta P h}{2\mu L}$$

$$\Rightarrow \boxed{u(y) = \frac{\Delta P}{2\mu L} (h-y)y} \quad (3)$$

