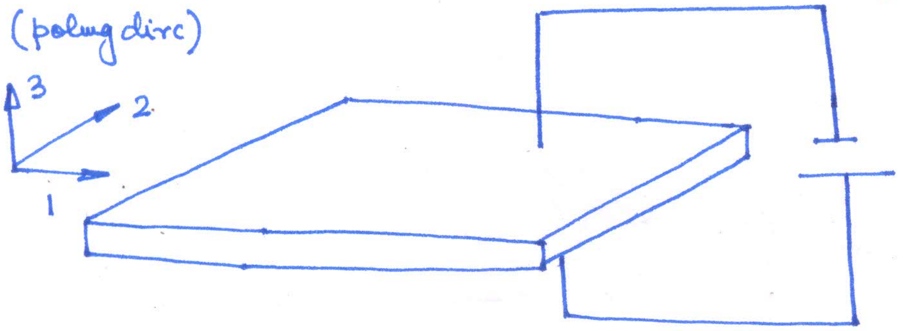
31 operational mode of PZT : actuator eqn



$E_1, E_2 = 0$

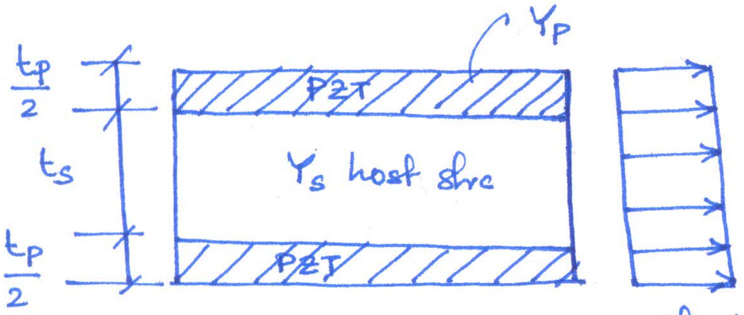
$\sigma_{22}, \sigma_{33} = 0$

$\tau_{ij} = 0$

$$\boxed{\epsilon_{11} = \frac{\sigma_{11}}{Y_p} + d_{13} E_3}$$

actuator eqn in 31 operational mode.

Extensional actuator



Strain distribution

Find the electric field to be applied to achieve a strain ϵ_s

Constitutive relation for the three layers

$$\epsilon_{11} = \begin{cases} \frac{1}{Y_p}\sigma_{11} + d_{13}E & \frac{t_s}{2} \le z \le \frac{1}{2}(t_s + t_p) \\ \frac{1}{Y_s}\sigma_{11} & -\frac{t_s}{2} \le z \le \frac{t_s}{2} \\ \frac{1}{Y_p}\sigma_{11} + d_{13}E & -\frac{1}{2}(t_s+t_p) \le z \le -\frac{t_s}{2} \end{cases}$$

Integrating both sides of the above eqn. w.r.t area, we get

$$\frac{bt_p}{2} Y_p \epsilon_{11} = \int_{\frac{t_s}{2}}^{\frac{t_s+t_p}{2}} \sigma_{11}\, dz\, dy + d_{13} Y_p \frac{t_p}{2} b E$$

$$b t_s Y_s \epsilon_{11} = \int_{-t_s/2}^{t_s/2} \sigma_{11}\, dz\, dy$$

$$\frac{bt_p}{2} Y_p \epsilon_{11} = \int_{-\left(\frac{t_s}{2}+\frac{t_p}{2}\right)}^{-t_s/2} \sigma_{11}\, dz\, dy + d_{13} Y_p \frac{t_p}{2} bE$$

Adding the three above equations, we get

$$(bt_pY_p + bt_sY_s)\epsilon_{11} = \underbrace{\int_{-\left(\frac{t_s+t_p}{2}\right)}^{\left(\frac{t_s+t_p}{2}\right)} \sigma_{11}\, dz\, dy}_{} + d_{13} Y_p t_p b E$$

$\Rightarrow$ $\parallel$ 0 (when no external force is applied)

$= F$ (applied force)

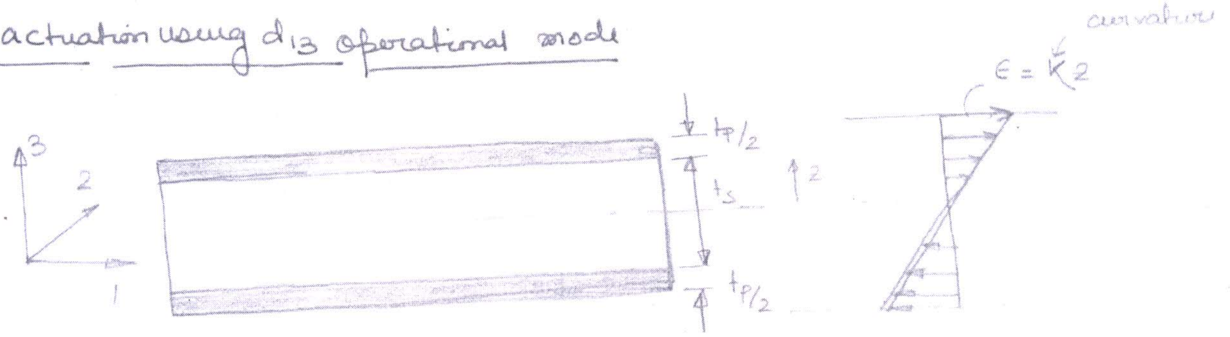
$$\Rightarrow \quad (t_pY_p + t_sY_s)\epsilon_{11} = d_{13} Y_p t_p E$$

$$\Rightarrow \quad \epsilon_{11} = \frac{d_{13} Y_p t_p E}{(t_pY_p + t_sY_s)} = \frac{d_{13}E}{\left(1 + \frac{Y_s t_s}{Y_p t_p}\right)}$$

free strain

$$\Rightarrow \quad \underset{\text{(electric field reqd)}}{E} = \left(1 + \frac{Y_s t_s}{Y_p t_p}\right)\frac{\epsilon_s}{d_{13}}$$

Bending actuation using d_{13} operational mode



Constitutive Relation

$$\epsilon(z) = \begin{cases} \frac{1}{Y_p}\sigma(z) - d_{13}E & \frac{t_s}{2} \le z \le \frac{t_s+t_p}{2} \\ \frac{1}{Y_s}\sigma(z) & -\frac{t_s}{2} \le z \le \frac{t_s}{2} \\ \frac{1}{Y_p}\sigma(z) + d_{13}E & -\frac{(t_s+t_p)}{2} \le z \le -\frac{t_s}{2} \end{cases} \quad \text{—— (i)}$$

For an Euler-Bernoulli beam,

$$\epsilon(z) = zK \leftarrow \text{curvature.}$$

$$= z \cdot \frac{1}{R} = z\frac{d\theta}{dx} = z\frac{\partial^2 w}{\partial x^2}.$$

Multiplying of both sides of Eqn. (i) with z and integrating w.r.t area, we get,

$$Y_p \int_{\frac{t_s}{2}}^{(t_s+t_p)/2} z\epsilon(z)\,dz\,dy = \int_{A_1} z\sigma(z)\,dz\,dy - \frac{Y_p d_{13}E}{8}\int_{\frac{t_s}{2}}^{\frac{(t_s+t_p)}{2}} z\,dz\,dy.$$

$$\Rightarrow \frac{Y_p K b}{3\times 8}\left[(t_s+t_p)^3 - t_s^3\right] = \int_{A_1} z\sigma(z)\,dz\,dy - \frac{Y_p d_{13}E}{8}\left[(t_s+t_p)^2 - t_s^2\right]$$

$$\Rightarrow \frac{Y_p K b}{24}\left[t_p^3 + 3t_p^2 t_s + 3t_p t_s^2\right] = \int_{A_1} z\sigma(z)\,dz\,dy - \frac{Y_p d_{13}E}{8}\left[t_p^2 + 2t_p t_s\right] \quad \text{—— (i).}$$

Similarly.

$$\frac{Y_s K b}{24}\left[t_s^3 + t_s^3\right] = \int_{A_2} z\sigma(z)\,dz\,dy \quad \text{—— (ii)}$$

and,

$$\frac{Y_p K b}{2 \cdot 3}\left[-\frac{t_s^3}{8}+\frac{(t_s+t_p)^3}{8}\right] = \int_{A_3} z\,\sigma(z)\,dz\,dy + \frac{Y_p d_{13} E}{8}\left[t_s^2-(t_s+t_p)^2\right] \qquad \text{——— (iii)}$$

Adding Eqns. (i), (ii) and (iii), we get,

$$\frac{Y_p K b}{12}\left[t_p^3+3t_p^2 t_s+3t_p t_s^2\right] = \underbrace{\int_A z\,\sigma(z)\,dz\,dy}_{\substack{\text{moment resultant}\\ (=0)\text{ if no external}\\ \text{moment is applied}}} - \frac{Y_p d_{13} E b}{4}\left[t_p^2+2t_p t_s\right]$$

$$\Rightarrow \frac{Kb}{12}\left[Y_p\left(t_p^3+3t_p^2 t_s+3t_p t_s^2\right)+Y_s t_s^3\right] = -\frac{Y_p d_{13} E b}{4}\left[t_p^2+2t_p t_s\right].$$

$$K = \frac{-3Y_p d_{13} E\left[t_p^2+2t_p t_s\right]}{\left[Y_p\left(t_p^3+3t_p^2 t_s+3t_p t_s^2\right)+Y_s t_s^3\right]}$$

$$= \frac{-3Y_p d_{13} E\left[1+\frac{2t_s}{t_p}\right]}{t_p\left[Y_p\left(1+3\frac{t_s}{t_p}+3\frac{t_s^2}{t_p^2}\right)+Y_s\frac{t_s^3}{t_p^3}\right]}$$