CHAPTER
ELEVEN
BASIC OPTICS

11.1 THE NATURE OF LIGHT

The phenomenon of light has attracted the attention of man from the earliest times. The ancient Greeks considered light to be an emission of small particles by a luminous body which entered the eye and returned to the body. Empedocles (484–424 B.C.) suggested that light takes time to travel from one point to another; however, Aristotle (384–322 B.C.) later rejected this idea as being too much to assume. The ideas of Aristotle concerning the nature of light persisted for approximately 2000 years.

In the seventeenth century, considerable effort was devoted to a study of the optical effects associated with thin films, lenses, and prisms. Huygens (1629–1695) and Hooke (1635–1703) attempted to explain some of these effects with a wave theory. In the wave theory, a hypothetical substance of zero mass, called the *ether*, was assumed to occupy all space. Initially, light propagation was assumed to be a longitudinal vibratory disturbance moving through the ether. The idea of secondary wavelets, in which each point on a wavefront can be regarded as a new source of waves, was proposed by Huygens to explain refraction. Huygens' concept of secondary wavelets is widely used today to explain, in a simple way, other optical effects such as diffraction and interference. At about the same time, Newton (1642–1727) proposed his corpuscular theory, in which light is visualized as a stream of small but swift particles emanating from shining bodies. The theory was able to explain most of the optical effects observed at the time, and, thanks to Newton's stature, was widely accepted for approximately 100 years.

A revival of interest in the wave theory of light began with the work of Young (1773–1829), who demonstrated that the presence of a refracted ray at an interface between two materials was to be expected from a wave theory while the corpuscular theory of Newton could explain the effect only with difficulty. His two-pinhole experiment, which demonstrated the interference of light, together with the work of Fresnel (1788–1827) on polarized light, which required transverse rather than longitudinal vibrations, firmly established the transverse ether wave theory of light.

The next major step in the evolution of the theory of light was due to Maxwell (1831–1879). His electromagnetic theory predicts the presence of two vector fields in light waves, an electric field and a magnetic field. Since these fields can propagate through space unsupported by any known matter, the need for the hypothetical ether of the previous wave theory was eliminated. The electromagnetic wave theory also unites light with all the other invisible entities of the electromagnetic spectrum, e.g., cosmic rays, gamma rays, x-rays, ultraviolet rays, infrared rays, microwaves, radio waves, and electric-power-transmission waves. The wide range of wavelengths and frequencies available for study in the electromagnetic spectrum has led to rapid development of additional theory and understanding.

Observation of the photoelectric effect by Hertz in 1887, which cannot be explained by a wave theory but is easily explained by a particle theory, led to Einstein's photon theory in 1907. The modern theory of wave mechanics successfully reconciles these two approaches, in which energy can be manifest in either particle or wave forms. For most of the effects to be described in later sections of this text the wave properties of light are important and the particle characteristics of individual photons have little application. For this reason, simple wave theory will be used in most of the discussions which follow.

11.2 WAVE THEORY OF LIGHT [1]

Electromagnetic radiation is predicted by Maxwell's theory to be a transverse wave motion which propagates with an extremely high velocity. Associated with the wave are oscillating electric and magnetic fields which can be described with electric and magnetic vectors **E** and **H**. These vectors are in phase, perpendicular to each other, and at right angles to the direction of propagation. A simple representation of the electric and magnetic vectors associated with an electromagnetic wave at a given instant of time is illustrated in Fig. 11.1. For simplicity and convenience of representation, the wave has been given sinusoidal form. Other wave forms such as the *sawtooth waveform* or the *square waveform* are often encountered in electronics. These complicated waveforms are frequently represented for mathematical analysis by a Fourier series; therefore, the simple sinusoidal representation provides the basic information needed for the analysis of more complicated shapes.

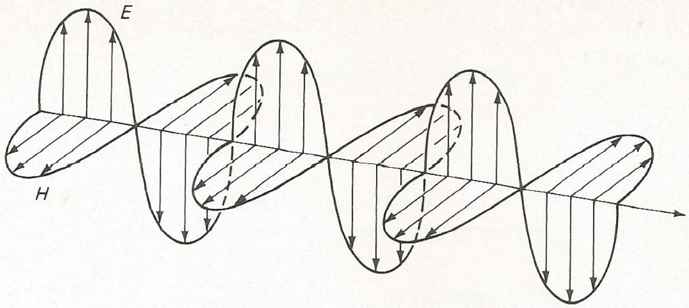

Figure 11.1 The electric and magnetic vectors associated with a plane electromagnetic wave.

All types of electromagnetic radiation propagate with the same velocity in free space (approximately 3×10^8 m/s, or 186,000 mi/s). Characteristics used to differentiate between the various radiations are wavelength and frequency. These two quantities are related to the velocity by the relationship

$$\lambda f = c \tag{11.1}$$

where λ = wavelength
f = frequency
c = velocity of propagation

The electromagnetic spectrum has no upper or lower limit of wavelength or frequency. The radiations observed to date have been classified in the broad general categories shown in Fig. 11.2.

Light is usually defined as radiation that can affect the human eye. From Fig. 11.2 it can be seen that the visible range of the spectrum is a small band centered about a wavelength of approximately 550 nm. The limits of the visible spectrum are not well defined because the eye ceases to be sensitive at both long and short wavelengths within the region; however, normal vision is usually

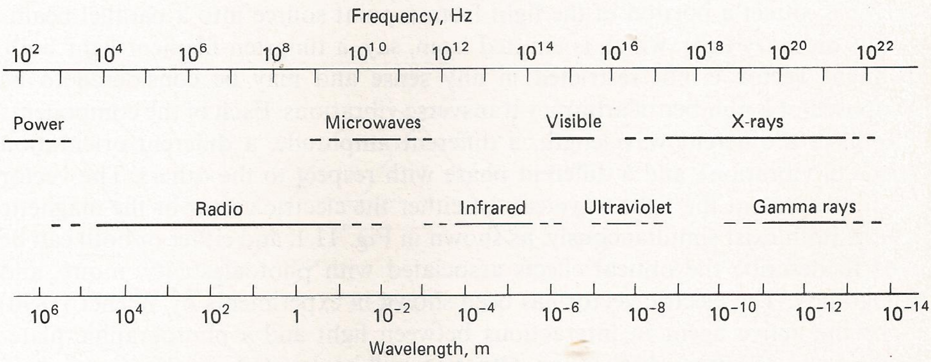

Figure 11.2 The electromagnetic spectrum.

Table 11.1 The visible spectrum

Wavelength range, nm	Color	Wavelength range, nm	Color
400–450	Violet	550–570	Yellow-green
450–480	Blue	570–590	Yellow
480–510	Blue-green	590–630	Orange
510–550	Green	630–700	Red

assumed to be the range from 400 to 700 nm. Within this range the eye interprets the wavelengths as the different colors listed in Table 11.1. Light from a source that emits a continuous spectrum with equal energy for every wavelength is interpreted as *white light*. Light of a single wavelength is known as *monochromatic light*.

Electromagnetic waves can be classified as one-, two-, or three-dimensional according to the number of dimensions in which they propagate energy. Light waves which emanate radially from a small source are three-dimensional. Two quantities associated with a propagating wave which will be useful in discussions involving geometrical and physical optics are wavefronts and rays. For a three-dimensional pulse of light emanating from a source, both the electric vector and the magnetic vector exhibit the periodic variation in magnitude shown in Fig. 11.1 along any radial line. The locus of points on different radial lines from the source exhibiting the same disturbance at a given instant of time, e.g., maximum or minimum values, is a surface known as a *wavefront*. As time passes, the surface moves and indicates how the pulse is propagating. If the medium is optically homogeneous and isotropic, the direction of propagation will be at right angles to the wavefront. A line normal to the wavefront, indicating the direction of propagation of the waves, is called a *ray*. When the waves are propagated out in all directions from a point source, the wavefronts are spheres and the rays are radial lines in all directions from the source. At large distances from the source, the spherical wavefronts have very little curvature, and over a limited region they can be regarded as plane. Plane wavefronts can also be produced by using a lens or mirror to direct a portion of the light from a point source into a parallel beam.

In ordinary light, which is emitted from, say, a tungsten-filament light bulb, the light vector is not restricted in any sense and may be considered to be composed of a number of arbitrary transverse vibrations. Each of the components may have a different wavelength, a different amplitude, a different orientation (plane of vibration), and a different phase with respect to the others. The vector used to represent the light wave can be either the electric vector or the magnetic vector. Both exist simultaneously, as shown in Fig. 11.1, and either or both can be used to describe the optical effects associated with photoelasticity, moiré, and holography. The electric vector has been shown in experiments by Wiener (1890) to be the active agent in interactions between light and a photographic plate; therefore, in all future discussions, attention will be devoted exclusively to vibrations associated with the electric vector.

A. The Wave Equation

Since the disturbance producing light can be represented by a transverse wave motion, it is possible to express the magnitude of the light (electric) vector in terms of the solution of the one-dimensional wave equation:

$$E = f(z - ct) + g(z + ct) \tag{11.2}$$

where
E = magnitude of light vector
z = position along axis of propagation
t = time
$f(z - ct)$ = wave motion in positive z direction
$g(z + ct)$ = wave motion in negative z direction

Most optical effects of interest in experimental stress analysis can be described with a simple sinusoidal or harmonic waveform. Thus, light propagating in the positive z direction away from the source can be represented by Eq. (11.2) as

$$E = f(z - ct) = \frac{K}{z} \cos \frac{2\pi}{\lambda} (z - ct) \tag{11.3}$$

where K is related to the strength of the source and K/z is an attenuation coefficient associated with the expanding spherical wavefront. At distances far from the source, the attenuation is small over short observation distances, and therefore it is frequently neglected. For plane waves, the attenuation does not occur since the beam of light maintains a constant cross section. Equation (11.3) can then be written as

$$E = a \cos \frac{2\pi}{\lambda} (z - ct) \tag{11.4}$$

where a is a constant known as the *amplitude* of the wave. A graphical representation of the magnitude of the light vector as a function of position along the positive z axis, at two different times, is shown for a plane light wave in Fig. 11.3. The length from peak to peak on the magnitude curve for the light vector is

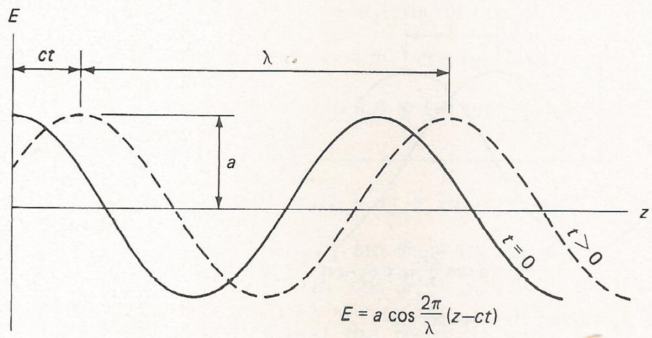

Figure 11.3 Magnitude of the light vector as a function of position along the axis of propagation at two different times.

defined as the *wavelength* λ. The time required for passage of two successive peaks at some fixed value of z is defined as the *period* T of the wave and is given by

$$T = \frac{\lambda}{c} \tag{11.5}$$

The *frequency* of the light vector is defined as the number of oscillations per second. Thus, the frequency is the reciprocal of the period, or

$$f = \frac{1}{T} = \frac{c}{\lambda} \tag{11.6}$$

The terms angular frequency and wave number are frequently used to simplify the argument in a sinusoidal representation of a light wave. The *angular frequency* ω and the *wave number* ξ are given by

$$\omega = \frac{2\pi}{T} = 2\pi f \tag{11.7}$$

$$\xi = \frac{2\pi}{\lambda} \tag{11.8}$$

Substituting Eqs. (11.7) and (11.8) into Eq. (11.4) yields

$$E = a \cos (\xi z - \omega t) \tag{a}$$

Two waves having the same wavelength and amplitude but a different phase are shown in Fig. 11.4. The two waves can be expressed by

$$E_1 = a \cos \frac{2\pi}{\lambda}(z + \delta_1 - ct) \qquad E_2 = a \cos \frac{2\pi}{\lambda}(z + \delta_2 - ct) \tag{11.9}$$

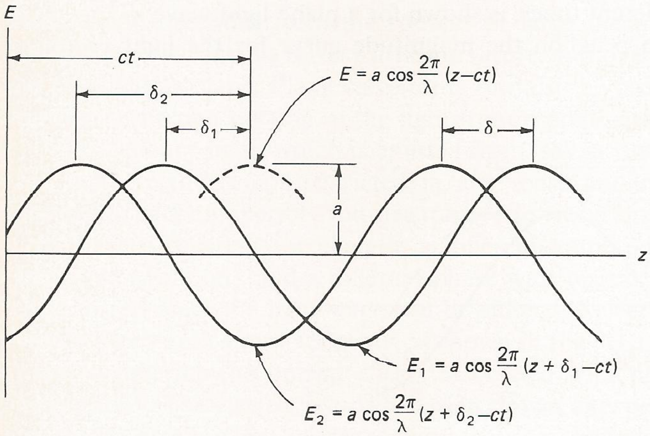

Figure 11.4 Magnitude of the light vector as a function of position along the axis of propagation for two waves with different initial phases.

where δ_1 = initial phase of wave E_1

δ_2 = initial phase of wave E_2

$\delta = \delta_2 - \delta_1$ = the *linear phase difference* between waves

The linear phase difference δ is often referred to as a *retardation* since wave 2 trails wave 1.

The magnitude of the light vector can also be plotted as a function of time at a fixed position along the beam. This representation is useful for many applications since the eye, photographic films, and other light-detecting devices are normally located at fixed positions for observations.

B. Superposition of Waves

In later chapters on photoelasticity and moiré, the phenomena associated with the superposition of two waves having the same frequency but different amplitude and phase will be encountered. At a fixed position z_0 along the light beam, where the observations will be made, the equations for the waves can be expressed as

$$E_1 = a_1 \cos \frac{2\pi}{\lambda}(z_0 + \delta_1 - ct) = a_1 \cos(\phi_1 - \omega t)$$

$$E_2 = a_2 \cos \frac{2\pi}{\lambda}(z_0 + \delta_2 - ct) = a_2 \cos(\phi_2 - \omega t) \tag{11.10}$$

where ϕ_1 = phase angle associated with wave E_1 at position z_0

ϕ_2 = phase angle associated with wave E_2 at position z_0

a_1 = amplitude of wave E_1

a_2 = amplitude of wave E_2

Consider first the case where the light vectors associated with the two waves oscillate in the same plane. The magnitude of the resulting light vector is simply

$$E = E_1 + E_2 \tag{b}$$

Substituting Eqs. (11.10) into Eq. (*b*) yields

$$\begin{aligned} E &= a_1(\cos \omega t \cos \phi_1 + \sin \omega t \sin \phi_1) \\ &\qquad + a_2(\cos \omega t \cos \phi_2 + \sin \omega t \sin \phi_2) \\ &= (a_1 \cos \phi_1 + a_2 \cos \phi_2) \cos \omega t + (a_1 \sin \phi_1 + a_2 \sin \phi_2) \sin \omega t \\ &= a \cos \phi \cos \omega t + a \sin \phi \sin \omega t \\ &= a \cos(\phi - \omega t) \end{aligned} \tag{c}$$

where
$$a^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos(\phi_2 - \phi_1) \tag{11.11}$$

$$\tan \phi = \frac{a_1 \sin \phi_1 + a_2 \sin \phi_2}{a_1 \cos \phi_1 + a_2 \cos \phi_2} \tag{11.12}$$

Equation (*c*) indicates that the resulting wave has the same frequency as the original waves but a different amplitude and a different phase angle. The above procedure can easily be extended to the addition of three or more waves.

A special case frequently arises where the amplitudes of the original waves are equal. In this case the amplitude of the resulting wave is given by Eq. (11.11) as

$$a = \sqrt{2a_1^2[1 + \cos(\phi_2 - \phi_1)]} \tag{d}$$

Since $\phi_2 - \phi_1 = 2\pi\delta/\lambda$,

$$a = \sqrt{2a_1^2\left(1 + \cos\frac{2\pi\delta}{\lambda}\right)} = \sqrt{4a_1^2 \cos^2\frac{\pi\delta}{\lambda}} \tag{e}$$

In most problems in optics the amplitude of the resulting wave is important, but the time variation is not. This results from the fact that the eye and other sensing instruments respond to the *intensity* of light (intensity is proportional to the square of the amplitude) but cannot detect the rapid time variations (for sodium light the frequency is 5.1×10^{14} Hz). Thus for the special case of two waves of equal amplitude the intensity is given by

$$I \sim a^2 = 4a_1^2 \cos^2\frac{\pi\delta}{\lambda} \tag{11.13}$$

Equation (11.13) indicates that the intensity of the light wave resulting from superposition of two waves of equal amplitude is a function of the linear phase difference δ between the waves. The intensity of the resultant wave assumes its maximum value when $\delta = n\lambda$, $n = 0, 1, 2, 3, \ldots$, or when the linear phase difference is an integral number of wavelengths. Under this condition

$$I = 4a_1^2 \tag{f}$$

which indicates that the intensity of the resultant wave is four times the intensity of one of the individual waves. The intensity of the resultant wave assumes its minimum value when $\delta = [(2n + 1)/2]\lambda$, $n = 0, 1, 2, 3, \ldots$, or when the linear phase difference is an odd number of half wavelengths. Under these conditions

$$I = 0 \tag{g}$$

The modification of intensity by superposition of light waves is referred to as an *interference effect.* The effect represented by Eq. (f) is *constructive interference.* The effect represented by Eq. (g) is *destructive interference.* Interference effects have important application in photoelasticity, moiré, and holography.

In previous discussions, the electric vector used to describe the light wave was restricted to a single plane. Light exhibiting this preference for a plane of vibration is known as *plane-* or *linearly polarized light.*

Two other important forms of polarized light arise as a result of the superposition of two linearly polarized light waves having the same frequency but mutually perpendicular planes of vibration, as shown in Fig. 11.5. At a fixed position z_0 along the light beam, the equations for the two waves can be expressed as

$$\begin{aligned} E_x &= a_x \cos\frac{2\pi}{\lambda}(z_0 + \delta_x - ct) = a_x \cos(\phi_x - \omega t) \\ E_y &= a_y \cos\frac{2\pi}{\lambda}(z_0 + \delta_y - ct) = a_y \cos(\phi_y - \omega t) \end{aligned} \tag{11.14}$$

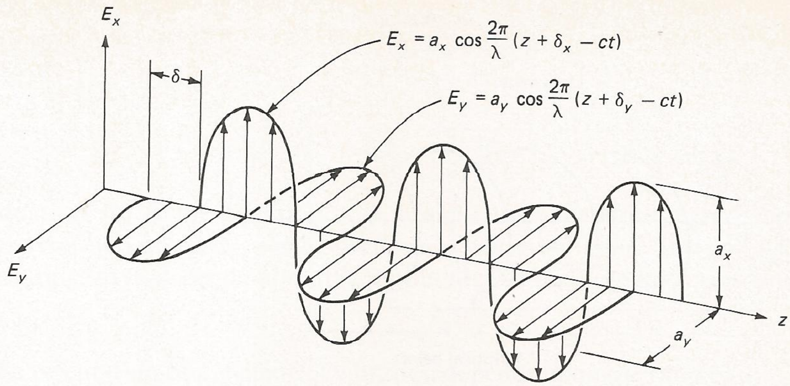

Figure 11.5 Two linearly polarized light waves having the same frequency but mutually perpendicular planes of vibration.

where ϕ_x = phase angle associated with wave in xz plane
ϕ_y = phase angle associated with wave in yz plane
a_x = amplitude of wave in xz plane
a_y = amplitude of wave in yz plane

The magnitude of the resulting light vector is given by

$$E = \sqrt{E_x^2 + E_y^2} \tag{h}$$

Considerable insight into the nature of the light resulting from the superposition of two mutually perpendicular waves is provided by a study of the trace of the tip of the resulting electric vector on a plane perpendicular to the axis of propagation at the point z_0. An expression for this trace can be obtained by eliminating time from Eqs. (11.14). Thus

$$\frac{E_x^2}{a_x^2} - 2\frac{E_x E_y}{a_x a_y}\cos(\phi_y - \phi_x) + \frac{E_y^2}{a_y^2} = \sin^2(\phi_y - \phi_x) \tag{i}$$

or since

$$\phi_y - \phi_x = \frac{2\pi}{\lambda}(\delta_y - \delta_x) = \frac{2\pi\delta}{\lambda}$$

$$\frac{E_x^2}{a_x^2} - 2\frac{E_x E_y}{a_x a_y}\cos\frac{2\pi\delta}{\lambda} + \frac{E_y^2}{a_y^2} = \sin^2\frac{2\pi\delta}{\lambda} \tag{11.15}$$

Equation (11.15) is the equation of an ellipse; therefore, light exhibiting this behavior is known as *elliptically polarized light.* At a fixed instant in time, the tips of the electric vectors at different positions along the z axis form an elliptical helix, as shown in Fig. 11.6. During an interval of time t, the helix will translate a distance ct in the positive z direction. As a result, the electric vector at position z_0 will rotate in a counterclockwise direction as the translating helix intersects the perpendicular plane at position z_0. The locus of points representing the trace of

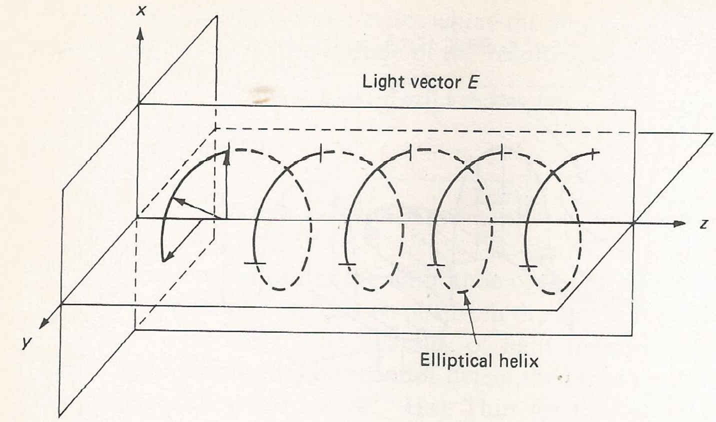

Figure 11.6 The elliptical helix formed by the tips of the light vectors along the axis of propagation at a fixed instant of time.

the tip of the light vector on the perpendicular plane is the ellipse described by Eq. (11.15) and illustrated in Fig. 11.7.

A special case of elliptically polarized light occurs when the amplitudes of the two waves E_x and E_y are equal and $\delta = [(2n + 1)/4]\lambda$, $n = 0, 1, 2, \ldots$, so that Eq. (11.15) reduces to

$$E_x^2 + E_y^2 = a^2 \tag{j}$$

Equation (*j*) is the equation of a circle; therefore, light exhibiting this behavior is known as *circularly polarized light*. In this case, the tips of the light vectors form a circular helix along the z axis at a given instant of time. For $\delta = \lambda/4, 5\lambda/4, \ldots$, the helix is a left circular helix, and the light vector at position z_0 rotates counter-clockwise with time when viewed from a distant position along the z axis. For $\delta = 3\lambda/4, 7\lambda/4, \ldots$, the helix is a right circular helix, and the electric vector at position z_0 rotates clockwise with time.

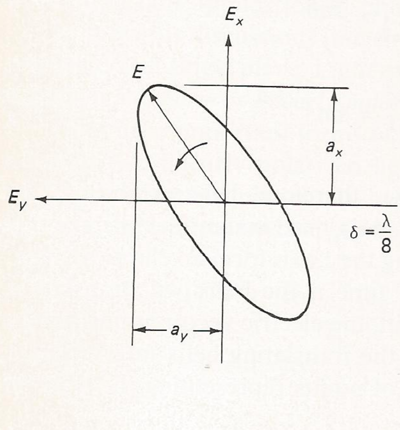

Figure 11.7 Trace of the tip of the light vector on the perpendicular plane at position z_0.

A second special case of elliptically polarized light occurs when the linear phase difference δ between the two waves E_x and E_y is an integral number of half wavelengths ($\delta = n\lambda/2$, $n = 0, 1, 2, \ldots$). For this case, Eq. (11.15) reduces to

$$E_y = \frac{a_y}{a_x} E_x \qquad (k)$$

Equation (k) is the equation of a straight line; therefore, light exhibiting this behavior is known as *plane-* or *linearly polarized light.* The amplitude of the resulting wave depends upon the amplitudes of the original waves since

$$a = \sqrt{a_x^2 + a_y^2} \qquad (l)$$

The orientation of the plane of vibration depends upon the ratio of the amplitudes of the original waves a_y/a_x and upon the linear phase difference δ between the waves. For $\delta = 0, \lambda, 2\lambda, \ldots$, the plane of vibration lies in the first and third quadrants. For $\delta = \lambda/2, 3\lambda/2, 5\lambda/2, \ldots$, the plane of vibration lies in the second and fourth quadrants.

Thus far in the discussions, light has been treated as a wave motion without beginning or end. The light emitted by a conventional light source, e.g., a tungsten-filament light bulb, consists of numerous short pulses originating from a large number of different atoms. Each pulse consists of a finite number of oscillations known as a *wavetrain.* Each wavetrain is thought to be a few meters long with a duration of approximately 10^{-8} s. Since the emissions occur in individual atoms which do not act together in a cooperative manner, the wavetrains may differ from each other in plane of vibration, frequency, amplitude, and phase. Such light is referred to as *incoherent light.* Light sources such as the *laser,* in which the atoms act cooperatively in emitting light, produce *coherent light,* in which the wavetrains are monochromatic, in phase, linearly polarized, and extremely intense. For the interference effects discussed previously, coherent wavetrains are required.

C. The Wave Equation in Complex Notation

A convenient way to represent both the amplitude and phase of a light wave, such as the one represented by Eq. (11.4), for calculations involving a number of optical elements is through the use of complex or exponential notation. Recall the *Euler Identity*

$$e^{i\theta} = \cos\theta + i\sin\theta \qquad (m)$$

where $i^2 = -1$. The sinusoidal wave of Eq. (11.4) is obviously the real part of the complex expression

$$\bar{E} = ae^{i(2\pi/\lambda)(z-ct)} = ae^{i(\phi-\omega t)} \qquad (n)$$

The imaginary part of Eq. (m) could also be used to represent the physical wave; however, it is normally assumed that the real part of a complex quantity is the one having physical significance.

If the amplitude of the wave is also considered to be a complex quantity, then

$$\bar{a} = a_r + ia_i = ae^{i[(2\pi/\lambda)\delta]}$$

where

$$a = \sqrt{a_r^2 + a_i^2} \tag{o}$$

and

$$\tan \frac{2\pi}{\lambda}\delta = \frac{a_i}{a_r} \tag{p}$$

A wave with an initial phase δ can be expressed in exponential notation as

$$\bar{E} = \bar{a}e^{i(2\pi/\lambda)(z-ct)} = ae^{i(2\pi/\lambda)(z+\delta-ct)} \tag{11.16}$$

The physical waves previously represented by Eqs. (11.9) are simply the real part of Eq. (11.16) when represented in exponential notation. Superposition of two or more waves having the same frequency but different amplitude and phase is easily performed with the exponential representation. The real and imaginary parts of the amplitudes of the individual waves are added separately in an algebraic manner. The resultant complex amplitude gives the amplitude and phase of a single wave equivalent to the sum of the individual waves. Extensive use will be made of this representation in Chap. 13, where the theory of photoelasticity is discussed.

The real and imaginary parts of a complex quantity such as the amplitude of a wave may also be written

$$a_r = \tfrac{1}{2}(\bar{a} + \bar{a}^*) \qquad a_i = \tfrac{1}{2}(\bar{a} - \bar{a}^*)$$

where

$$\bar{a}^* = a_r - ia_i$$

is the complex conjugate of the original complex amplitude

$$\bar{a} = a_r + ia_i$$

From Eq. (o),

$$a^2 = a_r^2 + a_i^2$$

Thus

$$a^2 = \bar{a}\bar{a}^* \tag{11.17}$$

This representation for the square of the amplitude of a complex quantity will be useful in future calculations dealing with the intensity of light.

11.3 REFLECTION AND REFRACTION [2]

In the previous section the electromagnetic wave nature of light was discussed, and wavefronts and rays were defined. The discussions were limited to light propagating in free space. Most optical effects of interest, however, occur as a result of the interaction between a beam of light and some physical material. In free space, light propagates with a velocity c, which is approximately 3×10^8 m/s. In any other medium, the velocity is less than the velocity in free space. The ratio of the velocity in free space to the velocity in a medium is a property of the medium known as the *index of refraction n*. The index of refraction for most gases is only slightly greater than unity (for air, $n = 1.0003$). Values for liquids range from 1.3 to 1.5 (for water, $n = 1.33$) and for solids range from 1.4 to 1.8 (for glass, $n = 1.5$). The index of refraction for a material is not constant but varies slightly with wavelength of the light being transmitted. This dependence of index of refraction on wavelength is referred to as *dispersion*.

Since the frequency of a light wave is independent of the material being traversed, the wavelength is shorter in a material than in free space. Thus a wave propagating in a material will develop a linear phase shift δ with respect to a similar wave propagating in free space. The magnitude of the phase shift, in terms of the index of refraction of the material, can be developed as follows. The time required for passage through a material of thickness h is

$$t = \frac{h}{v} \tag{a}$$

where h is the thickness of the material along the path of light propagation and v is the velocity of light in the material. The distance s traveled during the same time by a wave in free space is

$$s = ct = \frac{ch}{v} \tag{b}$$

Thus the distance δ by which the wave in the material trails the wave in free space is given by

$$\delta = s - h = \frac{ch}{v} - h = h(n - 1) \tag{11.18}$$

since the index of refraction of the material has been defined as $n = c/v$. The retardation δ, as given by Eq. (11.18), is a positive quantity since the index of refraction of a material is always greater than unity. The relative position of one wave with respect to another can be controlled by including the retardation in the phase of the appropriate wave equation.

When a beam of light strikes a surface between two transparent materials with different indices of refraction, it is experimentally observed that, in general, it is divided into a reflected ray and a refracted ray, as shown in Fig. 11.8. The reflected and refracted rays lie in the plane formed by the incident ray and the normal to the surface and known as the *plane of incidence*. The *angle of incidence* α, the *angle of reflection* β, and the *angle of refraction* γ are related as follows:

For reflection:
$$\alpha = \beta \tag{11.19}$$

For refraction:
$$\frac{\sin \alpha}{\sin \gamma} = \frac{n_2}{n_1} = n_{21} \tag{11.20}$$

where n_1 = index of refraction of material 1
n_2 = index of refraction of material 2
n_{21} = index of refraction of material 2 with respect to material 1

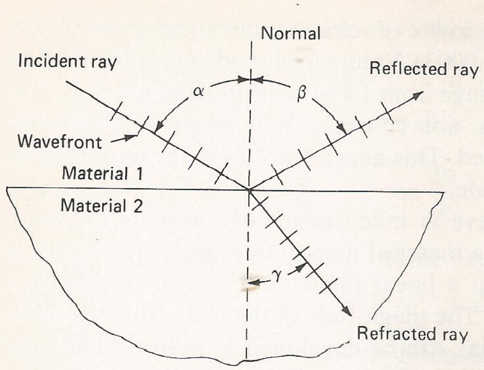

Figure 11.8 Reflection and refraction of a plane light wave at an interface between two transparent materials.

If the light beam originates in the material having the higher index of refraction, n_{21} will be a number less than unity. Under these conditions, some *critical angle of incidence* α_c is reached for which the angle of refraction is 90°. For angles of incidence greater than the critical angle, there is no refracted ray and *total internal reflection* occurs. Total internal reflection cannot occur when the beam of light originates in the medium with the lower index of refraction.

The laws of reflection and refraction give information about the direction of reflected and refracted rays but no information with regard to intensity. Intensity relationships, which can be derived from Maxwell's equations, indicate that the intensity of a reflected beam depends upon both the angle of incidence and the direction of polarization of the incident beam. Consider a completely unpolarized beam of light falling on a surface between two transparent materials, as shown in Fig. 11.9. The electric vector for each wavetrain in the beam can be resolved into two components, one perpendicular to the plane of incidence (the perpendicular component) and the other parallel to the plane of incidence (the parallel component). For completely unpolarized incident light, the two components would have equal intensity. The intensity of the reflected beam can be expressed as

$$I_r = RI_i \tag{11.21}$$

where I_i = intensity of incident beam
I_r = intensity of reflected beam
R = reflection coefficient

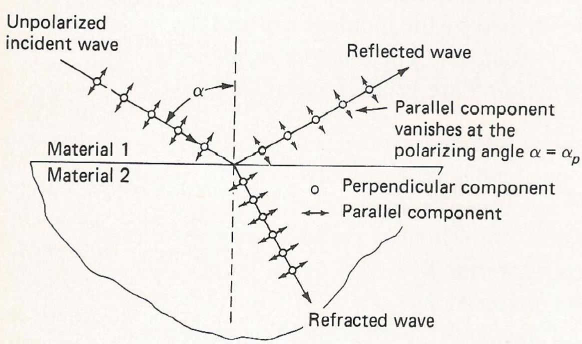

Figure 11.9 Reflection and refraction at the polarizing angle.

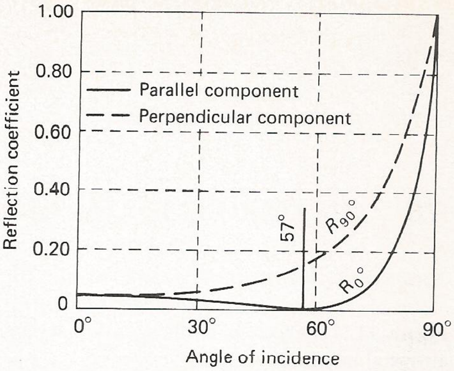

Figure 11.10 Reflection coefficients for an air-glass interface ($n_{21} = 1.5$).

For the perpendicular component

$$R_{90^\circ} = \frac{\sin^2 (\alpha - \gamma)}{\sin^2 (\alpha + \gamma)} \tag{11.22}$$

For the parallel component

$$R_{0^\circ} = \frac{\tan^2 (\alpha - \gamma)}{\tan^2 (\alpha + \gamma)} \tag{11.23}$$

Reflection coefficients for an air-glass interface ($n_{21} = 1.5$) are shown in Fig. 11.10. These data indicate that there is a particular angle of incidence for which the reflection coefficient for the parallel component is zero. This angle is referred to as the *polarizing angle* α_p. Since the parallel component is zero when the angle of incidence is equal to the polarizing angle, the beam reflected from the surface is plane-polarized with the plane of vibration perpendicular to the plane of incidence. Equation (11.23) indicates that the reflection coefficient for the parallel component is zero when $\tan (\alpha + \gamma) = \infty$ or when $\alpha + \gamma = 90^\circ$. Thus from Eq. (11.20),

$$\tan \alpha_p = \frac{n_2}{n_1} = n_{21}$$

It is also experimentally observed that phase changes occur during reflection. A phase change of $\delta = \lambda/2$ occurs when light is incident from the medium with the lower index of refraction. When light is incident from the medium with the higher index of refraction, no phase change occurs upon reflection.

Metal surfaces exhibit relatively large reflection coefficients, as shown in Fig. 11.11. At oblique incidence, the coefficients for light polarized parallel to the plane of incidence are less than the coefficients for the perpendicular component. A change of phase also occurs with metallic reflection. Unfortunately, the phase change varies with both angle of incidence and direction of polarization. As a result, plane-polarized light is changed by oblique reflection to elliptically polarized light.

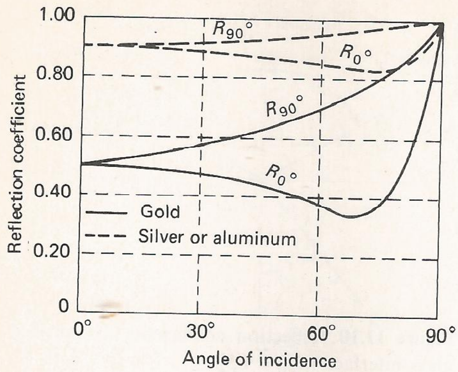

Figure 11.11 Reflection coefficients for several air-metal interfaces.

Previous discussions of reflection and refraction have dealt with materials that are optically isotropic (the index of refraction is the same for all directions in the material); therefore, light propagates with the same velocity in all directions. When a beam of ordinary light is directed at oblique incidence onto the surface of certain crystalline materials such as calcite and quartz, it is observed that, in general, a reflected beam and two refracted beams are produced. This phenomenon is known as *double refraction*, and the material is said to be *birefringent*. One of the refracted rays, known as the *ordinary ray*, propagates with the same velocity along all directions through the crystal and obeys Snell's law of refraction as given by Eq. (11.20). The second refracted ray, known as the *extraordinary ray*, propagates with different velocities along different directions through the crystal and is not governed by Eq. (11.20). Along one direction through the crystal, known as the *optic axis* (the optic axis is a direction and not a specific line), the ordinary and extraordinary rays travel with the same velocity.

If a ray of ordinary light is directed onto the face of a calcite crystal at normal incidence, the ordinary ray passes through the crystal with no deviation while the extraordinary ray is refracted at some angle with respect to the normal to the surface. Since opposite faces of the crystal are parallel, the two rays emerge as parallel beams. The two beams are observed to be plane-polarized in orthogonal directions. The vibrations associated with the ordinary ray are perpendicular to the plane containing the optic axis and the ordinary ray. The vibrations associated with the extraordinary ray lie in the plane containing the optic axis and the extraordinary ray. The ordinary ray and the extraordinary ray lie in the same plane when the plane of incidence coincides with the plane containing the optic axis and the normal to the surface.

For many optical applications, calcite crystals are cut into rectangular blocks with the faces parallel and perpendicular to the optic axis. Light entering the crystal at normal incidence on any of the faces does not deviate from the normal to the surface; therefore, the ordinary and extraordinary rays follow the same path through the crystal. For two of the faces (those perpendicular to the optic axis), the path is along the optic axis, and so the two rays travel with the same velocity. For the other four faces, the path is perpendicular to the optic axis, and so the two rays travel with different velocities. As a result, the waves emerge from the crystal with a linear phase difference δ. Plane-polarized incident light would emerge from such a crystal as elliptically polarized light. Optical elements which convert one form of polarized light into another are referred to as *retarders* or *wave plates*.

11.4 IMAGE FORMATION BY LENSES AND MIRRORS [2]

In the previous section, reflection and refraction of a plane light wave at a plane interface between two materials was considered. More complicated situations frequently arise in the optical systems used for experimental stress-analysis work. Since lenses and mirrors are widely used in many of these systems, a brief discussion of the significant features of these elements is provided here for future reference.

A. Plane Mirrors

Figure 11.12 shows an object O placed at a distance u in front of a plane mirror. The light from each point on the object (such as point A) is a spherical wave which reflects from the mirror in the manner discussed in Sec. 11.3. When the eye or other light-sensing instrument intercepts the reflected rays, they perceive an image I of the object O at a distance v behind the mirror, which can be determined by extending the reflected rays to the position A' as shown in Fig. 11.12. In this instance, the image I is a *virtual image* since light rays do not pass through image points such as A'. From the geometry of Fig. 11.12 it is obvious that the magnitudes of u and v are the same. The image is *erect* and has the same height as the object. One difference between the object and the image not apparent from Fig. 11.12 is that left and right are interchanged (an image of a left hand appears as a right hand in the mirror).

B. Spherical Mirrors

Figure 11.13 shows an object O placed at a distance u in front of a concave spherical mirror. The center of curvature of the mirror is located at C, and the focal point is at F. The light from each point on the object reflects in the manner

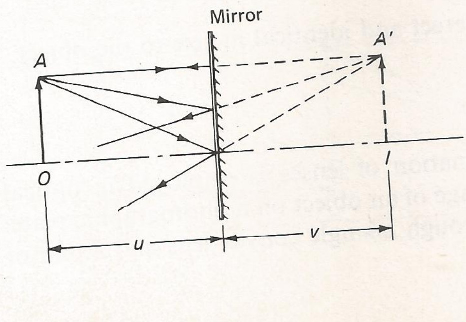

Figure 11.12 Image formation by a plane mirror.

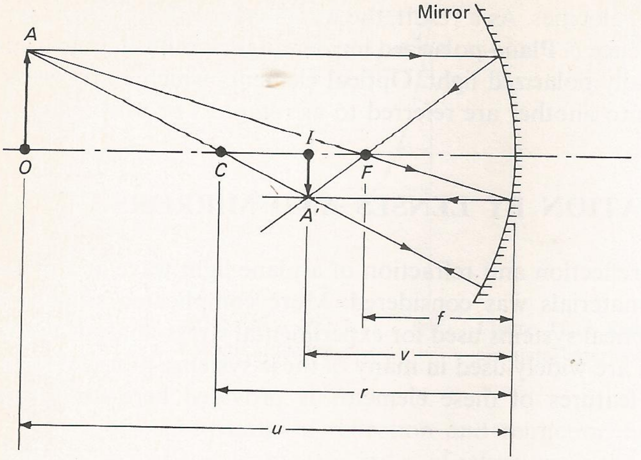

Figure 11.13 Image formation by a concave spherical mirror.

shown in Fig. 11.13 for point A. In this instance the image I is a *real image* since light rays pass through image points such as A'. From the geometry of Fig. 11.13 it can be shown that if all rays from the object make a small angle with respect to the axis of the mirror, the distance to the image satisfies the expression

$$\frac{1}{u}+\frac{1}{v}=\frac{2}{r}=\frac{1}{f} \tag{11.24}$$

For the case illustrated, both u and v are positive since both the object and the image are real. The distance v will be negative when dealing with a virtual image. With this sign convention, Eq. (11.24) applies to all concave, plane, and convex mirrors. The ratio of the size of the image to the size of the object is known as the magnification M and is given by the expression

$$M=-\frac{v}{u} \tag{11.25}$$

where the minus sign is used to indicate an inverted image. Equation (11.25) also applies to all concave, plane, and convex mirrors. For example, when Eqs. (11.24) and (11.25) are applied to the plane mirror of Fig. 11.12,

$$v=-u \qquad M=1$$

This indicates a virtual image which is erect and identical in size to the object.

C. Thin Lenses

At least one and often a series combination of lenses is employed in optical equipment to magnify and focus the image of an object on a photographic plate. For this reason, the passage of light through a single convex lens and a pair of convex lenses in series will be examined in detail. The discussion will be limited to instances where the thin-lens approximation can be applied; i.e., the thickness of the lens can be neglected with respect to other distances such as the focal length f of the lens and the object and image distances u and v.

D. Single-Lens System

The classical optical representation of a single-lens system is shown in Fig. 11.14. The light from each point on the object can be considered as a spherical wave which is reflected and refracted at the air-glass interface, as indicated in Sec. 11.3. In the study of mirrors, the reflected rays are of interest. In the case of lenses, the refracted rays produce the desired optical effects. From the geometry of Fig. 11.14 it can be shown that the object distance u, the image distance v, and the focal length f of the lens are related by

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \qquad (a)$$

Similarly the magnification is given by

$$M = -\frac{v}{u} \qquad (b)$$

Equations (a) and (b) are identical to Eqs. (11.24) and (11.25) for mirrors. In the development of Eq. (a), the assumption was made that all rays from the object make a small angle with respect to the axis of the lens. The image in Fig. 11.14 is a real image; therefore, the image distance v is positive even though the image is on the opposite side of the lens from the object. With v positive, Eq. (b) indicates that the magnification is negative (thus the image is inverted, as shown).

The situation illustrated in Fig. 11.14 occurs when the object is located beyond the focal point of the lens. The object may also be placed between the focal point and the lens surface, as illustrated in Fig. 11.15. In this case, the distance u is positive and v is negative since the image formed is virtual. Equation (b) then yields a positive magnification, which indicates that the image is erect (as shown).

A similar analysis for a concave lens indicates that Eqs. (11.24) and (11.25)

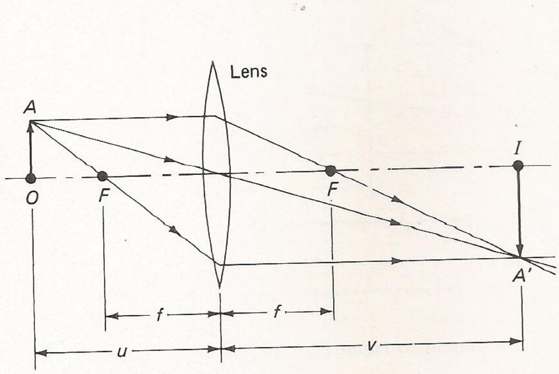

Figure 11.14 Image formation by a single convex lens (object outside the focal point).

In practice, plane-polarized light is produced with an optical element known as a plane or linear polarizer. Production of circularly polarized light or the more general elliptically polarized light requires the use of a linear polarizer together with an optical element known as a wave plate. A brief discussion of linear polarizers, wave plates, and their series combination follows.

A. Linear or Plane Polarizers [3–5]

When a light wave strikes a plane polarizer, this optical element resolves the wave into two mutually perpendicular components, as shown in Fig. 11.19. The component parallel to the axis of polarization is transmitted while the component perpendicular to the axis of polarization is either absorbed, as in the case of Polaroid, or suffers total internal reflection, as in the case of a calcite crystal such as the Nicol prism.

If the plane polarizer is fixed at some point z_0 along the z axis, the equation for the light vector can be written

$$E = a \cos \frac{2\pi}{\lambda}(z_0 - ct) \qquad (a)$$

Since the initial phase of the wave is not important in the developments which follow, Eq. (a) can be reduced through the use of Eqs. (11.6) and (11.7) to

$$E = a \cos 2\pi f t = a \cos \omega t \qquad (11.35)$$

where $\omega = 2\pi f$ is known as the circular frequency of the wave. The absorbed and transmitted components of the light vector are

$$E_a = a \cos \omega t \sin \alpha \qquad E_t = a \cos \omega t \cos \alpha$$

where α is the angle between the axis of polarization and the light vector.

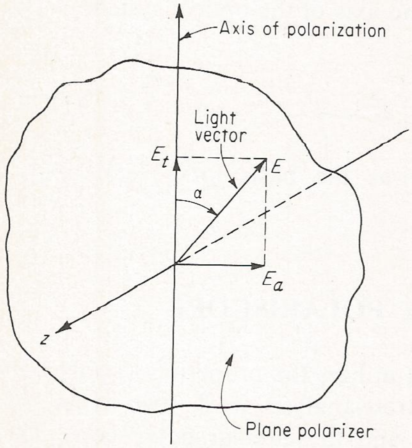

Figure 11.19 Absorbing and transmitting characteristics of a plane polarizer.

In the early days of photoelasticity the production of plane-polarized light was a difficult problem, and as a consequence a number of methods were employed, which included reflected light at a 57° angle of incidence, a glass pile, and the Nicol prism. However, these methods of producing plane-polarized light have been largely displaced with the advent of Polaroid filters, which have the advantage of providing a large field of very well polarized light at a relatively low cost. Most modern polariscopes containing linear polarizers employ Polaroid H sheet,† a transparent material with stained and oriented molecules. In the manufacture of H-type Polaroid films, a thin sheet of polyvinyl alcohol is heated, stretched, and immediately bonded to a supporting sheet of cellulose acetate butyrate. The polyvinyl face of the assembly is then stained by a liquid rich in iodine. The amount of iodine diffused into the sheet determines its quality, and the Polaroid Corporation produces three grades, denoted according to their transmittance of the light as HN-22, HN-32, and HN-38. Since the quality of a polarizer is judged by its transmission ratio, HN-22 (with a transmission ratio of the order of 10^5 at wavelengths normally employed in photoelasticity) is recommended for photoelastic purposes.

B. Wave Plates [3, 6, 7]

A wave plate has previously been defined as an optical element which has the ability to resolve a light vector into two orthogonal components and to transmit the components with different velocities. Such a material has been referred to as doubly refracting or birefringent. The doubly refracting plate illustrated in Fig. 11.20 has two principal axes labeled 1 and 2. The transmission of light along axis 1 proceeds at velocity c_1 and along axis 2 at velocity c_2. Since c_1 is greater than c_2, axis 1 is often called the *fast axis* and axis 2 the *slow axis*.

† Manufactured by the Polaroid Corporation, Cambridge, Mass.

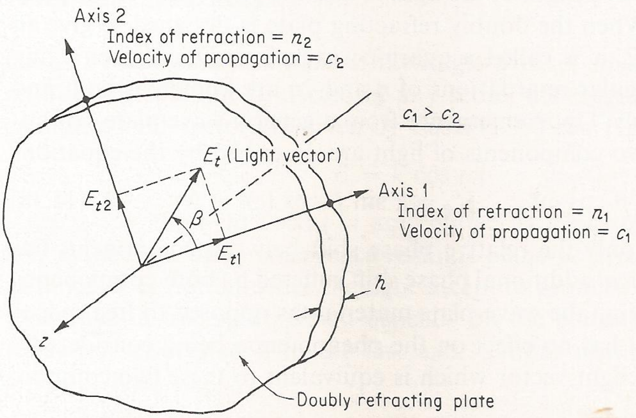

Figure 11.20 A plane-polarized light vector entering a doubly refracting plate.

If this doubly refracting plate is placed in a field of plane-polarized light so that the light vector E_t makes an angle β with axis 1 (the fast axis), then upon entering the plate the light vector is first resolved into two components E_{t1} and E_{t2} along axes 1 and 2, respectively. The magnitudes of the individual components E_{t1} and E_{t2} are given by

$$E_{t1} = E_t \cos \beta = a \cos \alpha \cos \omega t \cos \beta = k \cos \omega t \cos \beta$$

$$E_{t2} = E_t \sin \beta = a \cos \alpha \cos \omega t \sin \beta = k \cos \omega t \sin \beta$$

where $k = a \cos \alpha$. The light components E_{t1} and E_{t2} travel through the plate with different velocities c_1 and c_2, respectively. Because of this velocity difference, the two components will emerge from the plate at different times. In other words, one component is retarded in time relative to the other component. This retardation can be handled most effectively by considering the relative phase shift between the two components. From Eq. (11.18), the linear phase shifts for components E_{t1} and E_{t2} with respect to a wave in air can be expressed as

$$\delta_1 = h(n_1 - n) \qquad \delta_2 = h(n_2 - n)$$

where n is the index of refraction of air.

The relative linear phase shift is then computed simply as

$$\delta = \delta_2 - \delta_1 = h(n_2 - n_1) \tag{11.36}$$

The relative angular phase shift Δ between the two components as they emerge from the plate (recall from Sec. 11.2 that two mutually perpendicular components having the same frequency are equivalent to a rotating vector with angular frequency ω) is given by

$$\Delta = \frac{2\pi}{\lambda}\delta = \frac{2\pi h}{\lambda}(n_2 - n_1) \tag{11.37}$$

The relative phase shift Δ produced by a doubly refracting plate is dependent upon its thickness h, the wavelength of the light λ, and the properties of the plate as described by $n_2 - n_1$. When the doubly refracting plate is designed to give an angular retardation of $\pi/2$, it is called a quarter-wave plate. Doubly refracting plates designed to give angular retardations of π and 2π are known as half- and full-wave plates, respectively. Upon emergence from a general wave plate exhibiting a retardation Δ, the two components of light are described by the equations

$$E'_{t1} = k \cos \beta \cos \omega t \qquad E'_{t2} = k \sin \beta \cos (\omega t - \Delta) \tag{11.38}$$

With this representation, only the relative phase shift between components has been considered. The *identical* additional phase shift suffered by both components, as a result of passage through the wave-plate material (as opposed to free space), has been neglected since it has no effect on the phenomenon being considered.

The magnitude of the light vector which is equivalent to these two components can be expressed as

$$E'_t = \sqrt{(E'_{t1})^2 + (E'_{t2})^2} = k\sqrt{\cos^2 \beta \cos^2 \omega t + \sin^2 \beta \cos^2 (\omega t - \Delta)} \tag{11.39}$$

The angle that the emerging light vector makes with axis 1 (the fast axis) is given by

$$\tan \gamma = \frac{E'_{t2}}{E'_{t1}} = \frac{\cos(\omega t - \Delta)}{\cos \omega t} \tan \beta \tag{11.40}$$

Thus, it is clear that both the amplitude and the rotation of the emerging light vector can be controlled by the wave plate. Controlling factors are the relative phase difference Δ and the orientation angle β. Various combinations of Δ and β and their influence on the type of polarized light produced will be discussed in the next section.

Wave plates employed in a photoelastic polariscope may consist of a single plate of quartz or calcite cut parallel to the optic axis, a single plate of mica, a sheet of oriented cellophane, or a sheet of oriented polyvinyl alcohol. In recent years, as the design of the modern polariscope has tended toward a field of relatively large diameter, most wave plates employed have been fabricated from oriented sheets of polyvinyl alcohol. These wave plates are manufactured by the Polaroid Corporation by warming and unidirectionally stretching the sheet. Since the oriented polyvinyl alcohol sheet is only about 20 μm thick (for a quarter-wave plate), the commercial wave-plate filters are usually laminated between two sheets of cellulose acetate butyrate.

C. Conditioning of Light by a Series Combination of a Linear Polarizer and a Wave Plate

The magnitude and direction of the light vector emerging from a series combination of a linear polarizer and a wave plate are given by Eqs. (11.39) and (11.40). The light emerging from this combination of optical elements is always polarized; however, the type of polarization may be plane, circular, or elliptical. The factors which control the type of polarized light produced by this combination are the relative phase difference Δ imposed by the wave plate and the orientation angle β. Three well-defined cases exist.

Case 1: Plane-polarized light If the angle β is set equal to zero and the relative retardation Δ is not restricted in any sense, the magnitude and direction of the emerging light vector are given by Eqs. (11.39) and (11.40) as

$$E'_t = k \cos \omega t \qquad \gamma = 0$$

Since $\gamma = 0$, the light vector is not rotated as it passes through the wave plate; hence, the light upon emergence remains plane-polarized. The wave plate in this instance does not influence the light except to produce a retardation with respect to a wave in free space which depends on the plate thickness and the index of refraction associated with the fast axis. Similar results are obtained by letting $\beta = \pi/2$. Thus

$$E'_t = k \cos(\omega t - \Delta) \qquad \gamma = \frac{\pi}{2}$$

Case 2: Circularly polarized light If a wave plate is selected so that $\Delta = \pi/2$, that is, a quarter-wave plate, and β is set equal to $\pi/4$, the magnitude and direction of the light vector as it emerges from the plate are given by Eqs. (11.39) and (11.40) as

$$E_t' = \frac{\sqrt{2}}{2} k\sqrt{\cos^2 \omega t + \sin^2 \omega t} = \frac{\sqrt{2}}{2} k \qquad \gamma = \omega t$$

The light vector described by these expressions has a constant magnitude; therefore, the tip of the light vector traces out a circle as it rotates. The vector rotates with a constant angular velocity in a counterclockwise direction when viewed from a distant position along the path of propagation of the light beam. Such light is known as *left circularly polarized light. Right circularly polarized light* could be obtained by setting β equal to $3\pi/4$. The light vector would then rotate with a constant angular velocity in the clockwise direction.

Case 3: Elliptically polarized light If a quarter-wave plate ($\Delta = \pi/2$) is selected and β is permitted to be any angle other than $\beta = n\pi/4$ ($n = 0, 1, 2, 3, \ldots$), then by Eqs. (11.39) and (11.40), the magnitude and direction of the emerging light vector are

$$E_t' = k\sqrt{\cos^2 \beta \cos^2 \omega t + \sin^2 \beta \sin^2 \omega t} \qquad \tan \gamma = \tan \beta \tan \omega t$$

The light vector described by these expressions has a magnitude which varies with angular position in such a way that the tip of the light vector traces out an ellipse as it rotates. The shape and orientation of the ellipse and the direction of rotation of the light vector are controlled by the angle β.

Consider now the significance of Eq. (11.37) in the production of circularly polarized light

$$\Delta = \frac{2\pi h}{\lambda}(n_2 - n_1) \tag{11.37}$$

Recall that circularly polarized light requires the use of a quarter-wave plate; therefore, the phase difference Δ must be $\pi/2$. It is clear that the thickness h can be determined to give $\Delta = \pi/2$ once the plate material, $n_2 - n_1$, and the wavelength λ of the light are selected. However, a quarter-wave plate suitable for one wavelength of monochromatic light, i.e., a constant wavelength, will not be suitable for a different wavelength. Also, no quarter-wave plate can be designed for white light since it contains components of different wavelengths.

D. Arrangement of the Optical Elements in a Polariscope [8–12]

Plane polariscope The plane polariscope is the simplest optical system used in photoelasticity; it consists of two linear polarizers and a light source arranged as illustrated in Fig. 11.21*a*.

The linear polarizer nearest the light source is called the *polarizer*, while the second linear polarizer is known as the *analyzer*. In the plane polariscope the two axes of polarization are always crossed; hence no light is transmitted through the

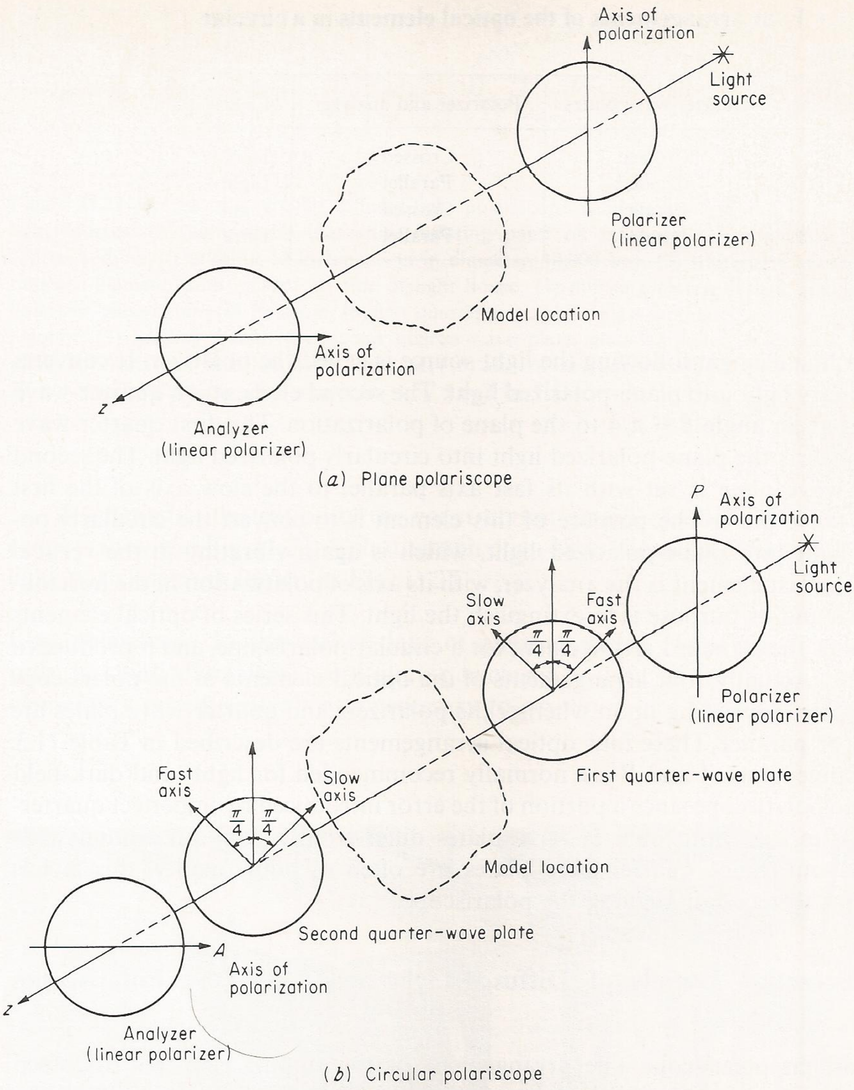

Figure 11.21 Arrangement of the optical elements in a plane polariscope and in a circular polariscope.

analyzer, and this optical system produces a dark field. In operation a photoelastic model is inserted between the two crossed elements and viewed through the analyzer. The behavior of the photoelastic model in a plane polariscope will be covered in Sec. 13.3.

Circular polariscope As the name implies, the circular polariscope employs circularly polarized light; consequently, the photoelastic apparatus contains four optical elements and a light source, which is illustrated in Fig. 11.21*b*.

Table 11.3 Four arrangements of the optical elements in a circular polariscope

Arrangement	Quarter-wave plates	Polarizer and analyzer	Field
A†	Crossed	Crossed	Dark
B	Crossed	Parallel	Light
C	Parallel	Crossed	Light
D	Parallel	Parallel	Dark

† Shown in Fig. 11.21.

The first element following the light source is called the polarizer. It converts the ordinary light into plane-polarized light. The second element is a quarter-wave plate set at an angle $\beta = \pi/4$ to the plane of polarization. This first quarter-wave plate converts the plane-polarized light into circularly polarized light. The second quarter-wave plate is set with its fast axis parallel to the slow axis of the first quarter-wave plate. The purpose of this element is to convert the circularly polarized light into plane-polarized light, which is again vibrating in the vertical plane. The last element is the analyzer, with its axis of polarization in the horizontal plane, and its purpose is to extinguish the light. This series of optical elements constitutes the standard arrangement for a circular polariscope, and it produces a dark field. Actually, four arrangements of the optical elements in the polariscope are possible, depending upon whether the polarizers and quarter-wave plates are crossed or parallel. These four optical arrangements are described in Table 11.3.

Arrangements *A* and *B* are normally recommended for light- and dark-field use of the polariscope since a portion of the error introduced by imperfect quarter-wave plates, i.e., both quarter-wave plates differ from $\pi/2$ by an amount ϵ, is canceled out. Since quarter-wave plates are often of poor quality, this fact is important to recall in aligning the polariscope.

E. Construction Details of Diffused-Light and Lens-Type Polariscopes [8–12]

Diffused-light polariscope The arrangement of the optical elements discussed previously is not sufficiently complete or detailed for the visualization of a working polariscope. The degree of complexity of a polariscope varies widely with the investigator and ranges from highly complex lens systems with servomotor drives on the four optical elements to very simple arrangements with no lenses and no provision for rotation of any element.

The diffused-light polariscope described here is one of the simplest and least expensive polariscopes available; yet it can be employed to produce very high quality photoelastic results. This polariscope requires only one lens; however, its field can be made very large since its diameter is dependent upon only the size of the available linear polarizers and quarter-wave plates. Actually, diffused-light polariscopes with field diameters up to 18 in (450 mm) can readily be constructed.

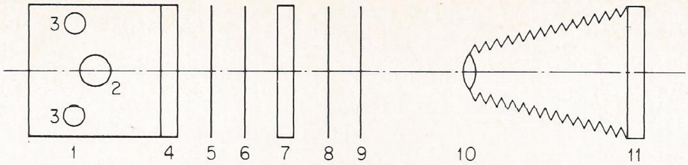

Figure 11.22 Design of a circular diffused-light polariscope with both white and monochromatic light sources: (1) light house (flat-white diffusing paint on interior): (2) monochromatic light source, sodium street lamp 12 in long, 3 in in diameter, 10,000 lm); (3) white-light source, 300 W tungsten-filament lamp located on side of light house; (4) diffusing plates, flashed opal glass; (5) polarizer, glass-laminated Polaroid; (6) first quarter-wave plate, glass-laminated orientated polyvinyl alcohol; (7) loading frame; (8) second quarter-wave plate, glass-laminated orientated polyvinyl alcohol; (9) analyzer glass-laminated Polaroid; (10) camera lens, good-quality process lens with about 20- to 24-in focal length; (11) camera for 4 by 5 or 5 by 7 film.

A schematic illustration of the construction details of a diffused-light polariscope is shown in Fig. 11.22. A photograph of an 18-in-diameter (450 mm) diffused-light polariscope is shown in Fig. 11.23.

Lens polariscope In the earlier days of photoelasticity, Nicol prisms (available only in small diameters) were almost exclusively used as the linear polarizing elements. Consequently, it was necessary to employ a lens system to expand the

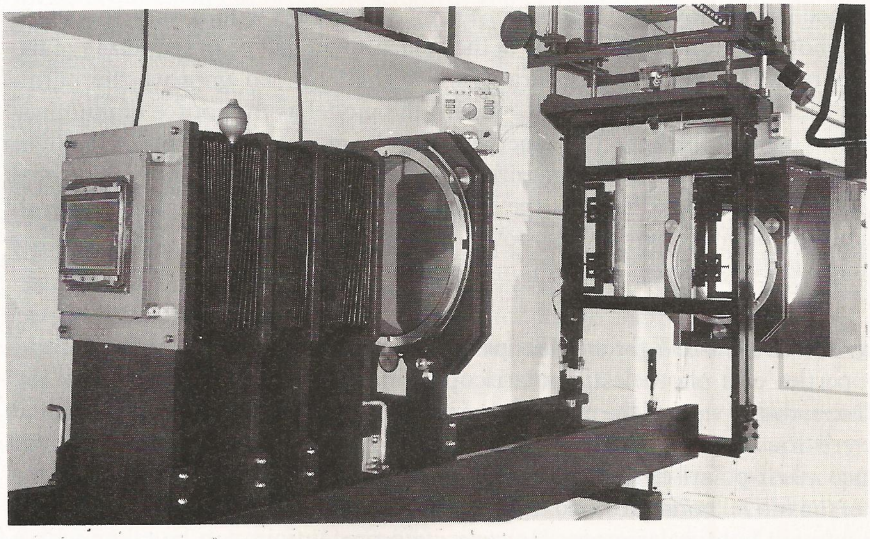

Figure 11.23 An 18-in-diameter diffused-light polariscope at IIT Research Institute; note that a portion of the polariscope has been suspended from the ceiling in order to open the working area behind the analyzer.

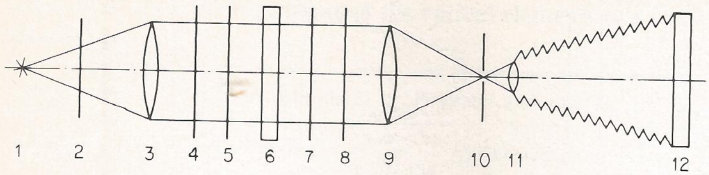

Figure 11.24 Construction details of a circular lens-type polariscope. (1) Light source (usually a small mercury arc), (2) color filter, (3) first field lens, (4) polarizer, (5) first quarter-wave plate, (6) loading frame and model, (7) second quarter-wave plate, (8) analyzer, (9) second field lens, (10) diaphragm stop, (11) camera lens, (12) camera back.

field of view so that reasonably sized models could be studied. However, with the advent of high-quality large-diameter sheets of Polaroid, it is no longer necessary to extend the diameter of the field through the use of a multiple-lens system. Instead, lens polariscopes should be employed only where parallel light over the whole field is a necessity. Instances where parallel light is important include applications where extremely precise definition of the entire model boundary is critical and where partial mirrors are to be employed in the photoelastic bench (for fringe sharpening and fringe multiplication).

Several variations of the lens systems are possible. The arrangement shown in Fig. 11.24 is one of the simplest types that can be employed to obtain parallel light. The polarizer, quarter-wave plates, and analyzer should be placed in the parallel beam between the two field lenses to achieve more complete polarization and to avoid problems associated with internal stresses in the field lenses. A point source of light is required for parallel light, and this point source is usually approached by employing a high-intensity mercury lamp with a very short arc. When a photoelastic model is placed in the field, a slight degree of scattering of the light occurs, which disturbs the parallelism of the light. To improve the parallelism of the light, it is appropriate to regard the model as the source of illumination and to control the light as it emerges from the model. The effects of the light scattered by the model can be minimized by placing a diaphragm stop at the focal point of the second field lens. As the diameter of the stop is reduced, the parallelism of the light is improved; however, the intensity of the light striking the camera back is decreased and film exposure times increase.

Comparison of diffused-light and lens polariscopes In this comparison the following properties of a photoelastic polariscope will be considered: definition of the boundaries, direct view of the model, light intensity, length of the unit, operational characteristics, and cost.

Since a lens polariscope employs a parallel beam of light, the definition of the image of the model boundaries on the camera back is sharper than that obtainable with a diffused-light polariscope. However, two precautions can be taken with a diffused-light polariscope to improve the image of the model boundaries. (1) A long-focal-length lens should be employed; moreover, it should be stopped down as far as possible. Results obtained by employing a lens with a focal length of 24 in (610 mm) stopped down to *f*: 45 are quite satisfactory in almost all applications. (2) If one region of the model is quite important (say a fillet or a hole), this region should be centered in the field of the polariscope. The width of the boundary shadow in a diffused-light polariscope is a function of the distance of the boundary from the centerline of the polariscope. Thus minimizing this distance for critical regions of the model, minimizes the boundary shadow where it may be detrimental.

For direct viewing of the model, the diffused-light polariscope is much more satisfactory than the lens polariscope. In a diffused-light polariscope the fringe pattern occurring in the model can be viewed by looking directly into the analyzer. Moreover, if the analyzer is advanced until it is quite close to the model, the investigator can load and align the model while viewing the pattern. With a lens polariscope the model fringe pattern cannot be viewed directly through the analyzer; instead, it is necessary to project the image on a ground-glass screen in a darkened room. This procedure is less suitable for rapid testing than that established with a diffusion polariscope.

Light intensity offers little or no difficulty for either polariscope provided they both are properly designed. High-intensity light sources and relatively fast lenses can be employed with both polariscopes to hold exposure times well below 1 min while using relatively slow high-contrast film.

The length of the polariscope is an item to be considered when laboratory space is limited. A lens polariscope is necessarily long, running from 10 to 15 ft (3 to 4.5 m), depending primarily upon the diameter of the field and the distance between the two field lenses. In a diffused-light polariscope the only component which requires any appreciable length is the camera. If the camera is eliminated, the system can be designed to occupy a length of less than 1 ft (0.3 m) and the image can be viewed directly. However, if a camera is required on a diffused-light polariscope, then a long-focal-length lens is necessary and the length of the camera will approach or exceed that of a lens polariscope.

The diffused-light polariscope is, in general, easier to operate than a lens polariscope, as the adjustments required are fewer and less precise. Another very important advantage of the diffused-light polariscope is that the surface finish of the model being tested does not require a high degree of polishing. The lens polariscope, on the other hand, has the advantage of a parallel beam of light which permits the utilization of partial mirrors for fringe sharpening and fringe multiplication. The fact that partial mirrors cannot be employed with a diffused-light polariscope is a serious disadvantage.

Finally, the cost of a large-field diffused-light polariscope is appreciably less than that of a large-field lens polariscope. The diffuser plate in a diffused-light polariscope has in effect replaced the lens system of a lens polariscope. As a consequence of the low cost of the diffuser plate in comparison with the expensive field lenses (i.e., the price of lenses increases roughly as the cube of their diameter), an appreciable savings in the purchase price of the diffused-light polariscope is effected.

11.7 OPTICAL INSTRUMENTS: THE INTERFEROMETER [1, 2, 13]

An interferometer is an optical device which can be used to measure lengths or changes in length with great accuracy by means of interference fringes. The modification of intensity of light by superposition of light waves was defined in Sec. 11.2 as an interference effect. The intensity of the wave resulting from the superposition of two waves of equal amplitude was shown by Eq. (11.13) to be a function of the linear phase difference δ between the waves. A fundamental requirement for the existence of well-defined interference fringes is that the light waves producing the fringes have a sharply defined phase difference which remains constant with time. When light beams from two independent sources are superimposed, interference fringes are not observed since the phase difference between the beams varies in a random way (the beams are incoherent). Two beams from the same source, on the other hand, interfere, since the individual wavetrains in the two beams have the same phase initially (the beams are coherent) and any difference in phase at the point of superposition results solely from differences in optical paths. Here optical-path length is defined as

$$\sum_{i=1}^{i=m} n_i L_i \qquad (a)$$

where L_i is the mechanical-path length in a material having an index of refraction n_i.

The concept of optical-path difference and its effect on the production of interference fringes can be illustrated by considering the reflection and refraction of light rays from a transparent plate having a thickness h, as shown in Fig. 11.25. Consider a plane wavefront associated with the light ray A which strikes the plate at an angle of incidence α. Ray B results from reflection at the front surface of the plate and, as discussed in Sec. 11.3, suffers a phase change of $\lambda/2$. A second ray is refracted at the front surface, reflected at the back surface, and refracted from the

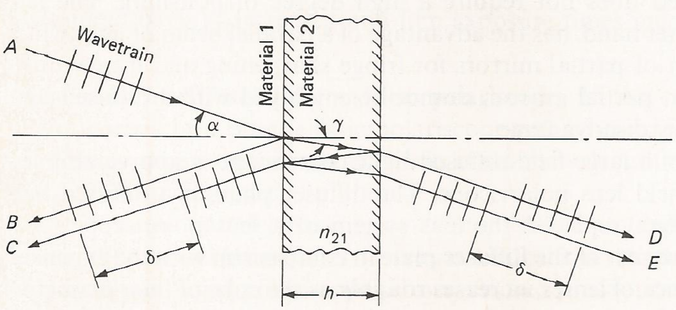

Figure 11.25 Reflection and refraction of light rays from a transparent plate ($n_2 > n_1$).

front surface before emerging from the front surface of the plate as ray C. The optical-path difference between rays B and C can be computed as

$$\delta = \frac{2h}{\cos\gamma} n_{21} - \frac{2h}{\cos\gamma} \sin\gamma \sin\alpha$$

But from Eq. (11.20)

$$\sin\alpha = n_{21} \sin\gamma$$

therefore
$$\delta = \frac{2h}{\cos\gamma} n_{21} - \frac{2h}{\cos\gamma} n_{21} \sin^2\gamma = 2hn_{21}\cos\gamma \tag{11.41}$$

Since ray B suffered a phase change of $\lambda/2$ on reflection, rays B and C will interfere destructively (produce minimum intensity or extinction when brought together) whenever

$$\delta = m\pi \qquad m = 0, 1, 2, 3, \ldots$$

If the light beam illuminates an extended area of the plate, and if the thickness of the plate varies slightly with position, the locus of points experiencing the same order of extinction will combine to form an interference fringe. The fringe spacings will represent thickness variations of approximately 7 μin or 180 nm (in glass with mercury light and a small angle γ).

Rays emerging from the back surface of the plate can also be used to produce interference effects. In Fig. 11.25, a third ray is refracted at both the front and back surfaces of the plate before emerging as ray D. A fourth ray undergoes two internal reflections before being refracted from the back surface of the plate as ray E. The optical-path difference between rays D and E is identical to the difference between rays B and C as given by Eq. (11.41). Since neither ray D nor E suffers a phase change on reflection, the two rays will interfere destructively when brought together whenever

$$\delta = (2m + 1)\frac{\lambda}{2} \qquad m = 0, 1, 2, 3, \ldots$$

The previous discussion serves to illustrate the principles associated with measurements employing interference effects. For the system illustrated in Fig. 11.25, the optical-path difference δ would be many wavelengths ($m \to \infty$). Such a system would involve high-order interference; therefore, it would require extreme coherence and long wavetrains such as those provided by a laser for successful operation. Other systems which utilize low-order interference ($m = 0, 1, 2, 3$) place less stringent requirements on the light source. Two low-order systems used in experimental stress analysis work are the Mach-Zehnder interferometer and the series interferometer.

A. Mach-Zehnder Interferometer

The essential features of a Mach-Zehnder interferometer are illustrated in Fig. 11.26. The light beam from a source is divided into a reference beam and an

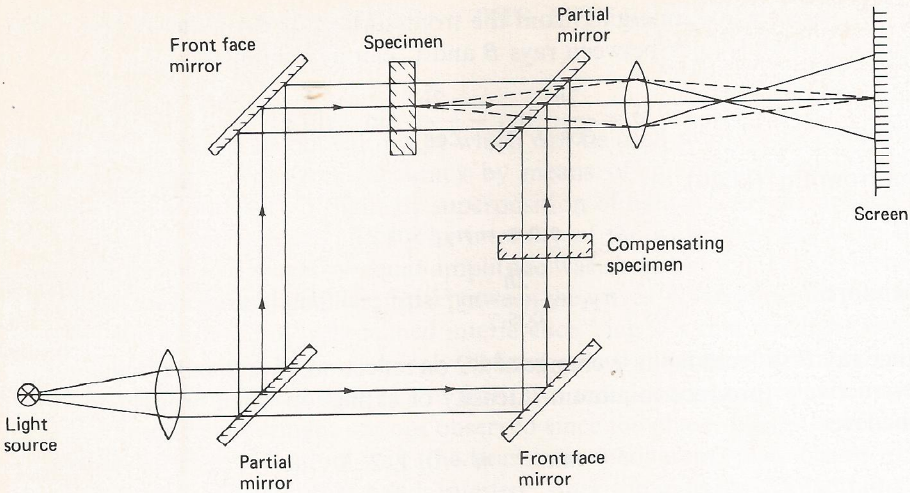

Figure 11.26 Light paths through a Mach-Zehnder interferometer.

active beam with a beam splitter (partial mirror). The beams are recombined after the active beam passes through the specimen of interest. Low-order interference can be obtained by inserting a compensating specimen in the reference beam to adjust for the difference in index of refraction between the specimen material and air.

B. Series Interferometer [13]

The essential features of the series interferometer are illustrated in Fig. 11.27. This instrument, developed by Post [13], is a relatively simple and stable instrument with a large field. It is well suited for photoelasticity applications. The series interferometer utilizes three partial mirrors in series. A fraction of the incident light is reflected and a fraction is transmitted at each of the mirrors. One portion of the light is transmitted directly through the mirrors, as depicted by ray *A* in Fig. 11.27. Other portions (see rays *B* and *C*) are characterized by only two reflections. A majority of the light, however, undergoes a variety of multiple reflections, as illustrated by ray *D*. When the optical path l_1 is nearly equal to the optical path l_2, rays which traverse paths similar to *B* and *C* interfere constructively and destructively to form an interference fringe pattern. This low-order fringe pattern gives the difference between l_1 and l_2 at any point in the field. Superimposed on this pattern is a uniform background intensity due to all the other rays transmitted through the three series mirrors. When a specimen is placed in the field between mirrors 1 and 2 and the optical path l_2 is adjusted to approximately equal l_1, a fringe pattern related to thickness variations in the model is obtained.

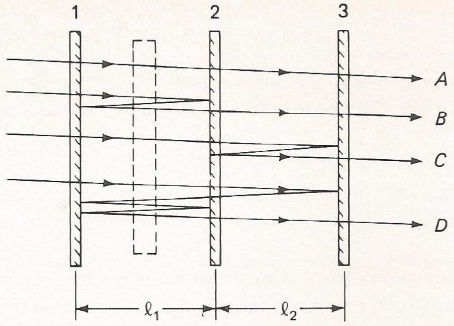

Figure 11.27 Typical forms of reflection as light passes through three partial mirrors in a series interferometer.

EXERCISES

11.1 Verify Eqs. (11.11) and (11.12).

11.2 The wavelength of light from a helium-neon laser is 632.8 nm. Determine:
(*a*) The frequency of this light
(*b*) The wavelength of this light in a glass plate ($n = 1.522$)
(*c*) The velocity of propagation in the glass plate
(*d*) The linear phase shift (in terms of wavelength in free space) after the light has passed through a 25-mm-thick glass plate

11.3 A plate of glass having an index of refraction of 1.57 with respect to air is to be used as a polarizer. Determine the polarizing angle and the angle of refraction of the transmitted ray.

11.4 Unpolarized light is directed onto a plane glass surface ($n = 1.57$) at an angle of incidence of 50°. Determine reflection coefficients associated with the parallel and perpendicular components of the reflected beam.

11.5 A light source is located 15 m below the surface of a body of water ($n = 1.33$). Determine the maximum distance (measured from a point directly above the source) at which the source will be visible from the air side of the air-water interface.

11.6 A ray of monochromatic light is directed at oblique incidence onto the surface of a glass plate. The ray emerges from the opposite side of the plate in a direction parallel to its initial direction but with a transverse displacement. Develop an expression for this transverse displacement in terms of the plate thickness h, the index of refraction of the glass n, and the angle of incidence α of the light beam.

11.7 The radius of a concave spherical mirror is 1000 mm. An object is located 1500 mm from the mirror. Determine the image location and the magnification. Show the results on a sketch similar to Fig. 11.13.

11.8 Solve Exercise 11.7 if the object is located 750 mm from the mirror.

11.9 Solve Exercise 11.7 if the object is located 200 mm from the mirror.

11.10 A concave mirror will be used to focus the image of an object onto a screen 1.50 m from the object. If a magnification of -2 is required, what radius of curvature must the mirror have?

11.11 Determine the image location and the magnification for an object located 750 mm from a mirror if the mirror is (*a*) a plane mirror and (*b*) a convex mirror with a radius of curvature of 1000 mm.

11.12 A convex mirror has a radius of curvature of 2500 mm. Determine the magnification and the image location for an object located 500 mm from the mirror. Show the results on a sketch similar to Fig. 11.13.

11.13 A thin convex lens has a focal length of 600 mm. An object is located 1200 mm to the left of the lens. Determine the image location and the magnification. Show the results on a sketch similar to Fig. 11.14.