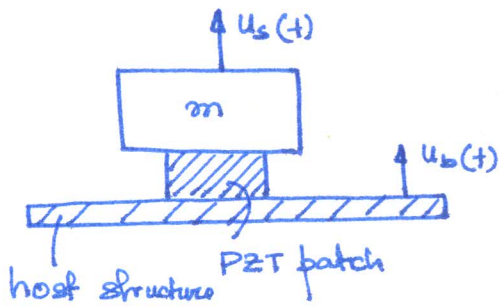
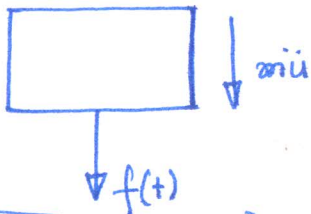


Accelerometer



Measurable quantity $q(t)$
 ↓
 Change in the electrodes

Free body diagram



$$m\ddot{u} + f(t) = 0 \quad (1)$$

$$\Rightarrow s^2 m u(s) + f(s) = 0$$

From piezoelectric sensor eqn (assuming $E=0$)

$$\frac{q(t)}{A_p} = \frac{d_{33} f(t)}{A_p}$$

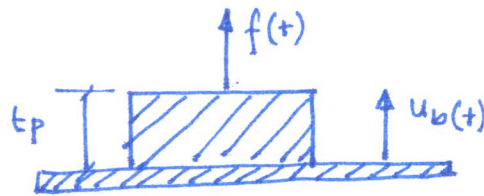
$$\Rightarrow q(t) = d_{33} f(t) \quad (3)$$

$$q(s) = d_{33} f(s)$$

Substituting eqn. (2) in eqn. (1), we get,

$$\frac{s^2 m f(s)}{k_p} + s^2 m u_b(s) + f(s) = 0$$

$$\Rightarrow \left(\frac{s^2 m}{k_p} + 1 \right) f(s) + s^2 m u_b(s) = 0 \quad (4)$$



Stress - strain relation

$$\frac{u(t) - u_b(t)}{t_p} = \frac{f(t)}{Y_p A_p}$$

$$\Rightarrow u(t) = \frac{f(t)}{k_p \left(\frac{Y_p A_p}{t_p} \right)} + u_b(t)$$

$$u(t) = \frac{f(t)}{k_p} + u_b(t) \quad (2)$$

$$u(s) = \frac{f(s)}{k_p} + u_b(s)$$

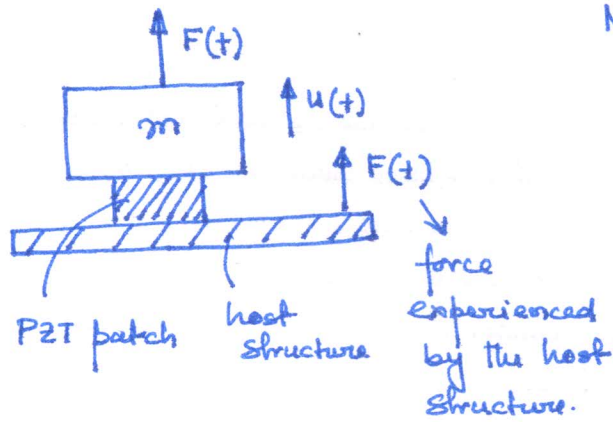
Substituting eqn. (2) in eqn. (4), we get,

$$\left(\frac{s^2 m}{k_p} + 1 \right) \frac{q(s)}{d_{33}} + s^2 m u_b(s) = 0$$

$$\Rightarrow q(s) = \frac{s^2 m d_{33} u_b(s)}{\left[\frac{s^2 m}{k_p} + 1 \right]}$$

$$q(s) = \frac{s^2 m d_{33} u_b(s)}{\left[\frac{s^2 m}{k_p} + 1 \right]}$$

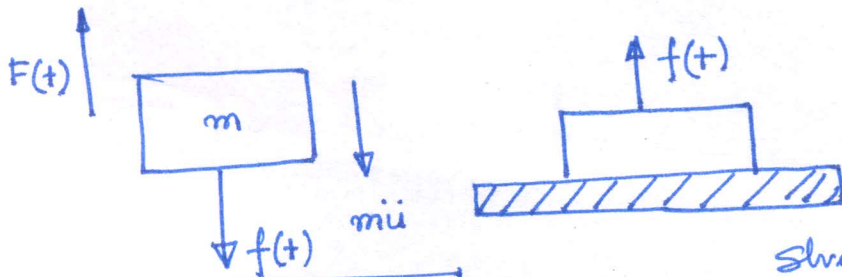
Force Sensor



Measurable parameter $q(t)$

Transfer function between $q(t)$ and $F(t)$

Free body diagram



$$\begin{aligned} m\ddot{u} + f(t) &= F(t) \\ s^2 m u(s) + f(s) &= F(s) \end{aligned} \quad (1)$$

Stress-strain relation

$$\frac{u}{t_p} = \frac{f(t)}{A_p Y_p}$$

$$\Rightarrow f(t) = u \left(\frac{A_p Y_p}{t_p} \right) k_p$$

$$\Rightarrow f(s) = k_p u(s) \quad (2)$$

From piezoelectric sensor eqn (assuming $E=0$)

$$\frac{q(t)}{A_p} = \frac{d_{33} f(t)}{A_p}$$

$$\Rightarrow q(s) = d_{33} f(s) \quad (3)$$

Substituting eqn. (3) in eqn. (2), we get,

$$q(s) = k_p d_{33} u(s) \quad (4)$$

Substituting eqns. (2) & (4) in eqn. (1), we get,

$$\frac{s^2 m k_p}{k_p d_{33}} q(s) + \frac{q(s)}{d_{33}} = F(s)$$

$$\Rightarrow q(s) = \frac{d_{33} F(s)}{\left[\frac{s^2 m}{k_p} + 1 \right]} \Rightarrow F(s) = \left[\frac{s^2 m}{k_p} + 1 \right] \frac{q(s)}{d_{33}}$$

Electrical boundary condn

Case 1 Constitutive relation (33 operational mode)

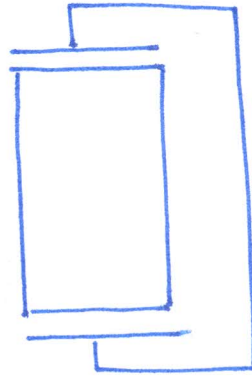
$$\epsilon_{33} = S \nabla_{33} + d_{33} E \stackrel{!}{=} 0$$

$$D_{33} = \epsilon E + d_{33} \nabla$$

⇓

$$\begin{aligned} \epsilon &= S \nabla \\ D &= d_{33} \nabla \\ \Rightarrow q &= d_{33} A_p \nabla \end{aligned}$$

Closed/short circuit condn
 $E=0$



Short circuit condn.
 $E=0$

$$\epsilon_{33} = S \nabla_{33} + d_{33} E$$

$$0 = D_{33} = \epsilon E + d_{33} \nabla$$

⇓

$$\epsilon_{33} = S \nabla_{33} + d_{33} E$$

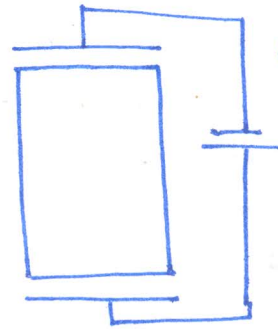
$$E = -\frac{d_{33} \nabla}{\epsilon}$$

⇒

$$\epsilon_{33} = S \left(1 - \frac{d_{33}^2}{\epsilon S}\right) \nabla$$

$$E = -\frac{d_{33} \nabla}{\epsilon}$$

} Open circuit condn.



Open circuit condn.

$E \neq 0$
 $D = 0$

$$\frac{d_{33}^2}{\epsilon S} = k^2$$

where k is the coupling coefficient
it is non-dimensional and $0 < k < 1$

Physical significance of the coupling coefficient

Coupling coefficient denotes ^{ratio} the energy transfer from mechanical domain to electrical domain.

