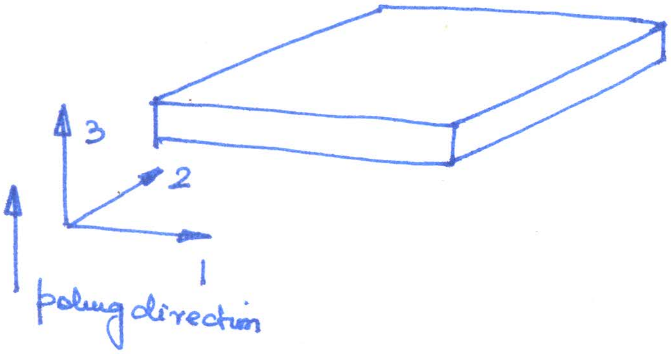
Piezoelectric Sensors

3-D Sensor constitutive relation



$$\begin{Bmatrix} D_1 \\ D_2 \\ D_3 \end{Bmatrix} = \begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{22} & 0 \\ 0 & 0 & \varepsilon_{33} \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix}^{=0}$$

$$+ \begin{bmatrix} 0 & 0 & 0 & 0 & d_{15} & 0 \\ 0 & 0 & 0 & d_{24} & 0 & 0 \\ d_{13} & d_{23} & d_{33} & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{Bmatrix}$$

Assuming no electric field acting between the electrodes, the above equation reduces to,

$$\Rightarrow \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & d_{15} & 0 \\ 0 & 0 & 0 & d_{24} & 0 & 0 \\ d_{13} & d_{23} & d_{33} & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{Bmatrix}$$

In general electrodes are placed in 12 planes, hence, the useful eqn is

$$\boxed{D_3 = d_{13}\sigma_{11} + d_{23}\sigma_{22} + d_{33}\sigma_{33}}$$

→ cannot be used to measure shear strains

D_3 → charge/area

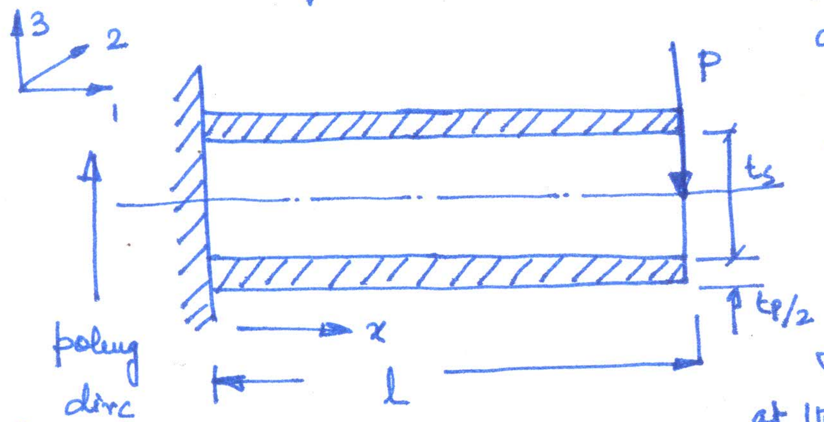
σ_{ii} → stress in the PZT

and $q = \iint D_3 \, dx \, dy$

Advantages of PZT sensors over conventional strain sensors

1. Compactness
2. Collocated sensing
3. Sensitivity over a large strain bandwidth
4. ease of embeddability.

Sensor: 31 operational mode.



$t_p << t_s$

Find the charge in the PZT due to a bending force P applied at the tip.

Bending moment @ x.

$$= M(x) = P(l-x)$$

$$\sigma_{11} \text{ at the interface} = \frac{M(x)\, z}{I_b} = \frac{P(l-x)\, t_s}{2 \cdot \frac{1}{12} b t_s^3}$$

(neglecting the moment of inertia due to the PZT patch)

$$\text{Strain at the interface} = \epsilon_{11} = \frac{6P(l-x)}{Y_s b t_s^2}$$

Assuming that there is no strain variation along the thickness of the PZT

$$\epsilon_{11\,(PZT)} = \frac{6P(l-x)}{Y_s b t_s^2}$$

$$\text{Stress in PZT} = \frac{6 Y_p P(l-x)}{Y_s b t_s^2}.$$

Electrical displacement D in the PZT

$$= \frac{6 d_{13} Y_p P(l-x)}{Y_s b t_s^2}.$$

$$\Rightarrow \frac{dq}{dA} = D = \frac{6 d_{13} Y_p P(l-x)}{Y_s b t_s^2}$$

$$\Rightarrow dq = \frac{6 d_{13} Y_p P(l-x)\, b\, dx}{Y_s b t_s^2}$$

$$\Rightarrow q = \int_0^l \frac{6 d_{13} Y_p P(l-x)\, dx}{Y_s t_s^2} = \frac{6 d_{13} Y_p P l^2}{2 Y_s t_s^2}$$

$$= \frac{3 d_{13} Y_p P l^2}{Y_s t_s^2}$$

Direction of the electric field produced ~~due~~ in PZT sensor w.r.t poling direction