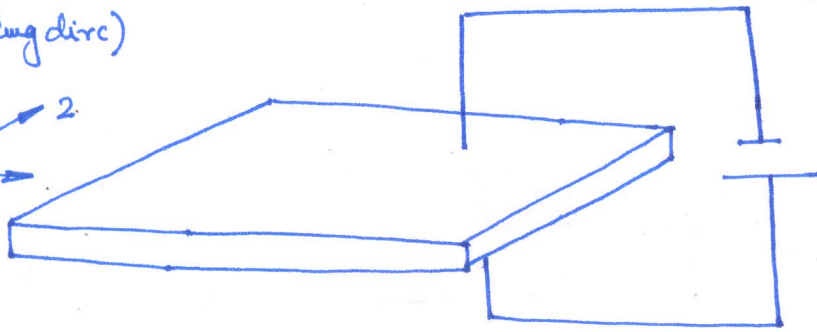
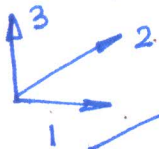


31 operational mode of PZT: actuator eqn

(poling dir)



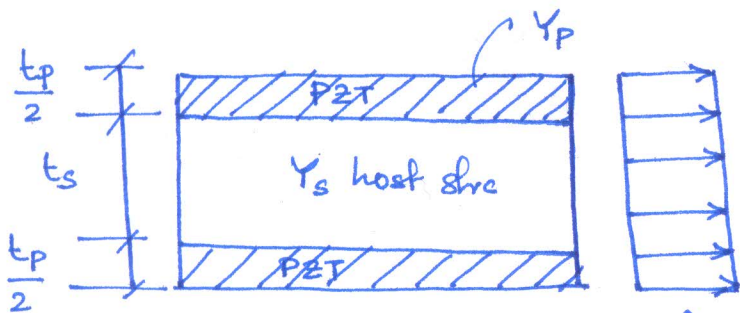
$$\begin{aligned} E_1, E_2 &= 0 \\ \sigma_{22}, \sigma_{33} &= 0 \\ \tau_{ij} &= 0 \end{aligned}$$

$$\epsilon_{11} = \frac{\sigma_{11}}{Y_P} + d_{13} E_3$$

Extensional actuator

actuator eqn in 31 operational mode.

Find the electric field to be applied to achieve a strain ϵ_s



Strain distribution

Constitutive relation for the three layers

$$\epsilon_{11} = \begin{cases} \frac{1}{Y_P} \sigma_{11} + d_{13} E & \frac{t_s}{2} \leq z \leq \frac{1}{2}(t_s + t_p) \\ \frac{1}{Y_S} \sigma_{11} & -\frac{t_s}{2} \leq z \leq \frac{t_s}{2} \\ \frac{1}{Y_P} \sigma_{11} + d_{13} E & -\frac{1}{2}(t_s + t_p) \leq z \leq -\frac{t_s}{2} \end{cases}$$

Integrating both sides of the above eqn. w.r.t area, we get

$$\frac{b t_p}{2} Y_P \epsilon_{11} = \int_{\frac{t_s}{2}}^{\frac{t_s+t_p}{2}} \sigma_{11} dz dy + d_{13} Y_P \frac{t_p b}{2} E$$

$$b t_s Y_S \epsilon_{11} = \int_{-\frac{t_s}{2}}^{\frac{t_s}{2}} \sigma_{11} dz dy$$

$$\frac{bt_p}{2} Y_p \epsilon_{11} = \int_{-\left(\frac{t_s}{2} + \frac{t_p}{2}\right)}^{-t_s/2} \sigma_{11} dz dy + d_{13} Y_p \frac{t_p}{2} b E$$

Adding the three above equations, we get

$$(bt_p Y_p + bt_s Y_s) \epsilon_{11} = \int_{-\left(\frac{t_s}{2} + \frac{t_p}{2}\right)}^{\left(\frac{t_s}{2} + \frac{t_p}{2}\right)} \sigma_{11} dz dy + d_{13} Y_p t_p b E$$

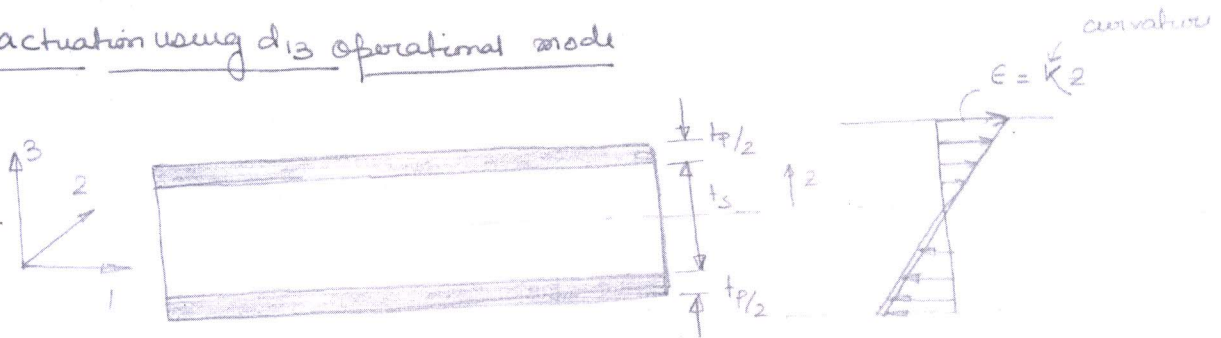
$$\Rightarrow \quad \begin{aligned} & \parallel \\ & 0 \text{ (when no external force is applied)} \\ & = F \text{ (applied force)} \end{aligned}$$

$$\Rightarrow (t_p Y_p + t_s Y_s) \epsilon_{11} = d_{13} Y_p t_p E$$

$$\Rightarrow \epsilon_{11} = \frac{d_{13} Y_p t_p E}{(t_p Y_p + t_s Y_s)} = \frac{\text{free strain } d_{13} E}{\left(1 + \frac{Y_s t_s}{Y_p t_p}\right)}$$

$$\Rightarrow \begin{aligned} E \\ \text{(electric field)} \\ \text{applied} \end{aligned} = \left(1 + \frac{Y_s t_s}{Y_p t_p}\right) \frac{E_s}{d_{13}}$$

Bending actuation using d_{13} operational mode



Constitutive Relation

$$\epsilon(z) = \begin{cases} \frac{1}{Y_p} \sigma(z) - d_{13} E & \frac{t_s}{2} \leq z \leq \frac{t_s+t_p}{2} \\ \frac{1}{Y_s} \sigma(z) & -\frac{t_s}{2} \leq z \leq \frac{t_s}{2} \\ \frac{1}{Y_p} \sigma(z) + d_{13} E - \frac{(t_s+t_p)}{2} & -\frac{t_s+t_p}{2} \leq z \leq -\frac{t_s}{2} \end{cases} \quad (i)$$

For an Euler-Bernoulli beam,

$$\begin{aligned} \epsilon(z) &= z \kappa \leftarrow \text{curvature.} \\ &= z \cdot \frac{1}{R} = z \frac{d\theta}{dx} = z \frac{\partial^2 w}{\partial x^2} \end{aligned}$$

Multiplying both sides of Eqn. (i) with z and integrating w.r.t area, we get,

$$Y_p \int_{\frac{t_s}{2}}^{\frac{(t_s+t_p)/2}{2}} z \epsilon(z) dz dy = \int_{A_1} z \sigma(z) dz dy - \frac{Y_p d_{13} E}{8} \int_{\frac{t_s}{2}}^{\frac{(t_s+t_p)/2}{2}} z dz dy$$

$$\Rightarrow \frac{Y_p \kappa b}{3 \times 8} \left[(t_s+t_p)^3 - t_s^3 \right] = \int_{A_1} z \sigma(z) dz dy - \frac{Y_p d_{13} E}{8} \left[(t_s+t_p)^2 - t_s^2 \right]$$

$$\Rightarrow \frac{Y_p \kappa b}{24} \left[t_p^3 + 3t_p^2 t_s + 3t_p t_s^2 \right] = \int_{A_1} z \sigma(z) dz dy - \frac{Y_p d_{13} E}{8} \left[t_p^2 + 2t_p t_s \right] \quad (i)$$

Similarly,

$$\frac{Y_s \kappa b}{24} \left[t_s^3 + t_s^3 \right] = \int_{A_2} z \sigma(z) dz dy \quad (ii)$$

