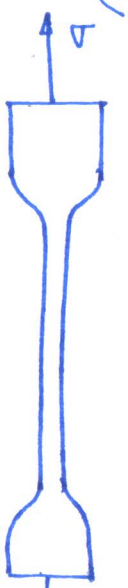
Constitutive Relation

Piezoelectric Material.

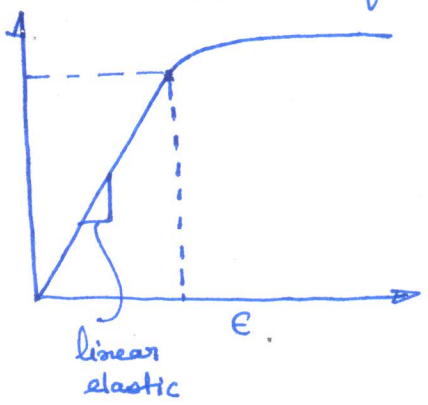
Direct piezoelectric effect → piezoelectric material produces an electrical displacement on application of mechanical strain

(sensing)

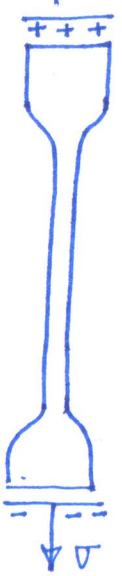


$\sigma = S\epsilon$

↙ mechanical compliance (m^2/N).



(non-piezoelectric material)



poling direc

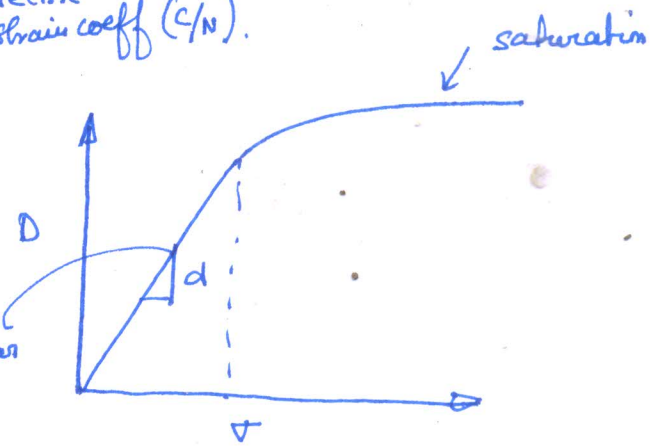
$\epsilon = S_{PZT}\,\sigma$

$D = d\,\sigma$ → piezoelectric strain coeff (C/N).

↙ electrical displacement (charge / area of electrode)

↘ N/m^2

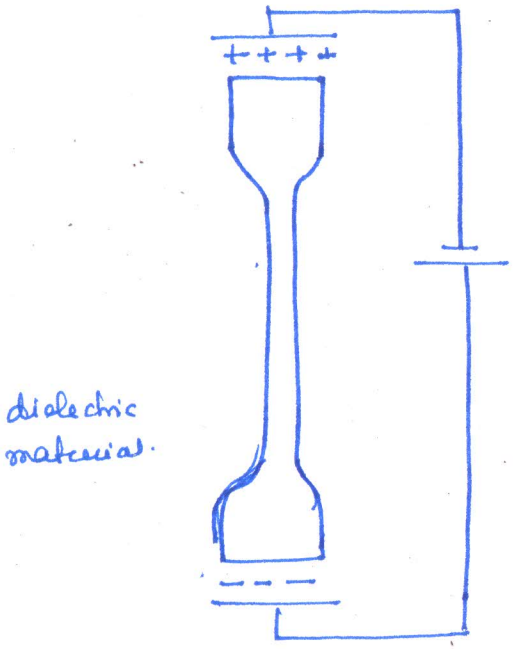
(piezoelectric material)



Tensile stress ~~along the polarization direc~~. generates a voltage of polarity opposing the poling direc.

Converse piezoelectric effect (actuation)

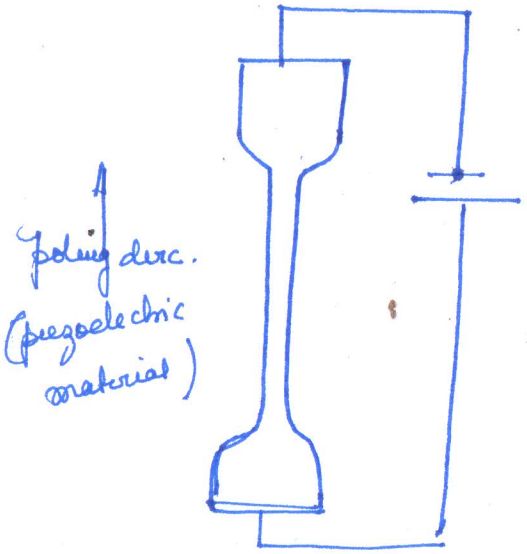
a mechanical strain is produced when an electric field is applied.



$D = \varepsilon E$ ← electric field (V/m, N/C)

D: electrical displacement

ε: permittivity (F/m)



$D = \varepsilon_{P2T} E$

$\epsilon = dE$ ← V/m

ϵ: mechanical strain

d: strain coeff. (m/V, C/N)

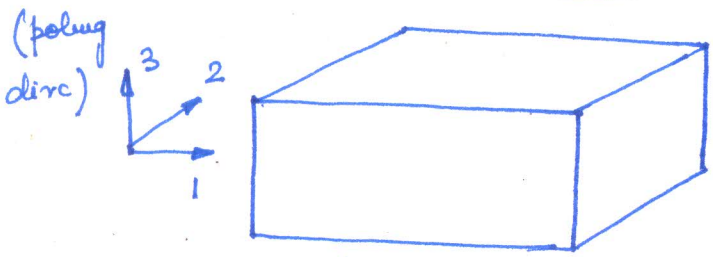
When voltage is applied in the poling direction, elongation occurs in poling direc.

One-dimensional constitutive relation

$$\begin{Bmatrix} \epsilon \\ D \end{Bmatrix} = \begin{bmatrix} S & d \\ d & \varepsilon \end{bmatrix} \begin{Bmatrix} \sigma \\ E \end{Bmatrix}$$ piezoelectric material

$$\begin{Bmatrix} \epsilon \\ D \end{Bmatrix} = \begin{bmatrix} S & 0 \\ 0 & \varepsilon \end{bmatrix} \begin{Bmatrix} \sigma \\ E \end{Bmatrix}$$ dielectric material.

Three-dimensional constitutive relation



Converse piezoelectric effect (actuation)

$$\underset{6\times1}{\{\epsilon\}} = \begin{Bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} \frac{1}{Y_p} & -\frac{\nu}{Y_p} & -\frac{\nu}{Y_p} & 0 & 0 & 0 \\ -\frac{\nu}{Y_p} & \frac{1}{Y_p} & -\frac{\nu}{Y_p} & 0 & 0 & 0 \\ -\nu/Y_p & -\nu/Y_p & \frac{1}{Y_p} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G_p & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_p & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_p \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{Bmatrix} + \begin{bmatrix} 0 & 0 & d_{13} \\ 0 & 0 & d_{23} \\ 0 & 0 & d_{33} \\ 0 & d_{24} & 0 \\ d_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix}$$

Direct piezoelectric effect (sensing)

$$\underset{3\times1}{\{D\}} = \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & d_{15} & 0 \\ 0 & 0 & 0 & d_{24} & 0 & 0 \\ d_{13} & d_{23} & d_{33} & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \sigma_{33} \\ \tau_{23} \\ \tau_{12} \\ \tau_{13} \end{Bmatrix} + \begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{22} & 0 \\ 0 & 0 & \varepsilon_{33} \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix}$$