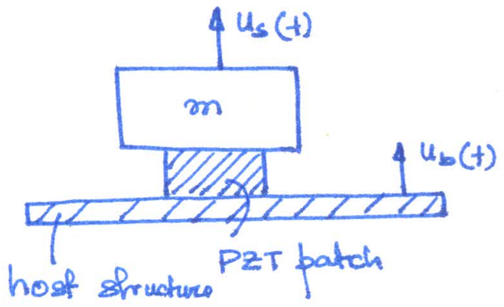
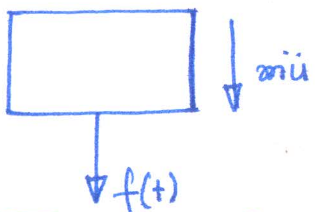
Accelerometer



Measurable quantity $q(t)$

Charge in the electrode

Free body diagram



$$\boxed{m\ddot{u} + f(t) = 0} \quad \text{———} (1)$$

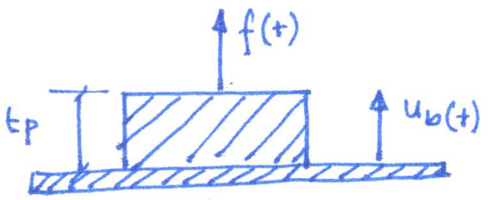
$$\Rightarrow \boxed{s^2 m u(s) + f(s) = 0}$$

From piezoelectric sensor eqn (assuming $E = 0$)

$$\frac{q(t)}{A_p} = \frac{d_{33} f(t)}{A_p}$$

$$\Rightarrow \boxed{q(t) = d_{33} f(t)} \quad \text{———} (3)$$

$$\boxed{q(s) = d_{33} f(s)}$$



Stress - strain relation

$$\frac{u(t) - u_b(t)}{t_p} = \frac{f(t)}{Y_p A_p}$$

$$\Rightarrow u(t) = \frac{f(t)}{\underbrace{\left(\frac{Y_p A_p}{t_p}\right)}_{k_p}} + u_b(t)$$

$$\Rightarrow \boxed{u(t) = \frac{f(t)}{k_p} + u_b(t)} \quad \text{———} (2)$$

$$\boxed{u(s) = \frac{f(s)}{k_p} + u_b(s)}$$

Substituting eqn. (2) in eqn. (1), we get,

$$\frac{s^2 m f(s)}{k_p} + s^2 m u_b(s) + f(s) = 0$$

$$\Rightarrow \boxed{\left(\frac{s^2 m}{k_p} + 1\right) f(s) + s^2 m u_b(s) = 0} \quad \text{———} (4)$$

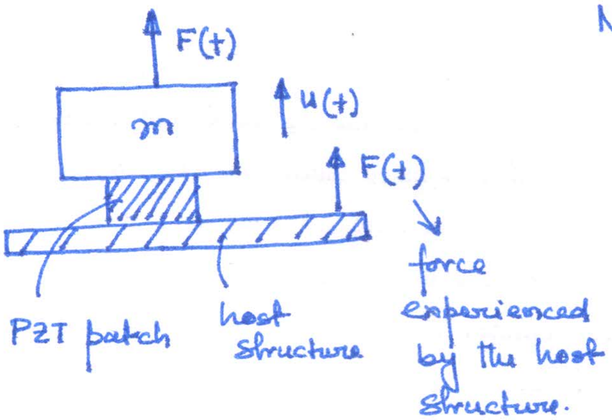
Substituting eqn. (3) in eqn. (4), we get,

$$\left(\frac{s^2 m}{k_p} + 1\right) \frac{q(s)}{d_{33}} + s^2 m u_b(s) = 0$$

$$\Rightarrow \cancel{u_b(s) =}$$

$$\Rightarrow \boxed{q(s) = \frac{s^2 m d_{33} u_b(s)}{\left[\frac{s^2 m}{k_p} + 1\right]}}$$

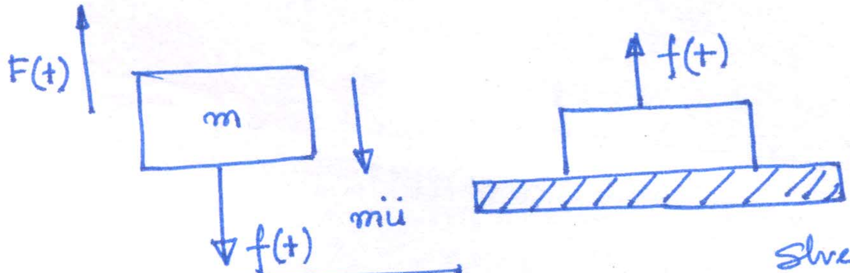
Force Sensor



Measurable parameter $q(t)$

Transfer function between $q(t)$ and $F(t)$

Free body diagram



$$\boxed{\begin{aligned} m\ddot{u} + f(t) &= F(t) \\ s^2 m u(s) + f(s) &= F(s) \end{aligned}} \quad \text{——(1)}$$

Stress-strain relation

$$\frac{u}{t_p} = \frac{f(t)}{A_p Y_p}$$

$$\Rightarrow f(t) = u \frac{A_p Y_p}{t_p} \; k_p$$

$$\Rightarrow \boxed{f(s) = k_p u(s)} \quad \text{——(2)}$$

From piezoelectric sensor eqn (assuming $E=0$)

$$\frac{q(t)}{A_p} = \frac{d_{33} f(t)}{A_p}$$

$$\Rightarrow \boxed{q(s) = d_{33} f(s)} \quad \text{——(3)}$$

Substituting eqn. (3) in eqn. (2), we get,

$$\boxed{q(s) = k_p d_{33} u(s)} \quad \text{——(4)}$$

Substituting eqns. (2) & (4) in eqn. (1), we get,

$$\cancel{s^2 m k_p} \quad \frac{s^2 m \, q(s)}{k_p d_{33}} + \frac{q(s)}{d_{33}} = F(s)$$

$$\Rightarrow q(s) = \frac{d_{33} F(s)}{\left[\frac{s^2 m}{k_p} + 1\right]} \Rightarrow \boxed{F(s) = \left[\frac{s^2 m}{k_p} + 1\right] \frac{q(s)}{d_{33}}}$$

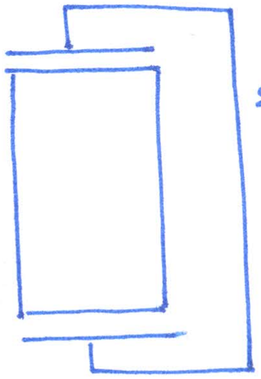
Electrical boundary condn

Case 1 Constitutive relation (33 operational mode)

$$\epsilon_{33} = s\sigma_{33} + d_{33}E^{\to 0}$$

$$D_{33} = \varepsilon E^{\to 0} + d_{33}\sigma$$

$$\Downarrow$$



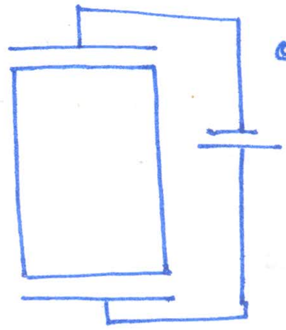
Short circuit condn.

$E = 0$

$$\boxed{\begin{aligned} \epsilon &= s\sigma \\ D &= d_{33}\sigma \\ \Rightarrow q &= d_{33}A_p\sigma \end{aligned}}$$

Closed/short circuit condn

$E = 0$



Open circuit condn.

$E \neq 0$

$D = 0$

$$\epsilon_{33} = s\sigma_{33} + d_{33}E$$

$$0 \leftarrow D_{33} = \varepsilon E + d_{33}\sigma$$

$$\Downarrow$$

$$\epsilon_{33} = s\sigma_{33} + d_{33}E$$

$$E = -\frac{d_{33}\sigma}{\varepsilon}$$

$$\Rightarrow \left.\begin{aligned} \epsilon_{33} &= s\left(1 - \frac{d^2}{s\varepsilon}\right)\sigma \\ E &= -\frac{d_{33}\sigma}{\varepsilon} \end{aligned}\right\} \text{ open circuit condn.}$$

$$\frac{d^2}{s\varepsilon} = k^2$$

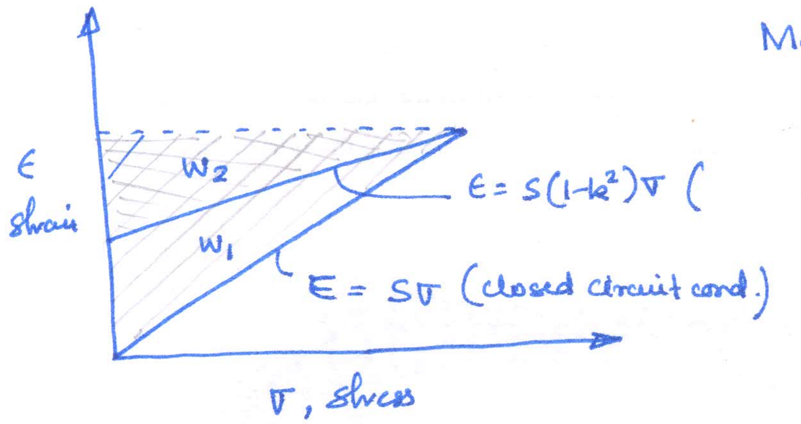
where k is the coupling coefficient

it is non-dimensional and is $0 < k < 1$

Physical significance of the coupling coefficient

Coupling coefficient denotes ratio the energy transfer from mechanical domain to electrical domain.

Let us consider a piezoelectric patch acted upon by a stress σ in closed circuit condition.



Mechanical work done

$$W_1 + W_2 = \frac{1}{2} s\sigma^2 .$$

Next, the PZT patch is unloaded to zero stress in open circuit condn. There will be a residual strain due to the electric field E which can be released under zero stress condn.

Mechanical work done $W_2 = \frac{1}{2} s(1-k^2)\sigma^2$.

Amount of energy transferred to the electrical domain $= W_1$

$$= \frac{1}{2} s\sigma^2 - \frac{1}{2} s(1-k^2)\sigma^2$$

$$= \frac{1}{2} k^2 s\sigma^2 .$$

Ratio of energy transferred to the total energy $= \dfrac{W_2}{W_1 + W_2} = k^2$

$\therefore \; 0 < k^2 < 1 \implies 0 < k < 1$