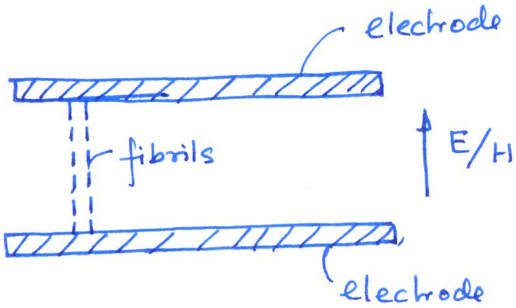
Modeling of ER/MR fluids

Bingham Plastic Model : Phenomenological model



Suspended particles align and form fibril/chain like structure which prevents fluid flow

Effect of fibrils on the viscosity of the fluid is negligible → produces a shear stress largely independent on the strain rate referred to yield stress and denoted by τ_y.

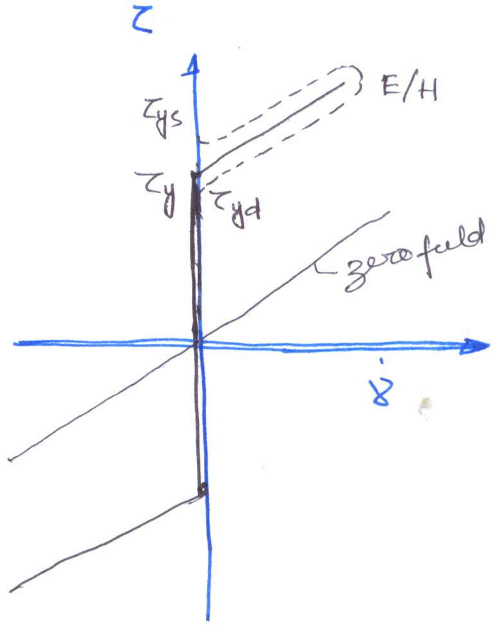
Adding this yield stress to Newtonian mode results in Bingham plastic model

most common model or slight extension of the model

$$\tau = \tau_y(E/H) + \mu\dot{\gamma}$$

$\tau_y(E/H)$ → strength of electric field/magnetic field

μ → zero field viscosity



$\tau_{y,s} > \tau_{y,d}$ → Reasons for this behavior may include reattachment to the walls of the field induced fibrils are broken near their ends by the bulk shear of the fluid accompanying flow

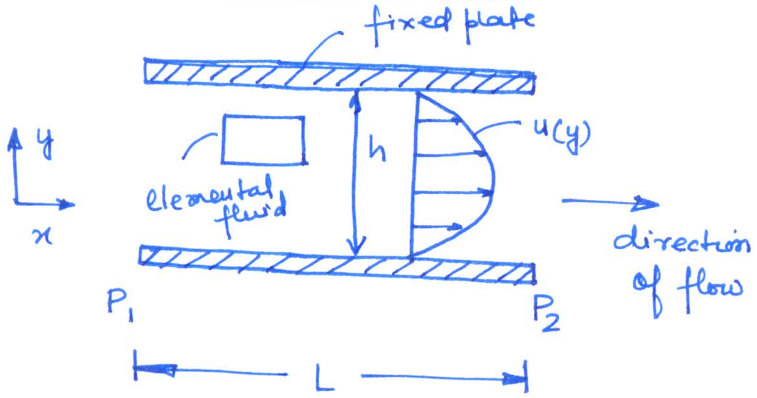
Slight variation from ideal behavior d can be neglected hen the transient response is unimportant

Pre-yield response

$\tau < \tau_y$ no fluid flow occurs (however in reality there is a small fluid flow)

Modelling of ER/MR fluid in flow mode/valve mode

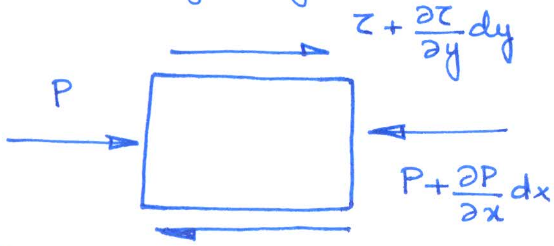
In absence of fluid flow



$$\Delta P = P_1 - P_2$$

$$\frac{\partial P}{\partial x} = -\frac{\Delta P}{L} = \text{constant}$$

Free body diagram



Balancing force in x-dirc, we get,

$$\frac{\partial \tau}{\partial y} dy\, dx \times b = \frac{\partial P}{\partial x} dx\, dy\, b$$

$$\Rightarrow \frac{\partial \tau}{\partial y} = \frac{\partial P}{\partial x} = -\frac{\Delta P}{L} \quad \text{—— (1)}$$

For Newtonian fluid,

$$\tau = \mu \frac{\partial u}{\partial y} \quad \text{—— (2)}$$

Substituting (2) in (1), we get,

$$\mu \frac{\partial^2 u}{\partial y^2} = -\frac{\Delta P}{L}$$

$$\Rightarrow u(y) = -\frac{\Delta P}{2\mu L} y^2 + Cy + D$$

Boundary condns on u(y)

$$u(y=0) = 0 \Rightarrow D = 0$$

$$u(y=h) = 0 \Rightarrow -\frac{\Delta P h^2}{2\mu L} + Ch = 0$$

$$\Rightarrow C = \frac{\Delta P h}{2\mu L}$$

$$\Rightarrow \boxed{u(y) = \frac{\Delta P}{2\mu L}(h-y)y} \quad \text{—— (3)}$$

From eqn. (3), τ can be related to $\frac{\Delta P}{L}$ as,

$$\tau = \mu \frac{\partial u}{\partial y} = \frac{\Delta P}{2L}(h - 2y)$$

$$= \frac{\Delta P}{L}\left(\frac{h}{2} - y\right)$$

ate of flow/m

$$\frac{Q}{b} = 2\int_0^{h/2} u(y)\, dy$$

$$= \frac{\Delta P}{\mu L}\int_0^{h/2} (hy - y^2)\, dy$$

$$= \frac{\Delta P h^3}{12 \mu L}$$

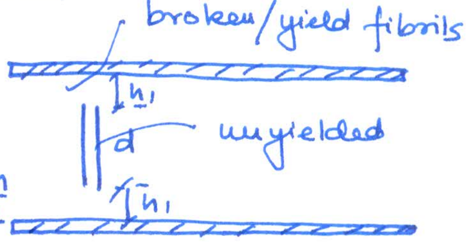
For an applied field $E/H \Rightarrow \tau_y \neq 0$

Critical value of pressure difference to start flow $= \frac{\Delta P_{cr}}{L}$

$$\frac{\Delta P_{cr}}{L}\left(\frac{h}{2}\right) = \tau_y \Rightarrow \boxed{\frac{\Delta P_{cr}}{L} = \frac{2\tau_y}{h}}$$

For $\frac{\Delta P}{L} > \frac{\Delta P_{cr}}{L}$ and $E/H \neq 0$ (τ_y)

$$\tau_y = \frac{\Delta P}{L}\left(\frac{h}{2} - h_1\right) \Rightarrow h_1 = -\frac{\tau_y L}{\Delta P} + \frac{h}{2}$$



Using Bingham Plastic model

$$\tau(y) = \tau_y + \mu \frac{\partial u}{\partial y}$$

$$\Rightarrow \mu \frac{\partial u}{\partial y} = \tau(y) - \tau_y$$

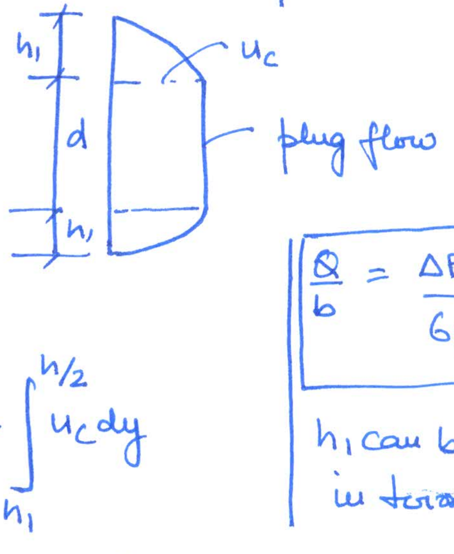
$$\frac{\partial u}{\partial y} = \frac{1}{\mu}\left[\frac{\Delta P}{L}\left(\frac{h}{2} - y\right) - \frac{\Delta P}{L}\left(\frac{h}{2} - h_1\right)\right]$$

$$\frac{\partial u}{\partial y} = \frac{\Delta P}{\mu L}(h_1 - y)$$

$$\Rightarrow u(y) = \frac{\Delta P}{\mu L}\left(h_1 y - \frac{y^2}{2}\right) + C = \frac{\Delta P}{2\mu L}(2h_1 - y)y + C$$

$u(y=0) = 0$
$\Rightarrow C = 0$

$$\Rightarrow \boxed{u(y) = \frac{\Delta P}{2\mu L}(2h_1 - y)y}$$



Constant velocity of the plug flow

$$u_c = u(y = h_1) = \frac{\Delta P}{2\mu L} h_1^2$$

Flow rate

$$\frac{Q}{b} = 2\int_0^{h_1} u(y)\,dy + 2\int_{h_1}^{h/2} u_c\,dy$$

$$= \frac{\Delta P}{\mu L}\left(h_1 y^2 - \frac{y^3}{3}\right)\Big|_0^{h_1} + \frac{\Delta P}{\mu L} h_1^2\left(\frac{h}{2} - h_1\right)$$

$$= \frac{\Delta P}{\mu L}\left(\frac{2h_1}{3}\right)h_1^2 + \frac{\Delta P}{2\mu L} h_1^2 h - \frac{\Delta P h_1^3}{\mu L}$$

$$= \frac{\Delta P}{2\mu L} h_1^2 h - \frac{\Delta P h_1^3}{3\mu L} = \frac{\Delta P h_1^2}{6\mu L}(3h - 2h_1)$$

$$\boxed{\frac{Q}{b} = \frac{\Delta P h_1^2}{6\mu L}(3h - 2h_1)}$$

h_1 can be expressed in terms of τ_y.