CHAPTER

EIGHT

STRAIN-GAGE CIRCUITS

8.1 INTRODUCTION

An electrical-resistance strain gage will change in resistance due to applied strain according to Eq. (6.5), which indicates

$$\frac{\Delta R}{R} = S_g \epsilon_{xx} \tag{6.5}$$

where the gage axis coincides with the x axis and $\epsilon_{yy} = -0.285\epsilon_{xx}$. In order to apply the electrical-resistance strain gage in any experimental stress analysis, the quantity $\Delta R/R$ must be measured and converted to the strain which produced the resistance change. Two electrical circuits, the potentiometer and the Wheatstone bridge, are commonly employed to convert the value of $\Delta R/R$ to a voltage signal (denoted here as ΔE) which can be measured with a recording instrument.

In this chapter the basic theory for the two circuits is presented, and the circuit sensitivities and ranges are derived. In addition, temperature compensation, signal addition, and loading effects are discussed in detail. Also covered are the constant-current circuits recommended for semiconductor strain gages. Methods of calibrating both the potentiometer and the Wheatstone bridge circuits are covered, and the effects of lead wires on noise, calibration, and temperature compensation are discussed.

Insofar as possible, this discussion has been kept fundamental without appreciable reference to commercially available circuits. The material presented here is applicable to all commercial circuits since their design is based on the fundamental principles covered in this chapter.

The methods of measuring the circuit output voltages ΔE are discussed in Chap. 9.

8.2 THE POTENTIOMETER AND ITS APPLICATION TO STRAIN MEASUREMENT [1]

The potentiometer circuit, which is often employed in dynamic strain-gage applications to convert the gage output $\Delta R/R$ to a voltage signal ΔE, is shown in Fig. 8.1. For fixed-value resistors R_1 and R_2 in the circuit, the open-circuit output voltage E can be expressed as

$$E = \frac{R_1}{R_1 + R_2} V = \frac{1}{1 + r} V \tag{8.1}$$

where V is the input voltage and $r = R_2/R_1$ is the resistance ratio for the circuit. If incremental changes ΔR_1 and ΔR_2 occur in the value of the resistors R_1 and R_2, the change ΔE of the output voltage E can be computed by using Eq. (8.1) as follows:

$$E + \Delta E = \frac{R_1 + \Delta R_1}{R_1 + \Delta R_1 + R_2 + \Delta R_2} V \tag{a}$$

Solving Eq. (a) for ΔE gives

$$\Delta E = \left(\frac{R_1 + \Delta R_1}{R_1 + \Delta R_1 + R_2 + \Delta R_2} - \frac{R_1}{R_1 + R_2} \right) V \tag{b}$$

which can be expressed in the following form by introducing $r = R_2/R_1$:

$$\Delta E = \frac{[r/(1 + r)^2](\Delta R_1/R_1 - \Delta R_2/R_2)}{1 + [1/(1 + r)][\Delta R_1/R_1 + r(\Delta R_2/R_2)]} V \tag{8.2}$$

Examination of Eq. (8.2) shows that the voltage signal ΔE from the potentiometer

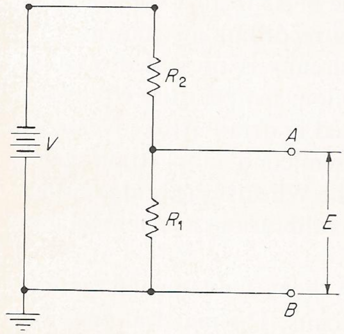


Figure 8.1 The potentiometer circuit.

circuit is a nonlinear function of $\Delta R_1/R_1$ and $\Delta R_2/R_2$. To inspect the nonlinear aspects of this circuit further, it is possible to rewrite Eq. (8.2) in the form

$$\Delta E = \frac{r}{(1+r)^2}\left(\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2}\right)(1-\eta)\,V \tag{8.3a}$$

where the nonlinear term η is expressed as

$$\eta = 1 - \frac{1}{1 + [1/(1+r)][\Delta R_1/R_1 + r(\Delta R_2/R_2)]} \tag{8.3b}$$

Equations (8.3) are the basic relationships which govern the behavior of the potentiometer circuit, and as such they can be used to establish the applicability of this circuit for strain-gage measurements.

A. Range and Sensitivity of the Potentiometer Circuit

The range of the potentiometer circuit is effectively established by the nonlinear term η, as expressed in Eq. (8.3b). If a strain gage of resistance R_g is used in the position of R_1, and if a fixed-ballast resistor R_b is employed in the position of R_2, then

$$R_1 = R_g \qquad \Delta R_1 = \Delta R_g \qquad R_2 = R_b \qquad \Delta R_2 = 0$$

and Eq. (8.3b) becomes

$$\eta = 1 - \frac{1}{1 + [1/(1+r)](\Delta R_g/R_g)} \tag{a}$$

By substituting Eq. (6.5) into Eq. (a), the nonlinear term is given in terms of strain ϵ, the gage factor S_g, and the resistance ratio $r = R_b/R_g$ as

$$\eta = 1 - \frac{1}{1 + [1/(1+r)]S_g\epsilon} \tag{b}$$

This expression can be evaluated by means of a series expansion where

$$\eta = 1 - \frac{1}{1+x} = x - x^2 + x^3 - x^4 + \cdots$$
$$x = \frac{1}{1+r} S_g \epsilon \tag{8.4}$$

The range of the potentiometer circuit depends upon the error which can be tolerated due to the nonlinearities introduced by the circuit. In Fig. 8.2, the nonlinear term η is given as a function of strain for various values of r measured with a metallic strain gage with a gage factor S_g equal to 2. Inspection of this figure shows that the error introduced by the nonlinearity of the circuit is sufficiently small to be negligible for normal strain-gage measurements of the order of a few thousand microinches per inch (micrometers per meter). The error does, however, approach

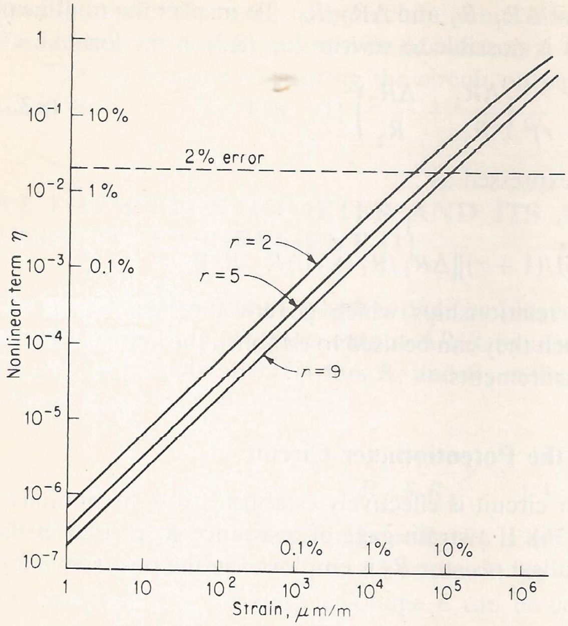


Figure 8.2 Dependence of the nonlinear term η on the strain level.

2 percent for strains between 1 and 10 percent, depending upon the value of the resistance ratio r. It is clear that increasing the value of r decreases the error due to circuit nonlinearities. The range of the potentiometer circuit, based on an allowable error of 2 percent, varies from 10 to 2 percent strain as r varies from a value of 9 to 2. Obviously the range of the potentiometer circuit is sufficient for determining elastic strains in metallic components. However, elastic-strain determinations in plastic materials and plastic-strain determinations in metallic materials may require corrections of the output signal.

The sensitivity of the potentiometer circuit is defined as the ratio of the output voltage divided by the strain:

$$S_c = \frac{\Delta E}{\epsilon} \tag{8.5}$$

It is apparent that the relative merit of a circuit can be judged by the magnitude of the circuit sensitivity S_c. In particular for dynamic applications, where relatively small strains are being measured, it is quite important that S_c be made as large as possible to reduce the degree of amplification necessary to read the output signal.

The sensitivity of the potentiometer circuit can be established by substituting Eq. (8.3a) into Eq. (8.5) to obtain

$$S_c = \frac{r}{(1+r)^2}\left(\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2}\right)\frac{V}{\epsilon} \tag{8.6}$$

where $\eta \ll 1$.

If a strain gage is placed in the circuit in place of R_1 and a ballast resistor is employed for R_2, then

$$R_g = R_1 \qquad \Delta R_g = \Delta R_1 \qquad R_b = R_2 \qquad \Delta R_2 = 0$$

and by Eqs. (6.5) and (8.6) the circuit sensitivity becomes

$$S_c = \frac{r}{(1+r)^2} S_g V \tag{c}$$

This equation indicates that the circuit sensitivity is controlled by the resistance ratio r and the circuit voltage V. However, if V can be varied, the limiting factor becomes the power which can be dissipated by the gage. This power must be held within the limits set forth in Sec. 6.6E. To account for the influence of the power P_g dissipated by the gage, the relationship between P_g and V given by Eq. (6.18) is applied to the potentiometer circuit to give

$$V = I_g R_g(1+r) = \sqrt{P_g R_g}(1+r) \tag{8.7}$$

Substituting Eq. (8.7) into Eq. (c) yields

$$S_c = \frac{r}{1+r} S_g \sqrt{R_g P_g} \tag{8.8}$$

This equation shows that the circuit sensitivity is controlled by two independent parameters, $r/(1+r)$ and $S_g\sqrt{R_g P_g}$. The first parameter $r/(1+r)$ is related to the circuit and is dictated by the selection of R_b, which fixes the value of r. It is clear that as r becomes very large, $r/(1+r)$ approaches 1 and maximum circuit efficiency is obtained. However, for very high values of r, the voltage required to drive the potentiometer becomes excessive. Values of r of about 9 are commonly used, which gives a circuit efficiency of approximately 90 percent. The circuit sensitivity is strongly influenced by the second parameter $S_g\sqrt{R_g P_g}$, which is determined entirely by the selection of the strain gage and is totally independent of the elements used in the design of the potentiometer circuit. In fact, if very high circuit sensitivities are required, much can be gained by careful gage selection, since $S_g\sqrt{R_g P_g}$ can be varied over wide limits, i.e., from 3 to about 700, while circuit efficiency can be varied over a much more limited range ($\frac{1}{2}$ to about 1).

The output voltage from any circuit containing a strain gage is inherently low, and as a consequence S_c is quite often as low as 5 or 10 μV per (μin/in) (μm/m) of strain. Exercise 8.3 shows typical values of S_c as well as the voltage required to drive a potentiometer circuit with a circuit efficiency of 90 percent.

B. Temperature Compensation and Signal Addition in the Potentiometer Circuit

It is possible to effect a certain degree of temperature compensation in the strain-measuring system by employing two strain gages in the potentiometer circuit. The gage used in place of R_1 is known as the *active strain gage* and is used to measure the strain at a given point and orientation on a specimen. The gage used to replace R_2 is known as the *dummy gage* and is mounted on a small block of material identical to that of the specimen and is exposed to the same thermal environment as the active gage. In the active gage the total change in resistance will be due to strain ΔR_ϵ and to changes in temperature ΔR_T. Hence,

$$\frac{\Delta R_1}{R_1} = \frac{\Delta R_\epsilon}{R_1} + \frac{\Delta R_T}{R_1} \tag{d}$$

However, in the dummy gage the change in resistance is due to a change in temperature alone; hence

$$\frac{\Delta R_2}{R_2} = \frac{\Delta R_T}{R_2} \tag{e}$$

Substituting Eqs. (*d*) and (*e*) into Eq. (8.3*a*) give

$$\Delta E = \frac{r}{(1+r)^2}\left(\frac{\Delta R_\epsilon}{R_1} + \frac{\Delta R_T}{R_1} - \frac{\Delta R_T}{R_2}\right)(1-\eta)\,V \tag{8.9}$$

Since $R_1 = R_2 = R_g$, it is evident that the ΔR_T terms cancel out of Eq. (8.9) and the output signal ΔE is due to the ΔR_ϵ term alone.

This type of temperature compensation is effective if the dummy and the active gages are from the same lot of gages, if the materials upon which each gage is mounted are identical, and if the temperature changes due to ambient conditions and gage currents on both gages are equal. If any one of these three conditions is violated, complete temperature compensation cannot be achieved.

A loss in circuit efficiency, as determined by $r/(1 + r)$, is the price which must be paid for temperature compensation. Since $R_1 = R_2 = R_g$ is a requirement for temperature compensation, $r = 1$, and the circuit efficiency is fixed at 50 percent. If circuit sensitivity is extremely important, temperature compensation by this means should not be specified, since temperature-compensated gages (discussed in Sec. 6.6B) can be employed to accomplish the same purpose. Moreover, the potentiometer circuit can be used only to record dynamic strains, and very little time is available during the readout period for the ambient temperature to change.

Resistance changes from two active strain gages placed in positions R_1 and R_2 can be used to increase the output signal in the potentiometer circuit in certain cases. As an example, consider the application shown in Fig. 8.3, where two gages

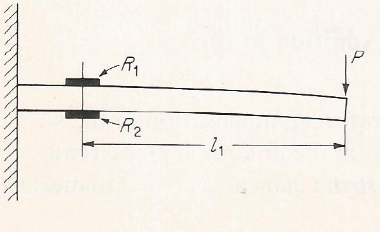


Figure 8.3 Strain gages mounted on the top and bottom surfaces of a cantilever beam.

are mounted on a beam in bending. The gage mounted on the top surface of the beam will exhibit a resistance change which can be indicated by

$$\frac{\Delta R_1}{R_1} = S_g f(P) \tag{f}$$

and the gage mounted on the bottom surface of the beam will exhibit a negative resistance change of

$$\frac{\Delta R_2}{R_2} = -S_g f(P) \qquad \text{where} \qquad f(P) = \frac{6Pl_1}{bh^2E} = \epsilon \tag{g}$$

The signal output from the potentiometer circuit containing these two active gages can be computed from Eqs. (8.3) as

$$\Delta E = \frac{r}{(1+r)^2}(2S_g\epsilon)V \tag{h}$$

which can be reduced to

$$\Delta E = \frac{r}{1+r}2S_g\sqrt{R_gP_g}\,\epsilon = S_g\sqrt{R_gP_g}\,\epsilon \tag{8.10}$$

where $r = 1$ since $R_1 = R_2 = R_g$. If one active gage had been employed, the relation for ΔE would have been

$$\Delta E = \frac{r}{1+r}S_g\sqrt{R_gP_g}\,\epsilon \tag{i}$$

Comparison of Eq. (8.10) and Eq. (i) indicates that the output signal has been doubled, based on a value of $r = 1$. However, if r is increased to, say, 9 in Eq. (i), the advantage of using two gages to increase circuit sensitivity is almost completely nullified.

Multiple-gage circuits are more important in transducer applications of strain gages, where the signals due to unwanted components of load are canceled in the circuit. This application is illustrated in Sec. 8.8.

C. Potentiometer Output

The output of a potentiometer circuit is measured across terminals A and B, as indicated in Fig. 8.1, to give a total voltage $E + \Delta E$. The output of a strain gage mounted to a specimen with a sinusoidally varying strain field is shown in Fig. 8.4.

Normally the voltage E is of the order of a few volts, say 2 to 10, and ΔE is measured in terms of microvolts. The voltage which is related to the strain is ΔE, and this quantity must be measured independently of the large superimposed voltage E. Unfortunately, voltage-measuring instruments are not usually available with sufficient range and sensitivity to measure ΔE when it is superimposed on E. Thus, the potentiometer circuit is inadequate for static strain-gage applications where both E and ΔE are constant with respect to time. The potentiometer circuit

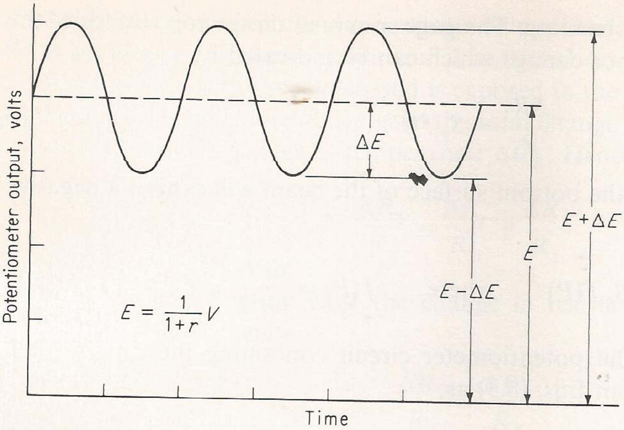


Figure 8.4 Potentiometer output as a function of time for a sinusoidally varying strain.

is quite useful, however, in dynamic applications where ΔE varies with time and E is a constant. In these applications it is possible to block the dc voltage E by the use of a suitable filter while passing the voltage ΔE without significant distortion. The filter serves to reduce the required range of the measuring instrument from $E + \Delta E$ to ΔE and permits the use of highly sensitive but limited-range measuring instruments. A typical filter used with a potentiometer circuit is illustrated in Fig. 8.5.

If R_M is large in comparison with $R_1 R_2/(R_1 + R_2)$, the voltage E' measured across the resistance of the measuring instrument R_M is

$$\frac{E'}{E + \Delta E} = \frac{R_M}{\sqrt{R_M^2 + 1/\omega^2 C^2}} \tag{8.11}$$

where C is the capacitance used in the filter and ω is the angular frequency of the voltage $E + \Delta E$. If one considers that the voltage $E + \Delta E$ is made up of many frequencies varying from zero (direct current) to several thousand hertz, it is

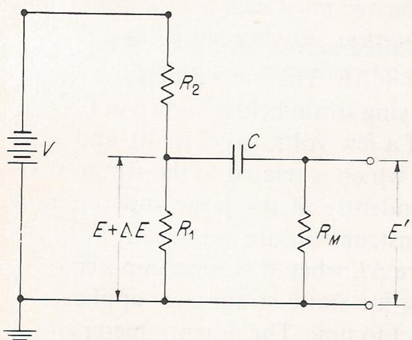


Figure 8.5 Filter for blocking out the dc component of the potentiometer-circuit output.

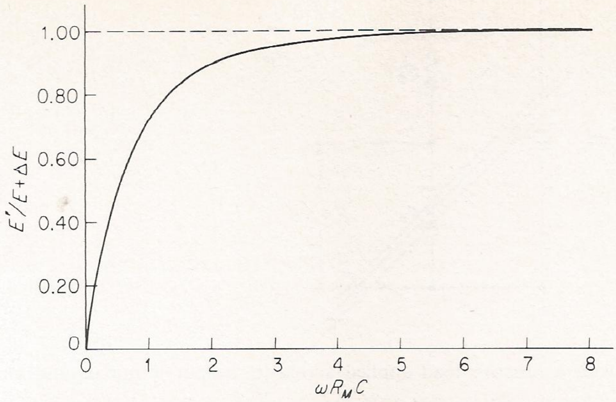


Figure 8.6 Typical response curve for a high-pass filter.

possible to plot $E'/(E + \Delta E)$ as a function of ω, as shown in Fig. 8.6. The student is referred to Exercise 8.6, where the validity of Fig. 8.6 is established.

Since the E component of the potentiometer output voltage $E + \Delta E$ is constant with time (that is, $\omega = 0$), this dc component of the signal is entirely blocked by the filter. If $R_M = 1\ \text{M}\Omega$ and $C = 0.1\ \mu\text{F}$, the RC constant for the filter is 10^{-1}, which indicates that frequency components associated with the ΔE voltage will pass through the filter with negligible distortion provided they exceed 30 rad/s or about 5 Hz. This filter arrangement can then be employed to successfully adapt the potentiometer circuit to dynamic strain-gage applications whenever the dynamic strains are composed of frequency components which exceed about 5 Hz.

D. Load Effects on the Potentiometer Circuit

In the derivation of the output voltage obtained from the potentiometer circuit given in Sec. 8.2, the effect of the resistance of the voltage-measuring instrument was neglected. That is to say, the measuring instrument was considered to have an infinite resistance. In practice, however, the voltage-measuring instrument does have a finite resistance, and this resistance can influence the output voltage of the potentiometer circuit since some of the current which normally passes through the gage will be diverted through the measuring instrument.

To show the effect of the resistance or, more correctly, the impedance of the measuring instrument, consider the circuit shown in Fig. 8.7, where a single active gage is used. This circuit can be reduced to an equivalent circuit with an equivalent resistor R_e replacing resistors R_M and R_g, as is also indicated in Fig. 8.7. If R_b is considered as a fixed-value ballast resistor and the nonlinear term η is much less than 1, then Eqs. (8.3) may be rewritten as

$$\Delta E\bigg|_{RM} = \frac{r}{(1+r)^2}\frac{\Delta R_e}{R_e}V = \frac{R_b R_e}{(R_e + R_b)^2}\frac{\Delta R_e}{R_e}V \tag{8.12}$$

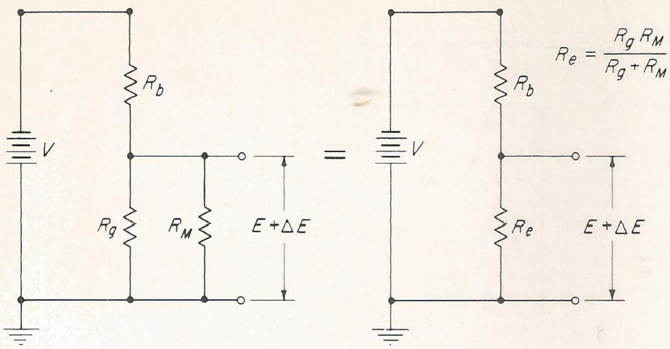


Figure 8.7 Potentiometer circuit with a resistive load applied across its output terminals and an equivalent circuit.

This value must be compared with the open-circuit (i.e., infinite resistance for the measuring instrument) output voltage of

$$\Delta E \bigg|_{R_M=\infty} = \frac{R_b R_g}{(R_g + R_b)^2} \frac{\Delta R_g}{R_g} V \tag{8.13}$$

To show the difference between these two equations, consider the expansion of the terms $R_b R_e/(R_e + R_b)^2$ and $\Delta R_e/R_e$ in terms of R_b, R_g, and R_M. By expanding $R_b R_e/(R_e + R_b)^2$, it is possible to show by using

$$R_e = \frac{R_g R_M}{R_g + R_M} \tag{j}$$

that

$$\frac{R_b R_e}{(R_e + R_b)^2} = \frac{R_b R_g(R_g/R_M + 1)}{[R_g + R_b(R_g/R_M) + R_b]^2} \tag{k}$$

Next, if $\Delta R_e/R_e$ is expanded, one obtains

$$\frac{\Delta R_e}{R_e} = \frac{(R_g + \Delta R_g)/(R_g + \Delta R_g + R_M) - R_g/(R_g + R_M)}{R_g/(R_g + R_M)}$$

$$= \frac{1}{R_g/R_M + 1} \frac{\Delta R_g}{R_g} \tag{l}$$

Substituting Eqs. (*k*) and (*l*) into Eq. (8.12) gives

$$\Delta E \bigg|_{R_M} = \frac{R_b R_g}{[R_g + R_b(R_g/R_M) + R_b]^2} \frac{\Delta R_g}{R_g} V \tag{8.14}$$

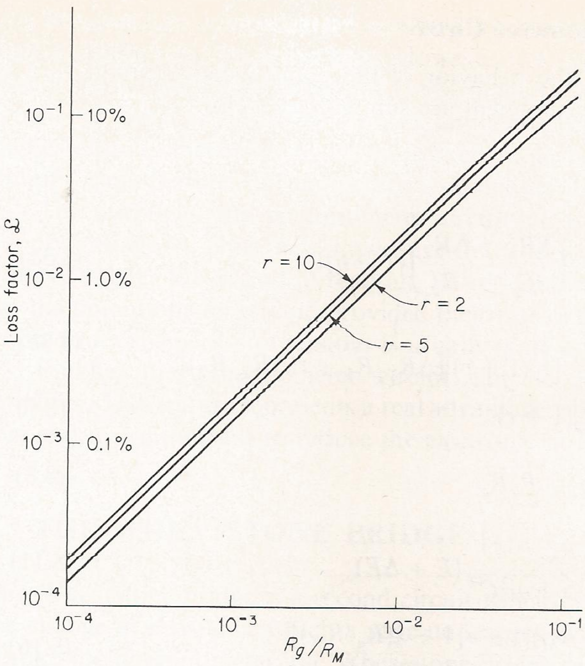


Figure 8.8 Loss factor $\mathscr{L}$ as a function of R_g/R_M.

If $\Delta E|_{R_M}$ is compared with $\Delta E|_{R_M=\infty}$ in the following manner:

$$\Delta E\Big|_{R_M} = \Delta E\Big|_{R_M=\infty}(1 - \mathscr{L}) \tag{8.15}$$

it is possible to determine an expression for $\mathscr{L}$ which represents the loss in voltage output ΔE due to the resistive load R_M.

Substituting Eqs. (8.13) and (8.14) into Eq. (8.15) and solving for $\mathscr{L}$, while noting that $r = R_b/R_g$, gives

$$\mathscr{L} = \frac{2r(R_g/R_M)\{1 + r[1 + \frac{1}{2}(R_g/R_M)]\}}{[1 + r(1 + R_g/R_M)]^2} \tag{8.16}$$

The loss in the output voltage, as expressed by the loss factor $\mathscr{L}$, is dependent on the resistance ratios R_b/R_g and R_g/R_M. The loss factor $\mathscr{L}$ is shown as a function of R_g/R_M for various values of R_b/R_g in Fig. 8.8. It is clear from the results shown in this figure that the loss in sensitivity is less than 2 percent if the resistance of the measuring instrument is about 100 times that of the gage resistance. Thus, if a 120-Ω strain gage is employed in the potentiometer circuit, the measuring instrument should have an input resistance of 12 kΩ. This value of resistance is not high for most measuring instruments; in fact, most cathode-ray oscilloscopes, which are frequently used with a potentiometer circuit, have input resistances of the order of 1 MΩ.

E. Summary of the Potentiometer Circuit

The equations which govern the behavior of the potentiometer circuit as it is applied to dynamic strain measurement are summarized below:

$$E = \frac{1}{1+r} V \tag{8.1}$$

$$\Delta E = \frac{r}{(1+r)^2}\left(\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2}\right)(1-\eta)V \tag{8.3a}$$

$$\eta = 1 - \frac{1}{1 + [1/(1+r)][\Delta R_1/R_1 + r(\Delta R_2/R_2)]} \tag{8.3b}$$

$$V = \sqrt{P_g R_g}(1+r) \tag{8.7}$$

$$S_c = \frac{r}{1+r} S_g \sqrt{P_g R_g} \tag{8.8}$$

$$E' = \frac{R_M}{\sqrt{R_M^2 + 1/\omega^2 C^2}}(E + \Delta E) \tag{8.11}$$

$$\mathscr{L} = \frac{2r(R_g/R_M)\{1 + r[1 + \frac{1}{2}(R_g/R_M)]\}}{[1 + r(1 + R_g/R_M)]^2} \tag{8.16}$$

The potentiometer circuit cannot be employed to measure the resistance change of a gage due to static strains because of the voltage E, which appears across its output terminals and swamps out the small voltage ΔE which is related to the resistance change. The circuit can, however, be employed to measure the resistance change in the gage due to dynamic strains if a suitable filter is used to block out the dc voltage E while passing without distortion the voltage pulse ΔE.

For most strain-gage applications the range over which the potentiometer circuit can respond is larger than the strains which are to be measured. However, for more specialized problems where strains of about 10 percent are to be measured, corrections must be made to account for nonlinearities introduced by the circuit.

The sensitivity of a potentiometer measuring system is controlled by the circuit efficiency given by $r/(1+r)$ and the gage selection which dictates the value of $S_g\sqrt{P_g R_g}$. Of the two factors, the gage selection is more important because of its greater variability (that is, 3 to about 700). The circuit efficiency can usually be maintained at about 90 percent by selection of $R_b = 9R_g$ and the provision of a sufficiently high voltage V to drive the allowable current through the gage.

The potentiometer circuit can be used to add or subtract signals from a multiple-strain-gage installation. However, the gain in circuit sensitivity by the addition of two strain-gage signals is usually not sufficient to warrant the use of two gages in place of one. For transducer applications, multiple-gage circuits should be considered to eliminate gage response from components (of, say, the load) which are not sought in the measurement.

Temperature compensation can be introduced into the circuit by employing both a dummy and an active gage. However, the circuit efficiency is reduced to 50 percent in this circuit arrangement. If low-magnitude strains are to be measured, it is usually more advisable to employ temperature-compensated gages and seek higher circuit efficiencies by using a fixed-value ballast resistor in place of the dummy gage required for temperature compensation within the circuit.

The output of the potentiometer circuit ΔE is directly proportional to the supply voltage V. Thus, it is imperative that the voltage supply provide a stable voltage V over the period of readout. Batteries serve as excellent power supplies for the potentiometer circuit, provided their rate of decay is small over the period of readout (this period of readout is usually quite short in dynamic applications).

Finally, the potentiometer circuit can be grounded together with the amplifier. This feature represents a real advantage when the signal-to-noise ratio is low and it is important to reduce the electronic noise level.

8.3 THE WHEATSTONE BRIDGE [2]

The Wheatstone bridge is a second circuit which can be employed to determine the change in resistance which a gage undergoes when it is subjected to a strain. Unlike the potentiometer, the Wheatstone bridge can be used to determine both dynamic and static strain-gage readings. The bridge may be used as a direct-readout device, where the output voltage ΔE is measured and related to strain. Also, the bridge may be used as a null-balance system, where the output voltage ΔE is adjusted to a zero value by adjusting the resistive balance of the bridge. In either of the modes of operation the bridge can be effectively employed in a wide variety of strain-gage applications.

To show the principle of operation of the Wheatstone bridge as a direct-readout device (where ΔE is measured to determine the strain), consider the circuit shown in Fig. 8.9. The voltage drop across R_1 is denoted as V_{AB} and is given as

$$V_{AB} = \frac{R_1}{R_1 + R_2} V \qquad (a)$$

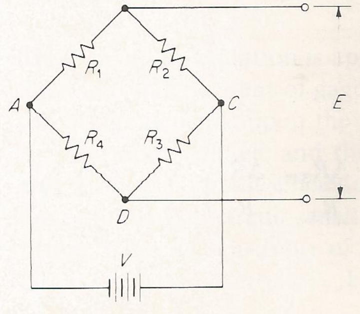


Figure 8.9 The Wheatstone bridge circuit.

and, similarly, the voltage drop across R_4 is denoted as V_{AD} and is given by

$$V_{AD} = \frac{R_4}{R_3 + R_4} V \tag{b}$$

The output voltage from the bridge E is equivalent to V_{BD}, which is

$$E = V_{BD} = V_{AB} - V_{AD} \tag{c}$$

Substituting Eqs. (a) and (b) into Eq. (c) and simplifying give

$$E = \frac{R_1 R_3 - R_2 R_4}{(R_1 + R_2)(R_3 + R_4)} V \tag{8.17}$$

The voltage E will go to zero and the bridge will be considered in balance when

$$R_1 R_3 = R_2 R_4 \tag{8.18}$$

It is this feature of balancing which permits the Wheatstone bridge to be employed for static strain measurements. The bridge is initially balanced before strains are applied to the gages in the bridge; thus the voltage E is initially zero, and the strain-induced voltage ΔE can be measured directly for both static and dynamic applications.

Consider an initially balanced bridge with $R_1 R_3 = R_2 R_4$ so that $E = 0$ and then change each value of resistance R_1, R_2, R_3, and R_4 by an incremental amount ΔR_1, ΔR_2, ΔR_3, and ΔR_4. The voltage output ΔE of the bridge can be obtained from Eq. (8.17), which becomes

$$\Delta E = V \frac{\begin{vmatrix} R_1 + \Delta R_1 & R_2 + \Delta R_2 \\ R_4 + \Delta R_4 & R_3 + \Delta R_3 \end{vmatrix}}{\begin{vmatrix} R_1 + \Delta R_1 + R_2 + \Delta R_2 & 0 \\ 0 & R_3 + \Delta R_3 + R_4 + \Delta R_4 \end{vmatrix}} = V \frac{A}{B} \tag{d}$$

where A is the determinant in the numerator and B is the determinant in the denominator. By expanding each of these determinants, neglecting second-order terms, and noting that $R_1 R_3 = R_2 R_4$, it is possible to show that

$$A = R_1 R_3 \left(\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4} \right) \tag{e}$$

$$B = \frac{R_1 R_3 (R_1 + R_2)^2}{R_1 R_2} \tag{f}$$

Combining Eqs. (d) to (f) yields

$$\Delta E = V \frac{R_1 R_2}{(R_1 + R_2)^2} \left(\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4} \right) \tag{8.19†}$$

† See Exercise 8.9.

By letting $R_2/R_1 = r$ it is possible to rewrite Eq. (8.19) as

$$\Delta E = V\frac{r}{(1+r)^2}\left(\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4}\right) \tag{8.20}$$

In reality, Eqs. (8.19) and (8.20) both carry a nonlinear term $1 - \eta$, as defined in Exercise 8.8. However, the influence of the nonlinear term is quite small and can be neglected, provided the strains being measured are less than 5 percent. Equation (8.20) thus represents the basic equation which governs the behavior of the Wheatstone bridge in strain measurement.

A. Wheatstone-Bridge Sensitivity

The sensitivity of the Wheatstone bridge must be considered from two points of view: (1) with a fixed voltage applied to the bridge regardless of gage current (a condition which exists in most commercially available instrumentation) and (2) with a variable voltage whose upper limit is determined by the power dissipated by the particular arm of the bridge which contains the strain gage. By recalling the definition for the circuit sensitivity given in Eq. (8.15) and using the basic bridge relationship given in Eq. (8.20), it is clear that the circuit sensitivity is

$$S_c = \frac{\Delta E}{\epsilon} = \frac{V}{\epsilon}\frac{r}{(1+r)^2}\left(\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4}\right) \tag{g}$$

If a multiple-gage circuit is considered with n gages (where $n = 1, 2, 3$, or 4) whose outputs sum when placed in the bridge circuit, it is possible to write

$$\sum_{m=1}^{m=n} \frac{\Delta R_m}{R_m} = n\frac{\Delta R}{R} \tag{h}$$

which by Eq. (6.5) becomes

$$\sum_{m=1}^{m=n} \frac{\Delta R_m}{R_m} = nS_g\epsilon \tag{i}$$

Substituting Eq. (*i*) into Eq. (*g*) gives the circuit sensitivity as

$$S_c = V\frac{r}{(1+r)^2}nS_g \tag{8.21}$$

This sensitivity equation is applicable in those cases where the bridge voltage V is fixed and independent of gage current. The equation shows that the sensitivity of the bridge depends upon the number n of active arms employed, the gage factor S_g, the input voltage, and the ratio of the resistances R_1/R_2. A plot of r versus $r/(1+r)^2$ (the circuit efficiency) in Fig. 8.10 shows that maximum efficiency and hence maximum circuit sensitivity occur when $r = 1$. With four active arms in this bridge, a circuit sensitivity of $S_g V$ can be achieved, whereas with one active gage a circuit sensitivity of only $S_g V/4$ can be obtained.

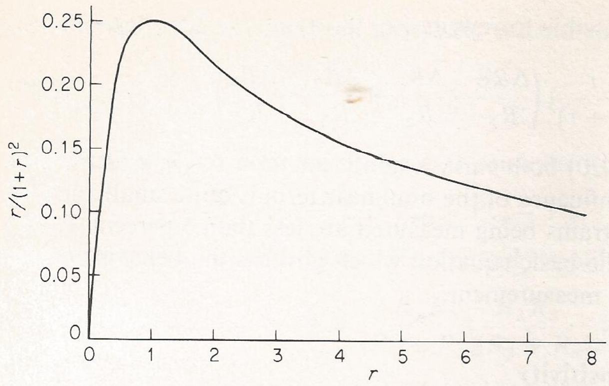


Figure 8.10 Circuit efficiency as a function of r for a fixed-voltage bridge.

When the bridge supply voltage V is selected to drive the gages in the bridge so that they dissipate the maximum allowable power, a different sensitivity equation must be employed. Since the gage current is a limiting factor in this approach, the number of gages used in the bridge and their relative position are important. To show this fact, consider the following four cases, illustrated in Fig. 8.11, which represent the most common bridge arrangements.

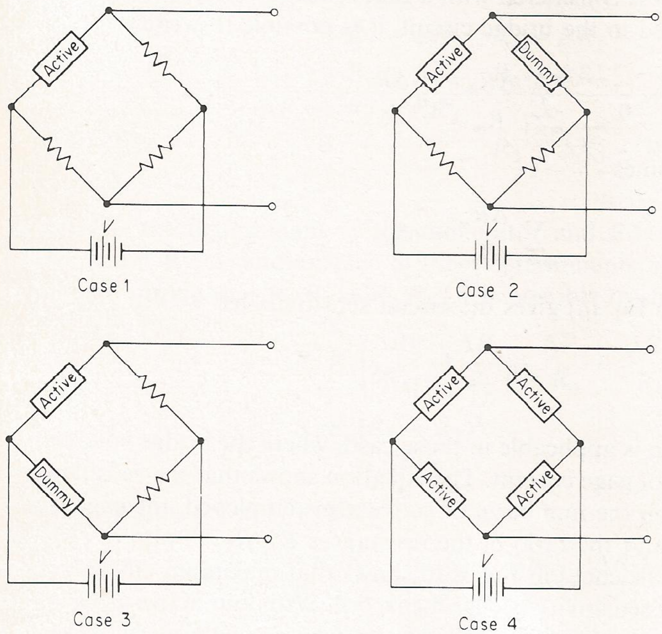


Figure 8.11 Four common bridge arrangements.

Case 1 This bridge arrangement consists of a single active gage in position R_1 and is often employed for many dynamic and some static strain measurements where temperature compensation in the circuit is not critical. The value of R_1 is, of course, equal to R_g, but the value of the other three resistors may be selected to maximize circuit sensitivity, provided the initial balance condition $R_1 R_3 = R_2 R_4$ is maintained. The power dissipated by the gage can be determined from

$$V = I_g(R_1 + R_2) = I_g R_g(1 + r) = (1 + r)\sqrt{P_g R_g} \tag{j}$$

By combining Eqs. (8.21) and (j) and recalling that $r = R_2/R_1$, it is possible to obtain the circuit sensitivity in the following form:

$$S_c = \frac{r}{1 + r} S_g \sqrt{P_g R_g} \tag{8.22}$$

Here it is evident that the circuit sensitivity of the bridge is due to two factors, namely, the circuit efficiency, which is given by $r/(1 + r)$, and the gage selection, represented by the term $S_g\sqrt{P_g R_g}$. For this bridge arrangement, r should be selected as high as possible to increase circuit efficiency but not high enough to increase the supply voltage beyond reasonable limits. A value of $r = 9$ will give a circuit efficiency of 90 percent and, with a 120-Ω gage dissipating 0.15 W requires a supply voltage $V = 42.4$ V.

Case 2 This bridge arrangement employs one active gage in arm R_1 and one dummy gage in arm R_2 which is utilized for temperature compensation (see Exercise 8.13). The value of the gage current which passes through both gages (note $R_1 = R_2 = R_g$) is given by

$$V = 2I_g R_g \tag{k}$$

Substituting Eq. (k) into Eq. (8.21) and noting for this case that $n = 1$ and $r = 1$ gives the circuit sensitivity as

$$S_c = \tfrac{1}{2} S_g \sqrt{P_g R_g} \tag{8.23}$$

In this instance the circuit efficiency is fixed at a value of $\frac{1}{2}$ since the condition $R_1 = R_2 = R_g$ requires that $r = 1$. Thus, it is clear that the placement of a dummy gage in position R_2 to effect temperature compensation reduces circuit efficiency to 50 percent. The gage selection, of course, maintains its importance in this arrangement since the term $S_g\sqrt{P_g R_g}$ again appears in the circuit-sensitivity equation. In fact, this term will appear in the same form in all four bridge arrangements covered in this section. It also appeared in the circuit sensitivity of the potentiometer circuit in Eq. (8.8).

Case 3 The bridge arrangement in this case incorporates an active gage in the R_1 position and a dummy gage in the R_4 position. Fixed resistors of any value are placed in positions R_2 and R_3. With these gage positions, the bridge is temperature-compensated since the temperature-introduced resistance changes

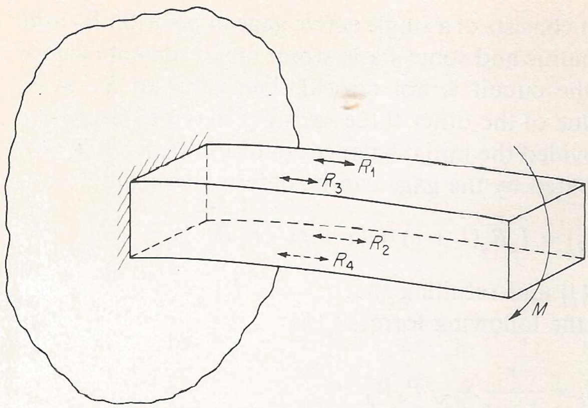


Figure 8.12 Positions of four gages employed on a beam in bending to give a bridge factor of 4.

are canceled out in the Wheatstone bridge circuit. The current through both the active gage and the dummy gage is given by

$$V = I_g(R_1 + R_2) = (1 + r)\sqrt{P_g R_g} \tag{l}$$

Substituting Eq. (l) into Eq. (8.21) and recalling for this case that $n = 1$ gives the circuit sensitivity as

$$S_c = \frac{r}{1 + r} S_g \sqrt{P_g R_g} \tag{8.24}$$

The circuit sensitivity for this bridge arrangement (case 3) is identical to that which can be achieved with the bridge circuit shown in case 1. Thus, temperature compensation can be obtained without any loss in circuit sensitivity if the dummy gage is placed in position R_4 rather than position R_2.

Case 4 In this bridge arrangement, four active gages are placed in the bridge, with one gage in each of the four arms. If the gages are placed on, say, a beam in bending, as shown in Fig. 8.12, the signals from each of the four gages will add and the value of n given in Eq. (8.21) will be equal to 4. The power dissipated by each of the four gages is given by

$$V = 2I_g R_g = 2\sqrt{P_g R_g} \tag{m}$$

Also, since the resistance is the same for all four gages,

$$R_1 = R_2 = R_3 = R_4 = R_g$$

and $r = 1$. The circuit sensitivity for this bridge arrangement is obtained by substituting Eq. (m) into Eq. (8.21):

$$S_c = 2I_g R_g S_g = 2S_g \sqrt{P_g R_g} \tag{8.25}$$

A four-active-arm bridge is slightly more than twice as sensitive as a single-active-arm bridge (case 1 or case 3). Also, this bridge arrangement is temperature-compensated. The four gages employed are a relatively high price to pay for this increased sensitivity. The two-active-arm bridge presented in Exercise 8.14 exhibits a sensitivity which approaches that given in Eq. (8.25).

Examination of these four bridge arrangements plus the fifth arrangement presented in Exercise 8.14 shows that the circuit sensitivity can be varied between $\frac{1}{2}$ and 2 times $S_g\sqrt{P_g R_g}$. Temperature compensation for a single active gage in position R_1 can be effected without loss in sensitivity by placing the dummy gage in position R_4. If the dummy gage is placed in position R_2, the circuit sensitivity is reduced by a factor of 2. Circuit sensitivities can be improved by the use of multiple-active-arm bridges, as illustrated in cases 4 and 5 (see Exercise 8.14). However, the cost of an additional gage or gages for a given strain measurement is rarely warranted except for transducer applications. For experimental stress analyses, single-active-arm bridges are normally employed, and the signal from the bridge is amplified from 10 to 1000 times before recording.

C. Null-Balance Bridges

The Wheatstone bridge arrangements described previously are normally employed in dynamic strain-gage applications where the bridge voltage ΔE is measured directly and related to the strain level. In static applications it is possible to employ a null-balance bridge where the resistance of one or more arms in the bridge is changed to match the effect of the change in resistance of the active gage. This null-balance system is usually more accurate than the direct-readout bridge and requires less expensive equipment for its operation.

A relatively simple null-balance type of Wheatstone bridge is illustrated in Fig. 8.13. A slide-wire resistance or helical potentiometer is placed across the bridge from points B to D. The center tap of the potentiometer is connected to point C. Active strain gages may be placed in any or all arms of the bridge. In the following discussion a single active gage will be considered in the R_1 position. A voltage-measuring instrument with a high sensitivity near the null point is also placed across the bridge between points B and D. Assume that the bridge is initially balanced with $R_1 R_3 = R_2 R_4$ and $R_5 = R_6$. The meter G is at null or zero

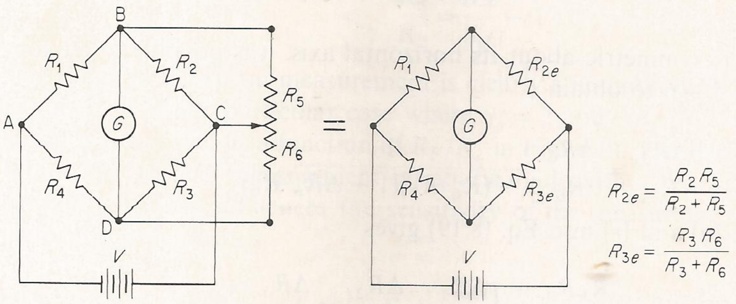


Figure 8.13 Parallel-balance circuit and an equivalent circuit for the null-balance Wheatstone bridge.

voltage. Now consider a resistance change in the arm R_1 which upsets this balance, causing a voltage indication on meter G. The slide wire or potentiometer is adjusted, making $R_5 \neq R_6$, until the bridge is again in balance. The potentiometer adjustment, which is calibrated, is proportional to the resistance change in the active gage. Thus the mechanical movement of the potentiometer serves as the readout means, and the voltage is measured only to establish a zero or null point. In this null-balance condition the adjustment of the potentiometer is independent of the input voltage V.

This circuit can be analyzed by considering the equivalent circuit, also shown in Fig. 8.13. In the equivalent circuit the equivalent resistances R_{2e} and R_{3e} are related to R_2, R_5 and R_3, R_6, respectively, by

$$R_{2e} = \frac{R_2 R_5}{R_2 + R_5} \qquad R_{3e} = \frac{R_3 R_6}{R_3 + R_6} \tag{n}$$

An adjustment of the potentiometer will produce a change in the resistance of R_5 equal to ΔR_5 and of R_6 equal to ΔR_6. Moreover, it is clear that

$$\Delta R_5 = -\Delta R_6 \tag{o}$$

since the total resistance of the potentiometer $R_p = R_5 + R_6$ is constant regardless of the adjustments.

The change in the effective resistance $\Delta R_{2e}/R_{2e}$ can be computed by employing finite differences, as shown below,

$$\frac{\Delta R_{2e}}{R_{2e}} = \frac{[R_2(R_5 + \Delta R_5)]/(R_2 + R_5 + \Delta R_5) - R_2 R_5/(R_2 + R_5)}{R_2 R_5/(R_2 + R_5)} \tag{p}$$

where the value of R_2 is considered fixed, which is consistent with the placement of a single active gage in position R_1 and a dummy gage in position R_4.

Simplifying this equation yields

$$\frac{\Delta R_{2e}}{R_{2e}} = \frac{\Delta R_5}{R_5} \frac{1}{1 + (R_5/R_2)(1 + \Delta R_5/R_5)} \tag{q}$$

Since the bridge is symmetric about its horizontal axis, it is possible to let subscripts $2 \rightarrow 3$ and $5 \rightarrow 6$ to obtain

$$\frac{\Delta R_{3e}}{R_{3e}} = \frac{\Delta R_6}{R_6} \frac{1}{1 + (R_6/R_3)(1 + \Delta R_6/R_6)} \tag{r}$$

Substituting Eqs. (q) and (r) into Eq. (8.19) gives

$$\Delta E = V \frac{R_1 R_{2e}}{(R_1 + R_{2e})^2} \left(\frac{\Delta R_1}{R_1} - \frac{\Delta R_{2e}}{R_{2e}} + \frac{\Delta R_{3e}}{R_{3e}} \right) = 0 \tag{s}$$

In Eq. (s), $\Delta R_{2e}/R_{2e}$ and $\Delta R_{3e}/R_{3e}$ are adjusted until $\Delta E \to 0$; hence, it is evident from Eq. (6.5) that

$$\frac{\Delta R_1}{R_1} = S_g\epsilon = \frac{\Delta R_{2e}}{R_{2e}} - \frac{\Delta R_{3e}}{R_{3e}}$$

$$= \frac{\Delta R_5}{R_5}\frac{1}{1 + (R_5/R_2)(1 + \Delta R_5/R_5)} - \frac{\Delta R_6}{R_6}\frac{1}{1 + (R_6/R_3)(1 + \Delta R_6/R_6)} \quad (t)$$

However, for an initially balanced bridge, where $R_5 = R_6$, $R_2 = R_3$, and $\Delta R_5 = -\Delta R_6$, Eq. (t) reduces appreciably, so that

$$\frac{\Delta R_1}{R_1} = S_g\epsilon = 2\frac{1 + R_5/R_2}{1 + 2(R_5/R_2) + (R_5/R_2)^2[1 - (\Delta R_5/R_5)^2]}\frac{\Delta R_5}{R_5} \quad (8.26)$$

Examination of Eq. (8.26) shows that the strain reading ϵ obtained by using a parallel-balance circuit is nonlinear in terms of the adjustment of R_5 and R_6, as indicated by the presence of the $(\Delta R_5/R_5)^2$ term. To minimize nonlinearities, it is necessary to limit the range of $\Delta R_5/R_5$, which in turn limits the overall range of the measurement of strain which can be achieved with this circuit.

If the value of $\Delta R_5/R_5$ is limited in range so that

$$0 < \frac{\Delta R_5}{R_5} < 0.1 \quad \text{and} \quad 0 < \left(\frac{\Delta R_5}{R_5}\right)^2 < 0.01 \quad (u)$$

the nonlinear term $(\Delta R_5/R_5)^2$ is negligible in comparison with 1 and Eq. (8.26) reduces to

$$\epsilon = \frac{2}{S_g}\frac{1}{1 + R_5/R_2}\frac{\Delta R_5}{R_5} \quad (8.27)$$

Equation (8.27) can be used to determine the strain directly from the adjustment of the potentiometer or slide wire if $\Delta R_5/R_5 < 0.1$. The range of measurement of strain which is possible with this parallel-balance circuit can be determined from Eq. (8.27) by setting $\Delta R_5/R_5 = 0.1$ and solving for R_5/R_2 in terms of strain to obtain

$$\frac{R_5}{R_2} = \frac{0.2}{S_g\epsilon} - 1 \quad (8.28)$$

The range in the strain measurement is clearly a function of R_5/R_2 and the gage factor S_g. For the particular case where $S_g = 2$ and $\Delta R_5/R_5 = 0.1$, the range of strain is illustrated as a function of R_5/R_2 in Fig. 8.14. The selection of R_5/R_2 to extend the range of measurement cannot be made without considering sensitivity, since R_5/R_2 also influences the sensitivity of the measurement. The sensitivity factor S_{pb} is defined as

$$S_{pb} = \frac{\Delta R_5/R_5}{\epsilon} \quad (8.29)$$

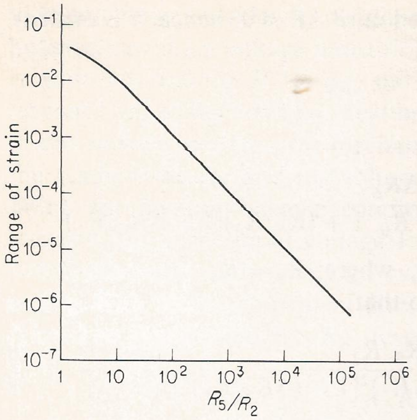


Figure 8.14 Range of strain which can be measured with a parallel-balance circuit.

and by Eq. (8.27), S_{pb} may be expressed as

$$S_{pb} = \frac{S_g}{2}\left(1 + \frac{R_5}{R_2}\right) \tag{8.30}$$

From Eq. (8.30) it is clear that the sensitivity of the parallel-balance circuit is increased by using higher values of R_5/R_2; however, as shown in Fig. 8.14, increasing R_5/R_2 results in a decrease in the allowable range of measurement. Thus a trade-off is necessary to establish a value of R_5/R_2 which gives a suitable range with an adequate sensitivity.

In practice, the parallel-balance arm is fabricated from two fixed resistors R_a and R_b and a potentiometer R_p, as shown in Fig. 8.15. A 20-turn potentiometer divided into 100 readable divisions per turn will permit $\Delta R_5/R_5$ to be established

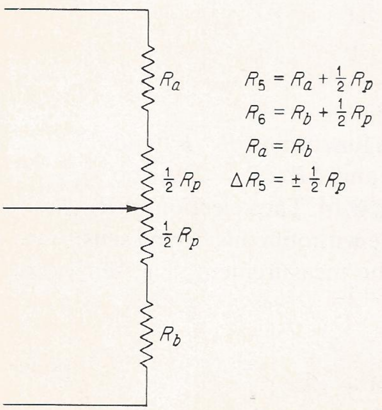


Figure 8.15 The parallel arm containing two fixed resistors and a potentiometer.

within ± 0.001 over the entire range of the potentiometer. It is reasonable then to require that

$$S_{pb}\epsilon = 0.001 \tag{8.31}$$

to fully utilize the sensitivity of the potentiometer readout.

By properly selecting R_a, R_b, R_p, and R_5/R_2, a number of different parallel bridge circuits can be designed. Those circuits with a high sensitivity are limited in range; however, this range can be increased if sensitivity is decreased or if the nonlinear effects are taken into account. Exercises 8.16 and 8.17 illustrate the design aspects of this parallel-balance bridge arrangement.

D. Commercial Strain Indicators

One of the most commonly employed bridge arrangements for static strain measurements is the reference bridge, which is used in several commercial instruments. A schematic illustration of the reference-bridge arrangement is presented in Fig. 8.16, where two bridges are used together to provide a null-balance system.

In this reference-bridge arrangement the bridge on the left-hand side is employed to contain the strain gage or gages, and the bridge on the right-hand side contains either fixed or variable resistors. When gages are placed in the left-hand bridge, the variable resistance in the right-hand bridge is adjusted to obtain initial balance. Strains producing resistance changes in the left-hand bridge cause an unbalance between the two bridges and an associated meter reading.

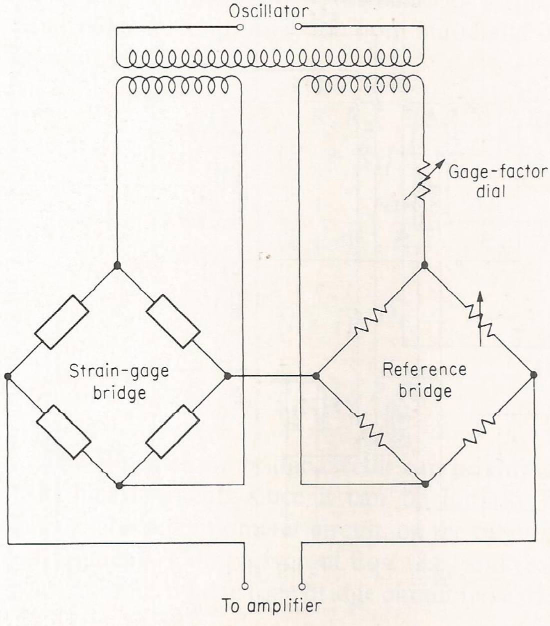
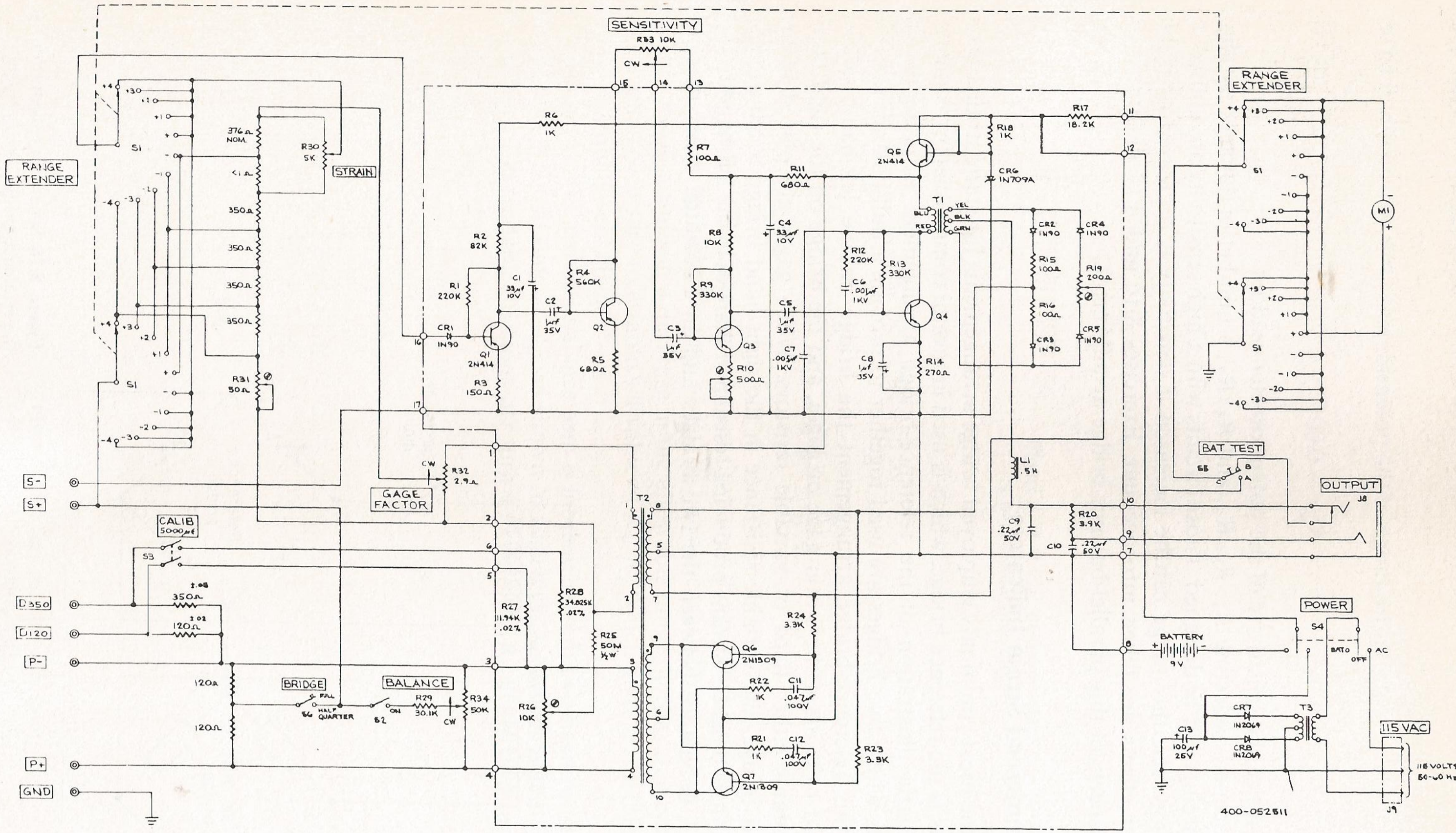


Figure 8.16 Schematic illustration of the reference-bridge arrangement.



NOTES:

1 UNLESS OTHERWISE SPECIFIED; ALL TRANSISTORS ARE 2N3638A, ALL RESISTORS ARE 1/4W, ±10%.

Figure 8.17 Circuit diagram for the Vishay Instrument Model P-350A strain indicator. (*Vishay Instruments, Inc.*)

This meter reading is nulled out by further adjustments of the variable resistor in the right-hand bridge. The advantage of the dual-bridge arrangement is that the left-hand bridge is left completely free for the strain gages and all adjustments for both initial and null balance are performed on the right-hand bridge.

In practice, the reference bridge is somewhat more complex than the schematic arrangement shown in Fig. 8.16. A more detailed circuit, used in Vishay Instrument's P-350A strain indicator, is shown in Fig. 8.17. While a detailed analysis of this circuit is beyond the scope of this text, its operating characteristics can be discussed in some detail. The strain-gage bridge is powered by an oscillator with a 1000-Hz square-wave output of 1.5 V (rms). There is no control over the magnitude of this input voltage; however, for null-balance systems, a fixed-value input voltage is not considered a serious limitation since in the null position the readout is independent of V. The indicator will function over a range of gage resistances from 50 to 2000 Ω. When the gage resistance is less than 50 Ω, the oscillator becomes overloaded. When the gage resistance is greater than 2000 Ω, the load on the bridge circuit due to the amplifier becomes excessive. The gage factor can be set to give readings which are directly calibrated in terms of strain provided $1.5 \leq S_g \leq 4.5$. The indicator can be read to ± 2 μin/in (μm/m) and is accurate to ± 0.1 percent of the reading or 5 μin/in (μm/m) whichever is greater. The range is $\pm 50{,}000$ μin/in (μm/m). The unit, shown in Fig. 8.18, is small, lightweight, and portable. It is simple to operate and is adequate for almost all static strain-gage applications.

E. Summary of the Wheatstone Bridge Circuit

The equations which govern the behavior of the Wheatstone bridge circuit, under initial balance conditions, for both static and dynamic strain measurements are summarized below:

$$\Delta E = V \frac{R_1 R_2}{(R_1 + R_2)^2} \left(\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4} \right) \tag{8.19}$$

$$\Delta E = V \frac{r}{(1 + r)^2} \left(\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4} \right) \tag{8.20}$$

$$S_c = V \frac{r}{(1 + r)^2} n S_g \tag{8.21}$$

$$S_c = \frac{r}{1 + r} S_g \sqrt{P_g R_g} \tag{8.24}$$

The Wheatstone bridge circuit can be employed for both static and dynamic strain measurements since it can be initially balanced to yield a zero output voltage. The potentiometer circuit, on the other hand, is suitable only for dynamic measurements. Comparison of Eqs. (8.8) and (8.24) shows that the potentiometer circuit and the Wheatstone bridge circuit have the same circuit sensitivity. For this

Figure 8.18 Model P-350A strain indicator. (*Vishay Instruments, Inc.*)

reason, the potentiometer circuit, because of its simplicity, is usually employed for dynamic measurements.

The output voltage ΔE from the Wheatstone bridge is nonlinear with respect to resistance change ΔR, and the measuring instrument used to detect the output voltage can produce loading effects which are similar to those produced with the potentiometer circuit. No emphasis was placed on nonlinearity or on loading effects since they are not usually troublesome in the typical experimental stress analysis. Nonlinear effects can become significant when semiconductor gages are used to measure relatively large strains because of the large ΔR's involved. In these applications, the constant-current circuits described in Sec. 8.4 should be used. Loading effects should never be a problem since high-quality, low-cost measuring instruments with high input impedances are readily available.

The Wheatstone bridge can be used to add or subtract signals from multiple-strain-gage installations. A four-active-arm bridge is slightly more than twice as sensitive as an optimum single-active-arm bridge. For transducer applications, the gain in sensitivity as well as cancellation of gage response from components of load which are not being measured are important factors; therefore, the Wheatstone bridge is used almost exclusively for this purpose.

Temperature compensation can be employed without loss of sensitivity provided a separate dummy gage or another active gage is employed in arm R_4 of the bridge. Grounding of the Wheatstone bridge can be accomplished only at point C (see Fig. 8.9); consequently, noise can become more of a problem with the Wheatstone bridge than with the potentiometer circuit. For further details on the treatment of noise in strain-gage circuits, see Sec. 8.7.

8.4 CONSTANT-CURRENT CIRCUITS [3–4]

The potentiometer and Wheatstone bridge circuits described in Secs. 8.2 and 8.3 were driven with a voltage source which ideally remains constant with changes in the resistance of the circuit. These voltage-driven circuits exhibit nonlinear outputs whenever $\Delta R/R$ is large [see Eq. (8.3*b*)]. This nonlinear behavior limits their applicability to semiconductor strain gages. It is possible to replace the constant-voltage source with a constant-current source, and it can be shown that improvements in both linearity and sensitivity result.

Constant-current power supplies with sufficient regulation for strain-gage applications are relatively new and have been made possible by advances in solid-state electronics. Basically, the constant-current power supply is a high-impedance (1 to 10 MΩ) device which changes output voltage with changing resistive load to maintain a constant current.

A. Constant-Current Potentiometer Circuit

Consider the constant-current potentiometer circuit shown in Fig. 8.19*a*. When a very high impedance meter is placed across resistance R_1, the measured output voltage E is

$$E = IR_1 \tag{8.32}$$

When resistances R_1 and R_2 change by ΔR_1 and ΔR_2, the output voltage becomes

$$E + \Delta E = I(R_1 + \Delta R_1) \tag{a}$$

Thus from Eqs. (8.32) and (*a*)

$$\Delta E = (E + \Delta E) - E = I\,\Delta R_1 = IR_1 \frac{\Delta R_1}{R_1} \tag{8.33}$$

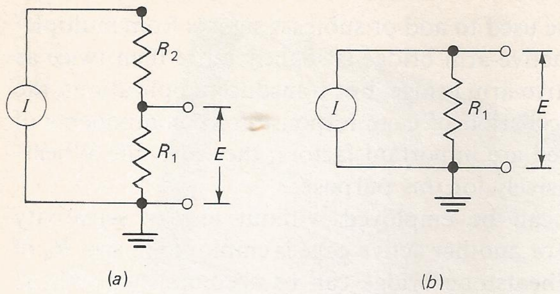


Figure 8.19 Constant-current potentiometer circuits.

It should be noted that ΔR_2 does not affect the signal output. Indeed, even R_2 is not involved in the output voltage, and hence it can be eliminated to give the very simple potentiometer circuit shown in Fig. 8.19*b*.

Substituting Eq. (6.5) into Eq. (8.33) yields

$$\Delta E = IR_g S_g \epsilon \tag{8.34}$$

Equations (8.33) and (8.34) show that the output signal ΔE is linear with respect to resistance change ΔR and strain ϵ. It is this feature of the constant-current potentiometer circuit which makes it more suitable for use with semiconductor strain gages than the constant-voltage potentiometer circuit.

The circuit sensitivity $S_c = \Delta E/\epsilon$ reduces to

$$S_c = IR_g S_g \tag{8.35}$$

If the constant-current source is adjustable, so that the current I can be increased to the power-dissipation limit of the strain gage, then $I = I_g$ and Eq. (8.35) can be rewritten as

$$S_c = \sqrt{P_g R_g}\, S_g \tag{8.36}$$

Thus, the circuit sensitivity is totally dependent on the strain-gage parameters P_g and R_g, and S_g and is totally independent of circuit parameters except for the capability to adjust the current source. Comparison of Eqs. (8.8) and (8.36) shows that the sensitivities differ by the $r/(1 + r)$ multiplier for the constant-voltage potentiometer; thus, S_c will always be higher for the constant-current potentiometer.

It was noted in deriving Eq. (8.33) that R_2 and ΔR_2 did not affect the signal output of the constant-current potentiometer. This indicates that temperature compensation by signal cancellation in the strain-gage circuit or signal addition cannot be performed. It is possible to maintain the advantages of high sensitivity and perfect linearity of this circuit and to obtain the capability of signal addition or subtraction by using a double constant-current potentiometer circuit.

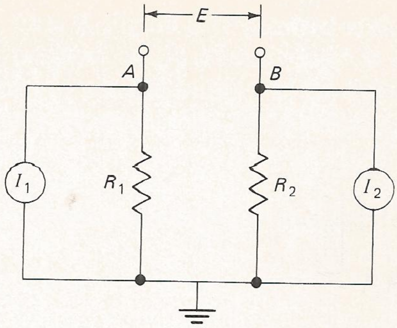


Figure 8.20 Double constant-current potentiometer circuit.

B. Double Constant-Current Potentiometer Circuit

Consider the double constant-current potentiometer circuit shown in Fig. 8.20. In this circuit, two constant-current generators I_1 and I_2 are employed, and the output voltage is measured with a very high impedance meter between points A and B. The voltage at points A and B will be

$$V_A = I_1 R_1 \qquad V_B = I_2 R_2$$

and the output voltage E is

$$E = V_A - V_B = I_1 R_1 - I_2 R_2 \tag{8.37}$$

The circuit can be balanced to give a null output ($E = 0$) initially if the current can be adjusted so that

$$I_1 R_1 = I_2 R_2 \tag{8.38}$$

If resistances R_1 and R_2 change by ΔR_1 and ΔR_2, Eq. (8.37) yields

$$E + \Delta E = I_1(R_1 + \Delta R_1) - I_2(R_2 + \Delta R_2) \tag{b}$$

For a circuit which is initially balanced, $E = 0$ and Eq. (b) reduces to

$$\Delta E = I_1 \, \Delta R_1 - I_2 \, \Delta R_2 = I_1 R_1 \frac{\Delta R_1}{R_1} - I_2 R_2 \frac{\Delta R_2}{R_2} \tag{8.39}$$

Examination of Eq. (8.39) shows that the output voltage ΔE is linear with respect to resistance change ΔR; thus, the circuit can be used with semiconductor gages where large values of $\Delta R/R$ are experienced. Also, since the circuit can be initially balanced, the voltage change ΔE can easily be measured and the circuit can be used for both static and dynamic measurements.

Application of the double potentiometer circuit to strain-gage measurements usually involves one of the four cases illustrated in Fig. 8.21.

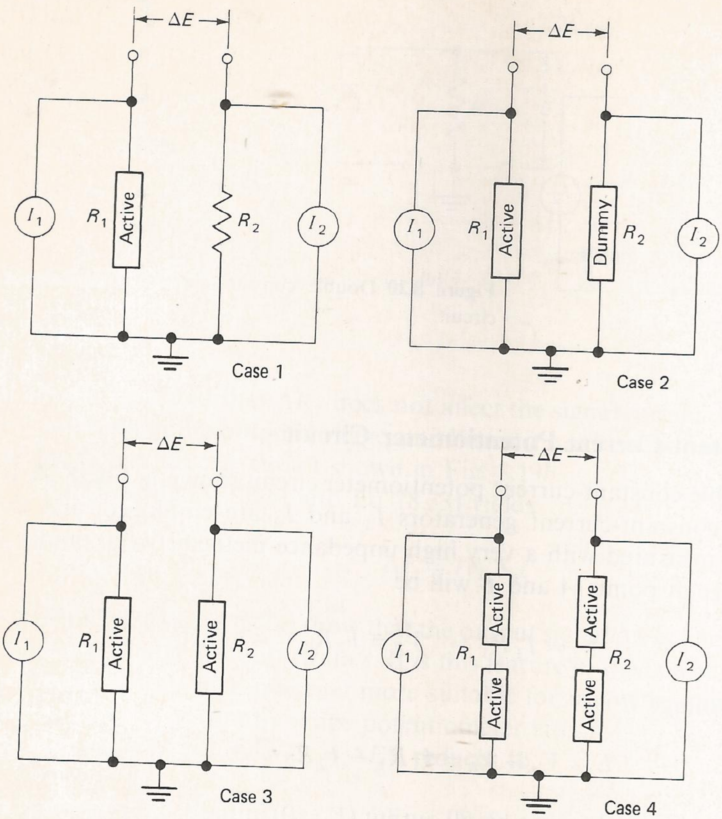


Figure 8.21 Four common double potentiometer circuits.

Case 1 This simple circuit incorporates an active gage in position R_1 and a fixed resistor in position R_2. The currents are adjusted so that $R_1 I_1 = R_2 I_2$. Since R_2 is fixed, $\Delta R_2 = 0$ and Eq. (8.39) reduces to

$$\Delta E = I_1 R_1 \frac{\Delta R_1}{R_1} = I_g R_g S_g \epsilon \qquad (c)$$

and the circuit sensitivity is

$$S_c = \frac{\Delta E}{\epsilon} = I_g R_g S_g = \sqrt{P_g R_g}\, S_g \qquad (8.40)$$

which is the same sensitivity as that achieved with the ordinary constant-current potentiometer circuit. The advantage of using the double constant-current potentiometer circuit in this case is to eliminate the output voltage E so that the voltage ΔE can be determined directly and related to the strain produced by static loadings.

Case 2 Figure 8.21 shows that this circuit incorporates an active gage in the R_1 position and a dummy gage in the R_2 position. The currents are adjusted to achieve initial balance of the circuit. In this case, $R_1 = R_2 = R_g$, and if both gages are exposed to the same thermal environment,

$$\Delta R_1 = \Delta R_\epsilon + \Delta R_T \qquad \Delta R_2 = \Delta R_T \tag{d}$$

Substitution of Eq. (d) into Eq. (8.39) gives

$$\Delta E = I_g R_g \left(\frac{\Delta R_\epsilon}{R_g} + \frac{\Delta R_T}{R_g} - \frac{\Delta R_T}{R_g} \right) \tag{e}$$

which reduces to

$$\Delta E = I_g R_g S_g \epsilon \tag{f}$$

From Eq. (f) it is clear that the circuit sensitivity is

$$S_c = I_g R_g S_g = \sqrt{P_g R_g}\, S_g \tag{8.41}$$

With active and dummy gages in the R_1 and R_2 positions, respectively, it is clear from Eq. (e) that temperature compensation is achieved. It is also evident by comparing Eqs. (8.40) and (8.41) that temperature compensation has been achieved with the double potentiometer circuit without loss in circuit sensitivity.

Case 3 Case 3, illustrated in Fig. 8.21, shows active gages in both positions R_1 and R_2 so that $R_1 = R_2 = R_g$. The currents are adjusted to achieve initial balance of the circuit. If the gages are placed on a beam in bending with the R_1 gage on the tension side and the R_2 gage on the compression side, then $\Delta R_1 = -\Delta R_2 = \Delta R_g$. For this case, $I_1 = I_2 = I_g$, and Eq. (8.39) becomes

$$\Delta E = 2R_g I_g S_g \epsilon \tag{g}$$

The circuit sensitivity becomes

$$S_c = 2R_g I_g S_g = 2\sqrt{P_g R_g}\, S_g \tag{8.42}$$

Examination of Eqs. (g) and (8.42) indicates that two active gages in the double potentiometer circuit provide a circuit sensitivity which is twice the value obtained with a single active gage. Temperature compensation is also achieved in this case.

Case 4 In certain applications, signal addition and the associated increase in output voltage are extremely important. Signal addition can be accomplished with a double constant-current potentiometer circuit by using multiple gages as shown in case 4 of Fig. 8.21. In this instance, $R_1 = R_2 = 2R_g$, $\Delta R_1 = 2\Delta R_g$, and $\Delta R_2 = -2\Delta R_g$. Substitution into Eq. (8.39) gives

$$\Delta E = 2I_g R_g \left(\frac{2\Delta R_g}{2R_g} + \frac{2\Delta R_g}{2R_g} \right) = 4I_g R_g \frac{\Delta R_g}{R_g} \tag{h}$$

and the circuit sensitivity becomes

$$S_c = 4I_g R_g S_g = 4\sqrt{P_g R_g}\, S_g \tag{8.43}$$

Clearly, multiple-gage installations can be used with the double constant-current potentiometer circuit to increase the output voltage to relatively high values in transducer applications. Signal addition or subtraction can be achieved and temperature compensation is automatic if identical gages are used and exposed to the same thermal environment.

The advantages of the double constant-current potentiometer circuit can be summarized as follows:

1. The output voltage ΔE is perfectly linear with respect to $\Delta R/R$, and, as such, the circuit is ideal for use with semiconductor strain gages where large values of $\Delta R/R$ may be encountered.
2. The sensitivity of the double constant-current potentiometer circuit is equal to or better than that of the Wheatstone bridge circuit (constant voltage supply) when one or two active gages are used. The sensitivity is twice that of the normal Wheatstone bridge when four active gages are employed.
3. Grounding is possible for noise elimination.
4. The circuit is the ultimate in simplicity. It can easily be used for either static or dynamic measurements if the current can be adjusted to achieve initial balance and to drive the gages at their maximum power capabilities.

The disadvantage of the circuit pertains to the ripple requirement for the constant-current power supply; however, with battery-driven constant-current supplies, ripple can be limited to 20 parts per million (ppm) for carefully designed circuits. Also, two constant-current sources are required, which increases the costs for an otherwise extremely simple circuit.

C. Constant-Current Wheatstone Bridge Circuits

Since Wheatstone bridges have been used to measure resistance changes in strain gages since the discovery of the basic phenomenon by Lord Kelvin in 1856, it is logical to consider a bridge driven by a constant-current supply, as shown in Fig. 8.22. The current I delivered by the supply divides at point A of the bridge into currents I_1 and I_2, where $I = I_1 + I_2$. The voltage drop between points A and B of the bridge is

$$V_{AB} = I_1 R_1 \tag{i}$$

and the voltage drop between points A and D is

$$V_{AD} = I_2 R_4 \tag{j}$$

Thus the output voltage E from the bridge can be expressed as

$$E = V_{BD} = V_{AB} - V_{AD} = I_1 R_1 - I_2 R_4 \tag{8.44}$$

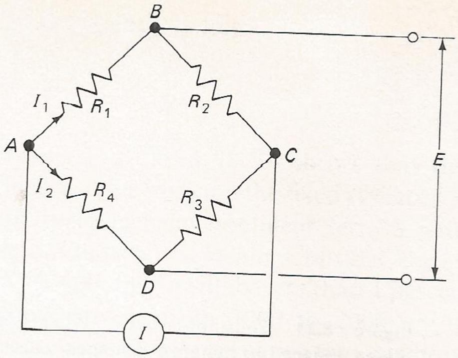


Figure 8.22 Wheatstone bridge with a constant current supply.

For the bridge to be in balance ($E = 0$) under no-load conditions,

$$I_1 R_1 = I_2 R_4 \tag{8.45}$$

Consider next the voltage V_{AC} and note that

$$V_{AC} = I_1(R_1 + R_2) = I_2(R_3 + R_4) \tag{k}$$

from which

$$I_1 = \frac{R_3 + R_4}{R_1 + R_2} I_2 \tag{l}$$

Since

$$I = I_1 + I_2 \tag{m}$$

Eq. (l) can be substituted into Eq. (m) to obtain

$$I_1 = \frac{R_3 + R_4}{R_1 + R_2 + R_3 + R_4} I \qquad I_2 = \frac{R_1 + R_2}{R_1 + R_2 + R_3 + R_4} I \tag{n}$$

Substituting Eqs. (n) into Eq. (8.44) yields

$$E = \frac{I}{R_1 + R_2 + R_3 + R_4}(R_1 R_3 - R_2 R_4) \tag{8.46}$$

From Eq. (8.46) it is evident that the balance condition ($E = 0$) for the constant-current Wheatstone bridge is the same as that for the constant-voltage Wheatstone bridge, namely,

$$R_1 R_3 = R_2 R_4 \tag{8.47}$$

If resistances R_1, R_2, R_3, and R_4 change by the amounts ΔR_1, ΔR_2, ΔR_3, and ΔR_4, the voltage $E + \Delta E$ measured with a very high impedance meter is

$$E + \Delta E = \frac{I}{\sum R + \sum \Delta R}[(R_1 + \Delta R_1)(R_3 + \Delta R_3) - (R_2 + \Delta R_2)(R_4 + \Delta R_4)] \tag{o}$$

where $\sum R = R_1 + R_2 + R_3 + R_4 \qquad \sum \Delta R = \Delta R_1 + \Delta R_2 + \Delta R_3 + \Delta R_4$

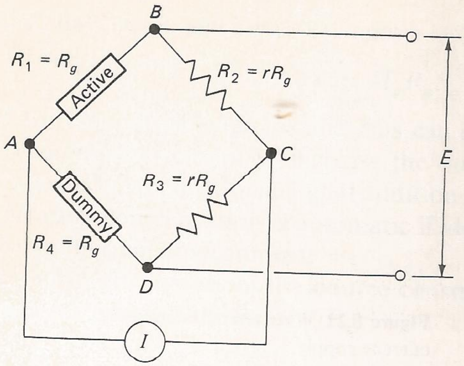


Figure 8.23 Constant-current Wheatstone bridge designed to minimize nonlinear effects.

Expanding Eq. (o) and simplifying after assuming the initial balance condition gives

$$\Delta E = \frac{IR_1R_3}{\sum R + \sum \Delta R}\left(\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4} + \frac{\Delta R_1}{R_1}\frac{\Delta R_3}{R_3} - \frac{\Delta R_2}{R_2}\frac{\Delta R_4}{R_4}\right) \tag{8.48}$$

Inspection of Eq. (8.48) shows that the output signal ΔE is nonlinear with respect to ΔR because of the term $\Sigma\, \Delta R$ in the denominator and because of the second-order terms $(\Delta R_1/R_1)(\Delta R_3/R_3)$ and $(\Delta R_2/R_2)(\Delta R_4/R_4)$ within the bracketed quantity. The nonlinearity of the constant-current Wheatstone bridge, however, is less than that with the constant-voltage bridge. Indeed, if the constant-current Wheatstone bridge is properly designed, the nonlinear terms can be made insignificant even for the large $\Delta R/R$'s encountered with semiconductor strain gages.

The nonlinear effects in a typical situation can be evaluated by considering the constant-current Wheatstone bridge shown in Fig. 8.23. A single active gage is employed in arm R_1, and a temperature-compensating dummy gage is employed in arm R_4. Fixed resistors are employed in arms R_2 and R_3. Thus

$$R_1 = R_4 = R_g \qquad R_2 = R_3 = rR_g \qquad \Delta R_2 = \Delta R_3 = 0$$

Under stable thermal environments, $\Delta R_1 = \Delta R_g$ and $\Delta R_4 = 0$. Equation (8.48) then reduces to

$$\Delta E = \frac{IR_g r}{2(1+r) + \Delta R_g/R_g}\frac{\Delta R_g}{R_g} \tag{8.49}$$

Again it is evident that Eq. (8.49) is nonlinear due to the presence of the term $\Delta R_g/R_g$ in the denominator. To determine the degree of the nonlinearity let

$$\frac{IR_g r}{2(1+r) + \Delta R_g/R_g}\frac{\Delta R_g}{R_g} = \frac{IR_g r}{2(1+r)}\frac{\Delta R_g}{R_g}(1-\eta)$$

where η, the nonlinear term, is

$$\eta = \frac{\Delta R_g/R_g}{2(1+r) + \Delta R_g/R_g} = \frac{S_g\epsilon}{2(1+r) + S_g\epsilon} \tag{8.50}$$

Inspection of Eq. (8.50) shows that the nonlinear term η can be minimized by increasing r (making the fixed resistors R_2 and R_3, say, nine times the value of R_g). In this case, the nonlinear term η will depend on the gage factor and on the magnitude of the strain. Consider, for example, a semiconductor strain gage with $S_g = 100$; then η will be less than 1 percent for strains less than 2000 μin/in (μm/m). Since strains from 1000 to 2000 μin/in (μm/m) represent the upper limit for semiconductor strain gages, they can be used with the constant-current Wheatstone bridge if it is properly designed ($r \geq 9$).

8.5 CALIBRATING STRAIN-GAGE CIRCUITS [5]

A strain-measuring system usually consists of a strain gage, a power supply (either constant voltage or constant current), circuit-completion resistors, an amplifier, and a recording instrument of some type. A schematic illustration of a typical system is shown in Fig. 8.24. It is possible to calibrate each component of the system and determine the voltage-strain relationship from the equations developed in Sec. 8.3. However, this procedure is time-consuming and subject to calibration errors associated with each of the components involved in the system. A more precise and direct procedure is to obtain a single calibration for the complete system so that readings from the recording instrument can be directly related to the strains which produced them.

Direct system calibration can be achieved by shunting a fixed resistor R_c across one arm (say R_2) of the Wheatstone bridge, as shown in Fig. 8.24. If the

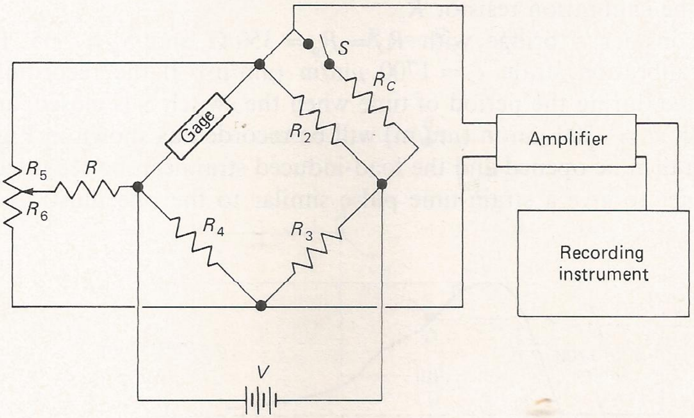


Figure 8.24 Typical strain-recording system.

bridge is initially balanced and the switch is closed to place R_c in parallel with R_2, the effective resistance of this arm of the bridge is

$$R_{2e} = \frac{R_2 R_c}{R_2 + R_c} \tag{a}$$

Because of the shunt resistance R_c, the ratio of the change in resistance to the original resistance in arm R_2 of the bridge is

$$\frac{\Delta R_2}{R_2} = \frac{R_{2e} - R_2}{R_2} \tag{b}$$

Combining Eqs. (*a*) and (*b*) gives

$$\frac{\Delta R_2}{R_2} = \frac{-R_2}{R_2 + R_c} \tag{c}$$

Substituting Eq. (*c*) into Eq. (8.19) gives the output voltage of the bridge produced by R_c. Thus

$$\Delta E = \frac{R_1 R_2}{(R_1 + R_2)^2} \frac{R_2}{R_2 + R_c} V \tag{d}$$

Note also that a single active gage in position R_1 of the bridge would produce an output due to a strain ϵ of

$$\Delta E = \frac{R_1 R_2}{(R_1 + R_2)^2} (S_g \epsilon) V \tag{e}$$

Equating Eqs. (*d*) and (*e*) gives

$$\epsilon_c = \frac{R_2}{S_g(R_2 + R_c)} \tag{8.51}$$

where ϵ_c is the calibration strain which would produce the same voltage output from the bridge as the calibration resistor R_c.

For example, consider a bridge with $R_2 = R_g = 350\ \Omega$ and $S_g = 2.05$. If $R_c = 100\ \text{k}\Omega$, the calibration strain $\epsilon_c = 1700\ \mu$in/in (μm/m). If the recording instrument is operated during the period of time when the switch S is closed, an instrument deflection $d_c = 1700\ \mu$in/in (μm/m) will be recorded as shown in Fig. 8.25. The switch can then be opened and the load-induced strain can be recorded in the normal manner to give a strain-time pulse similar to the one illustrated

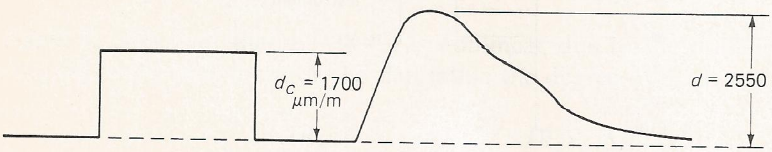


Figure 8.25 Calibrated strain-time trace.

where η, the nonlinear term, is

$$\eta = \frac{\Delta R_g/R_g}{2(1+r) + \Delta R_g/R_g} = \frac{S_g\epsilon}{2(1+r) + S_g\epsilon} \tag{8.50}$$

Inspection of Eq. (8.50) shows that the nonlinear term η can be minimized by increasing r (making the fixed resistors R_2 and R_3, say, nine times the value of R_g). In this case, the nonlinear term η will depend on the gage factor and on the magnitude of the strain. Consider, for example, a semiconductor strain gage with $S_g = 100$; then η will be less than 1 percent for strains less than 2000 μin/in (μm/m). Since strains from 1000 to 2000 μin/in (μm/m) represent the upper limit for semiconductor strain gages, they can be used with the constant-current Wheatstone bridge if it is properly designed ($r \geq 9$).

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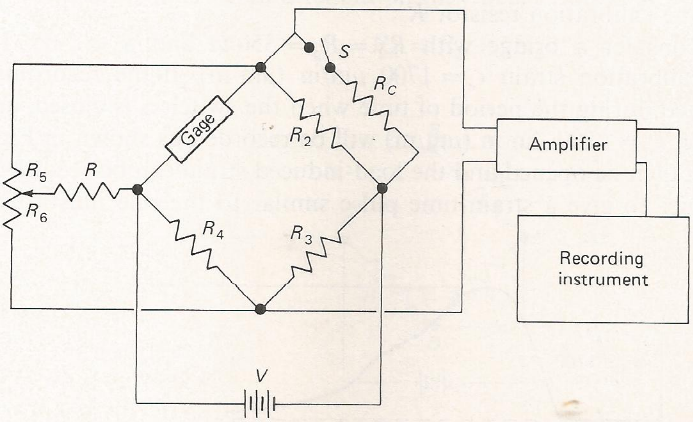

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$$R_{2e} = \frac{R_2 R_c}{R_2 + R_c} \tag{a}$$

Because of the shunt resistance R_c, the ratio of the change in resistance to the original resistance in arm R_2 of the bridge is

$$\frac{\Delta R_2}{R_2} = \frac{R_{2e} - R_2}{R_2} \tag{b}$$

Combining Eqs. (a) and (b) gives

$$\frac{\Delta R_2}{R_2} = \frac{-R_2}{R_2 + R_c} \tag{c}$$

Substituting Eq. (c) into Eq. (8.19) gives the output voltage of the bridge produced by R_c. Thus

$$\Delta E = \frac{R_1 R_2}{(R_1 + R_2)^2} \frac{R_2}{R_2 + R_c} V \tag{d}$$

Note also that a single active gage in position R_1 of the bridge would produce an output due to a strain ϵ of

$$\Delta E = \frac{R_1 R_2}{(R_1 + R_2)^2} (S_g \epsilon) V \tag{e}$$

Equating Eqs. (d) and (e) gives

$$\epsilon_c = \frac{R_2}{S_g(R_2 + R_c)} \tag{8.51}$$

where ϵ_c is the calibration strain which would produce the same voltage output from the bridge as the calibration resistor R_c.

For example, consider a bridge with $R_2 = R_g = 350\ \Omega$ and $S_g = 2.05$. If $R_c = 100\ \text{k}\Omega$, the calibration strain $\epsilon_c = 1700\ \mu\text{in/in}$ (μm/m). If the recording instrument is operated during the period of time when the switch S is closed, an instrument deflection $d_c = 1700\ \mu\text{in/in}$ (μm/m) will be recorded as shown in Fig. 8.25. The switch can then be opened and the load-induced strain can be recorded in the normal manner to give a strain-time pulse similar to the one illustrated

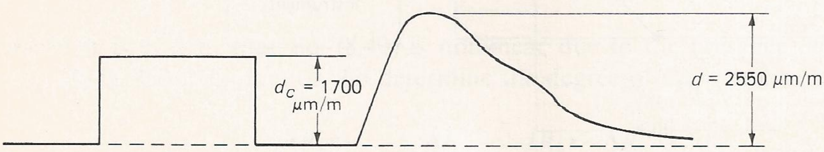

Figure 8.25 Calibrated strain-time trace.

in Fig. 8.25. The peak strain associated with this strain-time pulse produces an instrument deflection d which can be numerically evaluated as

$$\epsilon = \frac{d}{d_c}\epsilon_c \tag{8.52}$$

This method of shunt calibration is accurate and easy to employ. It provides a means of calibrating the complete system regardless of the number of components in the system. The calibration strain produces a reading on the recording instrument; all other readings are linearly related to this calibration value.

8.6 EFFECTS OF LEAD WIRES, SWITCHES, AND SLIP RINGS

The resistance change for a metallic foil strain gage is quite small [0.7 mΩ per μin/in (μm/m) of strain for a 350-Ω gage]. As a consequence, anything that produces a resistance change within the Wheatstone bridge is extremely important. The components within a Wheatstone bridge almost always include lead wires, soldered joints, and binding posts. Frequently, switches and slip rings are also included. The effects of each of these components on the output of the Wheatstone bridge circuit are discussed in the following sections.

A. Effect of Lead Wires [6, 7]

Consider first a two-lead wire system, illustrated in Fig. 8.26, where a single active gage is positioned on a test structure at a location remote from the bridge and recording system. If the length of the two-lead wire system is long, three detrimental effects occur: signal attenuation, loss of balancing capability, and loss of temperature compensation.

To show that signal attenuation may occur note that

$$R_1 = R_g + 2R_L \tag{a}$$

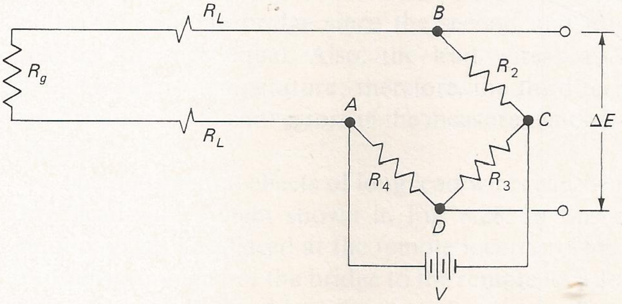

Figure 8.26 Two-lead wire system.

where R_L is the resistance of a single lead wire. Note also that

$$\frac{\Delta R_1}{R_1} = \frac{\Delta R_g}{R_g + 2R_L} = \frac{\Delta R_g/R_g}{1 + 2R_L/R_g} \tag{b}$$

Equation (b) may be expressed in terms of a signal loss factor $\mathscr{L}$. Thus

$$\frac{\Delta R_1}{R_1} = \frac{\Delta R_g}{R_g}(1 - \mathscr{L}) \tag{c}$$

From Eqs. (b) and (c), the signal loss factor $\mathscr{L}$ for the two-lead wire system can be expressed as

$$\mathscr{L} \begin{cases} = \dfrac{2R_L/R_g}{1 + 2R_L/R_g} \\ \approx \dfrac{2R_L}{R_g} \quad \text{if } \dfrac{2R_L}{R_g} \ll 1 \end{cases} \tag{8.53}$$

The signal loss factor $\mathscr{L}$ is shown as a function of the ratio of lead resistance to gage resistance in Fig. 8.27. It is clear from this relationship that $\mathscr{L}$ increases rapidly as R_L becomes a significant fraction of R_g. In order to limit lead-wire losses to less than 2 percent, $R_L/R_g \leq 0.01$. If test conditions dictate long leads, then large-gage wire must be employed to limit R_L and it is advantageous to use 350-Ω gages in place of 120-Ω gages. The resistance of a 100-ft (30.5-m) length of lead wire as a function of wire gage size is listed in Table 8.1 for solid copper wire.

The second detrimental effect of the two-lead wire system is loss of the ability to initially balance the bridge. For the bridge shown in Fig. 8.26,

$$R_1 = R_g + 2R_L \qquad R_2 = R_3 = rR_g \qquad \text{fixed resistors}$$
$$R_4 = R_g \qquad \Delta R_2 = \Delta R_3 = 0$$

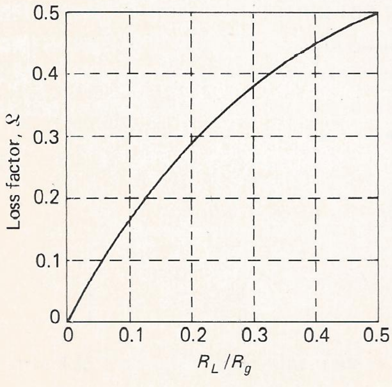

Figure 8.27 Loss factor as a function of the ratio of lead wire to gage resistance, two-lead wire system.

Table 8.1 Resistance of solid-conductor copper wire, Ω per 100 ft (30.5 m)

Gage size	Resistance	Gage size	Resistance	Gage size	Resistance
12	0.159	22	1.614	32	16.41
14	0.253	24	2.567	34	26.09
16	0.402	26	4.081	36	41.48
18	0.639	28	6.490	38	65.96
20	1.015	30	10.310	40	104.90

With the above resistances present in the bridge, it is obvious that the initial balance condition $R_1 R_3 = R_2 R_4$ is not satisfied. Of course, a parallel-balance resistor similar to the one shown in Fig. 8.24 is available in most commercial bridges to obtain initial balance; however, if $R_L/R_g > 0.02$, the range of the balance potentiometer is exceeded and initial balance of the bridge cannot be achieved.

The third detrimental effect of the two-lead wire system is that temperature compensation of the measuring circuit is lost. First, consider a temperature-compensating dummy gage with relatively short leads in arm R_4 of the bridge shown in Fig. 8.26. The output voltage due to resistance changes in arms R_1 and R_4 as obtained from Eq. (8.20) is

$$\Delta E = V\frac{r}{(1+r)^2}\left(\frac{\Delta R_1}{R_1} - \frac{\Delta R_4}{R_4}\right) \tag{d}$$

If the gages are subjected to a temperature difference ΔT at the same time that the active gage is subjected to a strain ϵ, then Eq. (d) becomes

$$\Delta E = V\frac{r}{(1+r)^2}\left[\left(\frac{\Delta R_g}{R_g + 2R_L}\right)_\epsilon + \left(\frac{\Delta R_g}{R_g + 2R_L}\right)_{\Delta T} + \left(\frac{2\Delta R_L}{R_g + 2R_L}\right)_{\Delta T} - \left(\frac{\Delta R_g}{R_g}\right)_{\Delta T}\right] \tag{8.54}$$

Examination of Eq. (8.54) shows that temperature compensation is not achieved in the Wheatstone bridge since the second and fourth terms in the bracketed quantity are not equal. Also, the lead wires can suffer significant resistance changes due to temperature; therefore, the third term in the bracketed quantity can produce significant errors in the measurement of strain with the two-lead-wire system.

The detrimental effects of long lead wires can be minimized by employing the three-lead-wire system shown in Fig. 8.28. In this circuit, both the active and dummy gages are placed at the remote location. One of the three wires is used to transfer terminal A of the bridge to the remote location. It is not considered a lead wire since it is not within either arm R_1 or arm R_4 of the bridge. The active and dummy gages each have one long lead wire with resistance R_L and one short lead

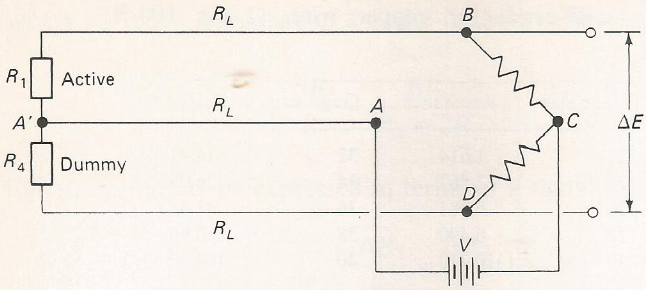

Figure 8.28 Three-lead wire system.

wire with negligible resistance (see Fig. 8.28). With the three-lead-wire system, the signal loss factor $\mathscr{L}$ is reduced to

$$\mathscr{L}\begin{cases} = \dfrac{R_L/R_g}{1 + R_L/R_g} \\ \approx \dfrac{R_L}{R_g} \quad \text{if } \dfrac{R_L}{R_g} \ll 1 \end{cases} \tag{8.55}$$

The bridge retains its initial balance capability since the resistance of both arms R_1 and R_4 is increased by R_L. With the three-lead-wire system, Eq. (8.54) becomes

$$\Delta E = V\frac{r}{(1+r)^2}\left[\left(\frac{\Delta R_g}{R_g + R_L}\right)_\epsilon + \left(\frac{\Delta R_g}{R_g + R_L}\right)_{\Delta T} + \left(\frac{\Delta R_L}{R_g + R_L}\right)_{\Delta T} - \left(\frac{\Delta R_g}{R_g + R_L}\right)_{\Delta T} - \left(\frac{\Delta R_L}{R_g + R_L}\right)_{\Delta T}\right]$$

Temperature compensation is achieved since all the temperature-related terms in the bracketed quantity cancel.

B. Effect of Switches [8]

In most strain-gage applications, many strain gages are installed and are monitored several times during the test. When the number of gages is large, it is not economically feasible to employ a separate recording instrument for each gage. Instead, a single recording system is used, and the gages are switched in and out of the instrument system. Two different methods of switching are commonly used in multiple-gage installations.

The first method, illustrated in Fig. 8.29, involves switching one side of each active gage, in turn, into arm R_1 of the bridge. The other side of each of the active gages is connected to terminal A of the bridge by a common lead wire. A single dummy gage or fixed resistor is used in arm R_4 of the bridge. With this arrangement, the switch is located within arm R_1 of the bridge; therefore, an extremely

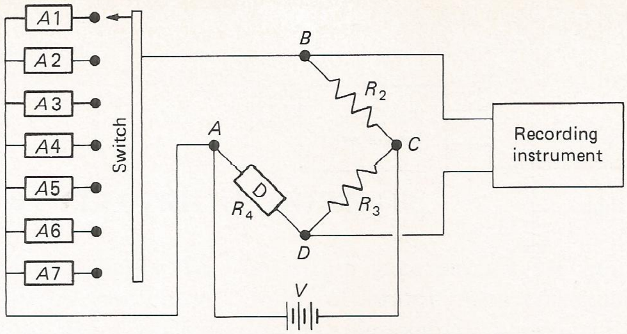

Figure 8.29 Switching active gages in arm R_1 of the Wheatstone bridge.

high quality switch with negligible resistance (less than 500 $\mu\Omega$) must be used. If the switching resistance is not negligible, the switch resistance adds to ΔR_g to produce an error in the strain measurement. The quality of a switch can be checked quite easily, since excessive switch resistance makes it impossible to reproduce zero strain readings.

The second method, illustrated in Fig. 8.30, involves switching the complete bridge. In this method, a three-pole switch is employed in the leads between the bridge and the power supply and the recording instrument. Since the switch is not located in the arms of the bridge, switching resistance is not particularly important. Switching the complete bridge is more expensive, however, since a separate dummy gage and two bridge-completion resistors are required for each active gage.

C. Effect of Slip Rings [9]

Strain gages are frequently used on rotating machinery, where it is impossible to use ordinary lead wires to connect the active gages to the recording instrument. Slip rings are employed in these applications to provide lead-wire connections. The rings are mounted on a shaft which is attached to the rotating member in such a way that the axis of the shaft coincides with the axis of rotation of the member. The shell of the slip-ring assembly is stationary and usually carries several brushes for each slip ring. Lead wires from the strain-gage bridge, which rotates with the member, are connected to the slip rings. Lead wires from the power supply and recording instrument, which are stationary, are connected through the brushes to the appropriate slip ring. Depending on the design of the slip-ring assembly, satisfactory operation at rotary speeds to 24,000 r/min can be achieved.

Dirt collecting on the slip rings and brush jump tends to generate electrical noise in this type of strain-gage system. The use of multiple brushes in parallel helps to minimize the noise; however, the resistance changes between the rings and the brushes are usually so large that slip rings are not recommended for use in an arm of the bridge. Instead, a complete bridge should be assembled on the

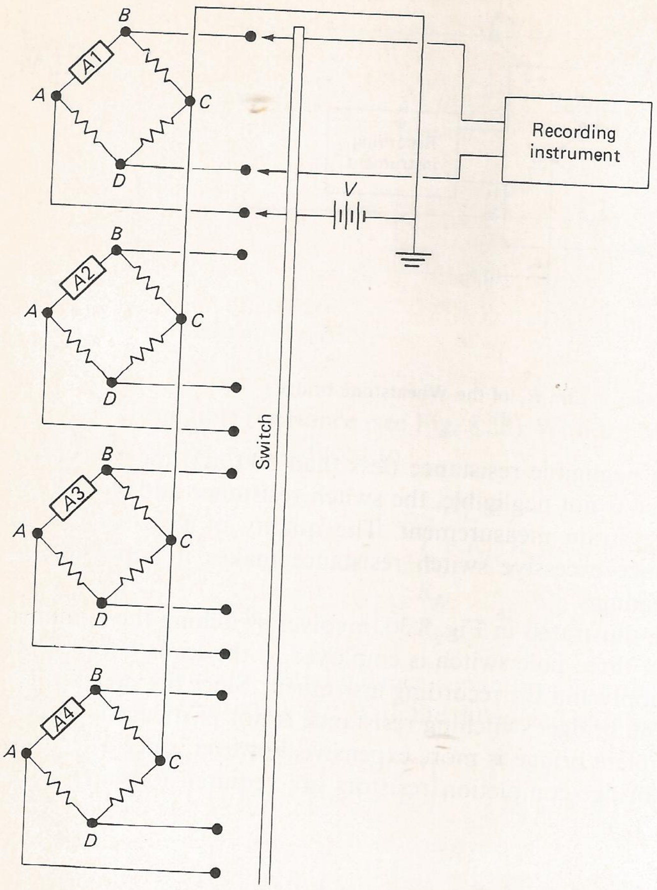

Figure 8.30 Switching the complete bridge.

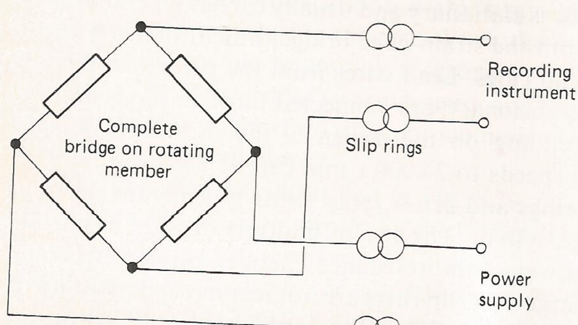

Figure 8.31 Slip rings connecting a complete bridge to the recording instrument and power supply.

rotating member, as shown in Fig. 8.31, and the slip rings should be used to connect the bridge to the power supply and recording instrument. In this way, the effects of slip-ring resistance changes on the strain measurements are minimized.

8.7 ELECTRICAL NOISE [10]

The voltages ΔE from strain gage circuits are quite small [usually less than 10 μV per μin/in (μm/m) of strain]. As a consequence, electrical noise is an important consideration in strain-gage circuit design. Electrical noise in strain-gage circuits is produced by the magnetic fields generated when currents flow through disturbing wires in close proximity to the strain-gage lead wires, as shown in Fig. 8.32. When an alternating current flows in the disturbing wire, a time-varying magnetic field is produced which cuts both wires of the signal circuit and induces a voltage in the signal loop. The induced voltage is proportional to the current I and the area enclosed by the signal loop but inversely proportional to the distance from the disturbing wire to the signal circuit.

Since the distances d_1 and d_2 in Fig. 8.32 are not equal, the difference in magnetic fields at the two signal leads induces a noise voltage E_N which is superimposed on the strain-gage signal voltage ΔE. In certain instances, where the disturbing fields are large, the noise voltage becomes significant and makes separation of the true strain-gage signal from the noise signal quite difficult.

There are three procedures which should normally be employed to reduce noise to a minimum.

Procedure 1 All lead wires should be tightly twisted together to minimize the area in the signal loop and make the distances d_1 and d_2 equal. In this way, the noise voltage is minimized.

Procedure 2 Only shielded cables should be used, and the shields should be connected only to the signal ground as indicated in Fig. 8.33. If the shield is connected

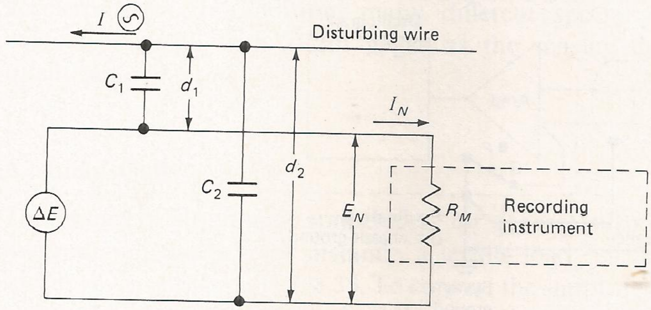

Figure 8.32 Generation of electrical noise.

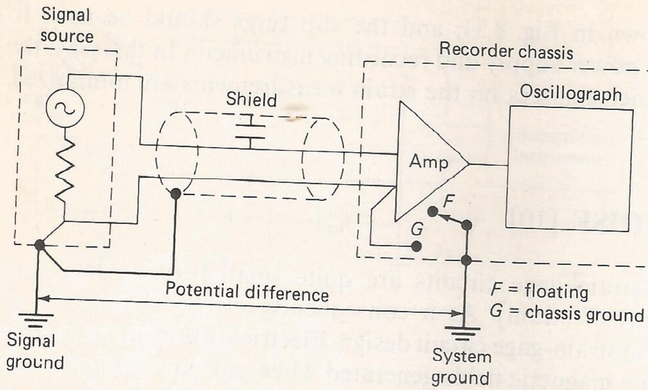

Figure 8.33 Proper method of grounding a shielded cable.

to both the signal ground and the system ground, as shown in Fig. 8.34, a ground loop is formed. Since two different grounds are seldom at the same absolute voltage, a noise signal can be generated by the potential difference which exists between the two to combine with the strain-gage signal in the lower lead of Fig. 8.34. A second ground loop, from the signal source through the cable shield to the amplifier, also occurs with the grounding method shown in Fig. 8.34. Alternating currents in the shield, due to this second ground loop, are coupled to the signal pair through the distributed capacitance in the signal cable (see Fig. 8.34). Either of these ground loops is capable of generating a noise signal 100 times larger than the strain gage signal.

In most recording instruments, the third conductor in the power cord is used to provide the system ground. Since this ground is connected to the electronic enclosure for the instrument, care should be exercised to insulate the enclosure

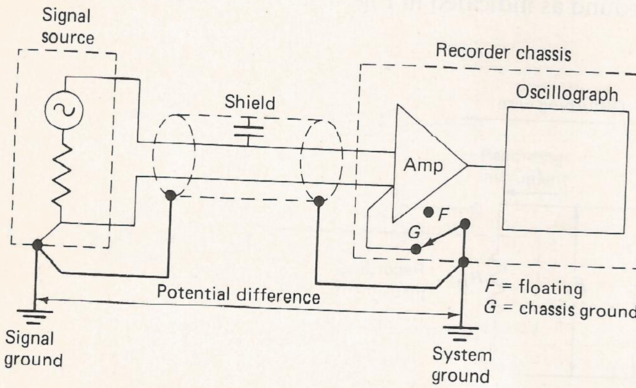

Figure 8.34 Incorrect method of grounding a shielded cable, resulting in a ground loop.

from any other building ground. In most modern recording instruments, the amplifier can be operated in either a floating mode or a grounded mode, as shown in Figs. 8.33 and 8.34, respectively. In the floating mode, the amplifier is insulated from the system ground; and, with the signal source also isolated from the system ground, the arrangement is correct for minimizing the noise signal. The proper point of attachment for the signal ground on both the potentiometer circuit and the Wheatstone bridge circuit is at the negative terminal of the power supply. The power supply itself should be floated relative to the system ground to avoid a ground loop at the supply.

Procedure 3 The third way to eliminate noise is by common-mode rejection. Here the lead wires are arranged so that any noise signals will appear equally and simultaneously in both the lead wires. If a differential amplifier is employed in the recording system, the noise signals are rejected and the strain signals are amplified. Unfortunately, the common-mode rejection of the very best differential amplifiers is not perfect, and a very small portion of the common-mode voltage is transmitted by the amplifier. The common-mode rejection for good low-level data amplifiers is about 10^6 to 1 at 60 Hz; thus, it is evident that significant noise suppression can be achieved in this manner.

8.8 TRANSDUCER APPLICATIONS

Since the electrical-resistance strain gage is such a remarkable measuring device—small, lightweight, linear, precise, and inexpensive—it is used as the sensor in a wide variety of transducers. In a transducer such as a load cell, the load is measured by subjecting a mechanical member to the load and measuring the strain developed in the mechanical member. Since the load is linearly related to the strain, as long as the mechanical member remains elastic, the load cell can be calibrated so that the output signal is interpreted as a load reading.

Transducers of many different types and models are commercially available. Included are load cells, torque meters, pressure gages, displacement gages, and accelerometers. In addition, many different special-purpose transducers are custom-designed, with strain gages as the sensing device, to measure other quantities.

A. Load Cells

The design of strain-gage transducers for general-purpose measurement is relatively easy. Consider, for instance, a tensile load cell fabricated from a tension specimen, as shown in Fig. 8.35. To convert the simple tension bar into a load cell, four strain gages are mounted on the central region of the bar with two opposite gages in the axial direction and two opposite gages in the transverse direction, as

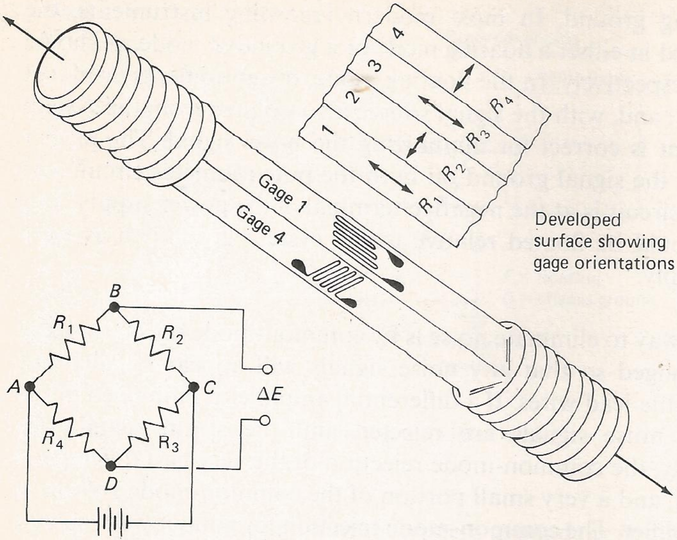

Figure 8.35 Strain gages mounted on a simple tension specimen to produce a load cell.

shown in Fig. 8.35. When a load P is applied to the tension member, the axial and transverse strains produced are

$$\epsilon_a = \frac{P}{AE} \qquad \epsilon_t = -\frac{\nu P}{AE} \tag{8.56}$$

where A = cross-sectional area of tension member
E = modulus of elasticity of the material used to fabricate member
ν = Poisson's ratio of material used to fabricate member

If the four gages are positioned in the Wheatstone bridge as shown in Fig. 8.35, the ratio of output voltage to supply voltage $\Delta E/V$ is given by Eq. (8.20) as

$$\frac{\Delta E}{V} = \frac{1}{4}\left(\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4}\right) \tag{8.57}$$

The changes in resistance of the four gages on the tension member are obtained from Eqs. (6.5) and (8.56) as

$$\frac{\Delta R_1}{R_1} = \frac{\Delta R_3}{R_3} = S_g\epsilon_a = \frac{S_g P}{AE} \qquad \frac{\Delta R_2}{R_2} = \frac{\Delta R_4}{R_4} = S_g\epsilon_t = -\frac{\nu S_g P}{AE}$$

Substituting into Eq. (8.57) gives

$$\frac{\Delta E}{V} \begin{cases} = \dfrac{S_g P}{2AE}(1+\nu) \\ \approx \dfrac{P}{AE}(1+\nu) \qquad \text{when } S_g \approx 2.00 \end{cases} \tag{8.58}$$

From Eq. (8.58), it is evident that the output signal $\Delta E/V$ is linearly related to the load P. The magnitude of $\Delta E/V$ will depend upon the design of the tension member, i.e., its cross-sectional area A and the material constants E and ν. In most commercial load cells $\Delta E/V$ varies between 0.001 and 0.003. Steel with $E = 30 \times 10^6$ lb/in^2 (207 GPa) and $\nu = 0.30$ is usually used to fabricate the tension member. The range of the load cell P_R is then

$$P_R = A \frac{\Delta E}{V} \frac{E}{1+\nu} \tag{8.59}$$

The upper limit on the output signal $\Delta E/V$ is determined by the strength of the tensile member and the fatigue limit of the strain gages. The maximum stress in the tension member is obtained from Eq. (8.59) as

$$\sigma = \frac{P}{A} = \frac{\Delta E}{V} \frac{E}{1+\nu} \tag{8.60}$$

With steel tension members and $\Delta E/V = 0.003$, the stress $\sigma = 69{,}000$ lb/in^2 (476 MPa) is well within the fatigue limit of heat-treated alloy steel such as 4340. However, the axial strain $\epsilon_a = 2300$ μin/in (μm/m) is near the fatigue limit of most strain gages.

Placement of the strain gages on the four sides of the tension member, as shown in Fig. 8.35, provides a load cell which is essentially independent of either bending or torsional loads. Consider a bending moment M applied to the tension member either by a transverse load or by an eccentrically applied axial load. The moment M may have any direction relative to the axes of symmetry of the cross section as shown in Fig. 8.36. The components M_1 and M_2 of the moment M will produce resistance changes in the gages as follows:

$$\left.\frac{\Delta R_2}{R_2}\right|_{M_1} = -\left.\frac{\Delta R_4}{R_4}\right|_{M_1} \quad \text{and} \quad \left.\frac{\Delta R_1}{R_1}\right|_{M_1} = \left.\frac{\Delta R_3}{R_3}\right|_{M_1} = 0$$

$$\left.\frac{\Delta R_3}{R_3}\right|_{M_2} = -\left.\frac{\Delta R_1}{R_1}\right|_{M_2} \quad \text{and} \quad \left.\frac{\Delta R_2}{R_2}\right|_{M_2} = \left.\frac{\Delta R_4}{R_4}\right|_{M_2} = 0$$

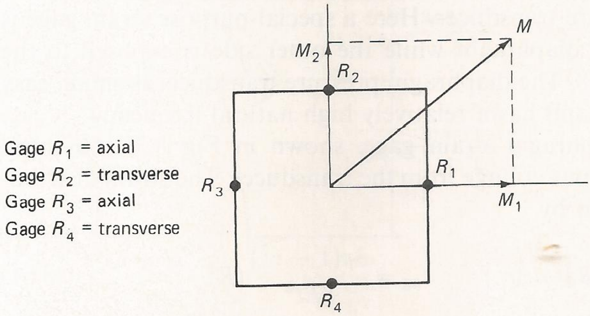

Figure 8.36 Resolution of the moment M into components M_1 and M_2.

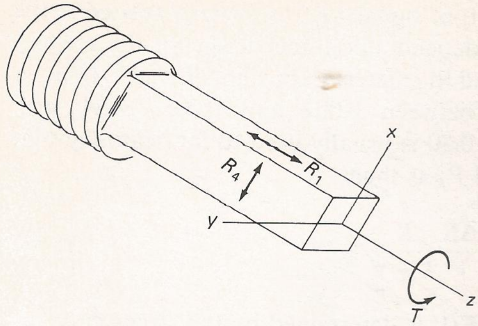

Figure 8.37 Torque T applied to the tension member of an axial-load cell.

Substitution of these relations into Eq. (8.57) shows that the effects of bending moments applied to the load cell are canceled in the Wheatstone bridge since $\Delta E/V$ vanishes for both M_1 and M_2.

Consider next the tension member subjected to a torque T, as shown in Fig. 8.37. The state of stress in the tension member for this form of loading has been shown to be

$$\tau_{max} = \frac{4.81T}{a^3} \qquad \sigma_{xx} = \sigma_{yy} = \sigma_{zz} = 0$$

Thus

$$\epsilon_{xx} = \epsilon_{yy} = \epsilon_{zz} = 0 \tag{a}$$

When Eqs. (*a*) are substituted into Eq. (6.5),

$$\frac{\Delta R_1}{R_1} = \frac{\Delta R_3}{R_3} = S_g \epsilon_{zz} = 0 \qquad \frac{\Delta R_2}{R_2} = \frac{\Delta R_4}{R_4} = S_g \epsilon_{xx} = 0 \tag{b}$$

Substitution of Eqs. (*b*) into Eq. (8.57) indicates that the output of the tensile load cell is independent of the applied torque since $\Delta E/V$ is again equal to zero. Temperature compensation is also achieved with the four active strain gages in the bridge.

B. Diaphragm Pressure Transducers [11–13]

A second type of transducer which utilizes a strain gage as the sensing element is the diaphragm type of pressure transducer. Here a special-purpose strain gage is mounted on one side of the diaphragm while the other side is exposed to the pressure, as shown in Fig. 8.38. The diaphragm pressure transducer is small, easy to fabricate, and inexpensive and has a relatively high natural frequency.

The special-purpose diaphragm strain gage, shown in Fig. 8.39, has been designed to maximize the output voltage from the transducer. The strain distribution in the diaphragm is given by

$$\epsilon_{rr} = \frac{3p(1-\nu^2)}{8Et^2}(R_o^2 - 3r^2) \qquad \epsilon_{\theta\theta} = \frac{3p(1-\nu^2)}{8Et^2}(R_o^2 - r^2) \tag{8.61}$$

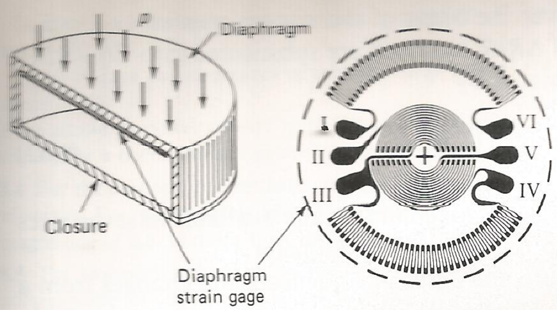

Figure 8.38 Diaphragm-type pressure transducer.

where p = pressure
t = thickness of diaphragm
R_o = outside radius of diaphragm
r = position parameter

Examination of this strain distribution indicates that the circumferential strain $\epsilon_{\theta\theta}$ is always positive and assumes its maximum value at $r = 0$. The radial strain ϵ_{rr} is positive in some regions but negative in others and assumes its maximum negative value at $r = R_o$. Both these distributions are shown in Fig. 8.39. The special-purpose diaphragm strain gage has been designed to take advantage of this distribution. Circumferential grids are employed in the central region of the diaphragm, where $\epsilon_{\theta\theta}$ is a maximum. Similarly, radial grids are employed near the edge of the diaphragm, where ϵ_{rr} is a maximum. It should also be noted that the circumferential and radial grids are each divided into two parts so that the special-purpose gage actually consists of four separate gages. Terminals are provided which permit the individual gages to be connected into a bridge with the circumferential elements in arms R_1 and R_3 and the radial elements in arms R_2 and R_4.

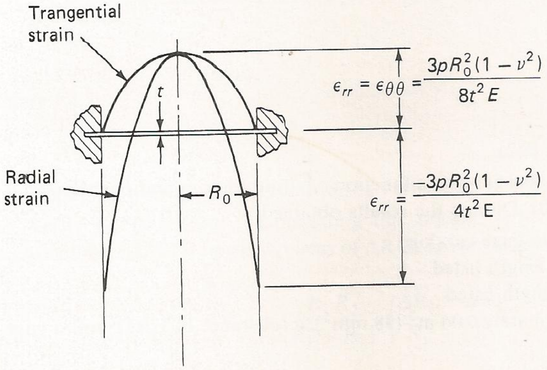

Figure 8.39 Distribution of strain over a diaphragm.

If the strains are averaged over the areas of the circumferential and radial grids, and if the average values of $\Delta R/R$ obtained are substituted into Eq. (8.57), the signal output can be approximated by

$$\frac{\Delta E}{V} = 0.82\,\frac{pR_o^2(1-\nu^2)}{t^2E} \tag{8.62}$$

Special-purpose diaphragm strain gages are commercially available in seven sizes ranging from 0.187 to 1.25 in (4.75 to 31.8 mm) in diameter.

Under the action of the pressure, the diaphragm deflects and changes from a flat circular plate to a segment of a large-radius shell. As a consequence, the strain in the diaphragm is nonlinear with respect to the applied pressure. Acceptable linearity can be maintained by limiting the deflection of the diaphragm. The center deflection w_c of the diaphragm can be expressed as

$$w_c = \frac{3pR_o^4(1-\nu^2)}{16t^3E} \tag{8.63}$$

If $w_c \leq t/4$, $\Delta E/V$ will be linear to within 0.3 percent over the pressure range of the transducer.

Frequently, diaphragm pressure transducers are employed to measure pressure transients. In these dynamic applications, the natural frequency of the diaphragm should be considerably higher than the highest frequency present in the pressure pulse. Depending upon the degree of damping incorporated in the design of the transducer, an undamped natural frequency 5 to 10 times greater than the highest applied frequency should be sufficient to avoid resonance effects. The natural frequency f_n of the diaphragm can be expressed as

$$\omega_n = 2\pi f_n = \frac{10.21t}{R_o^2}\sqrt{\frac{gE}{12(1-\nu^2)\gamma}} \tag{8.64}$$

where γ is the density of the diaphragm material and g is the gravitational constant. If the thickness of the diaphragm cannot be determined accurately, the natural frequency can be determined experimentally by tapping the transducer on the center of the diaphragm and recording the oscillatory signal on an oscilloscope. The peak-to-peak period is the reciprocal of the natural frequency.

EXERCISES

8.1 Verify Eqs. (8.3*a*) and (8.3*b*) from Eq. (8.2).

8.2 Determine the magnitude of the nonlinear term as a function of strain for a potentiometer circuit ($r = 9$) with a single active gage ($S_g = 2.00$). Discuss the results obtained.

8.3 Select the following gages from a strain-gage catalog:

(*a*) A gage having the smallest gage length listed

(*b*) A gage having the largest gage length listed

(*c*) A gage having an area of approximately 0.06 in² (38 mm²), a resistance of 120 Ω, and a gage factor of 2.00

(*d*) A gage having an area of approximately 0.03 in^2 (19 mm^2), a resistance of 1000 Ω, and a gage factor of 2.00

(*e*) A gage having an area of approximately 0.03 in^2 (19 mm^2), a resistance of 350 Ω, and a gage factor of 3.5

If the allowable power density is 0.5 W/in^2 (0.78 mW/mm^2) for each of these gages, determine S_c, V, and R_b for a potentiometer circuit with $r = 9$. Discuss the results.

8.4 For the best and poorest cases in Exercise 8.3, construct a plot of output voltage ΔE as a function of strain. For strain levels associated with the yield strength of low-carbon steels, determine the magnitude of ΔE. Is this a high- or low-level signal?

8.5 The sawtooth voltage pulse shown in Fig. E8.5 is fed into the filter as indicated. Resolve this voltage pulse into its Fourier components and consider the pulse distortion as the first five components pass through the filter. Discuss the results obtained.

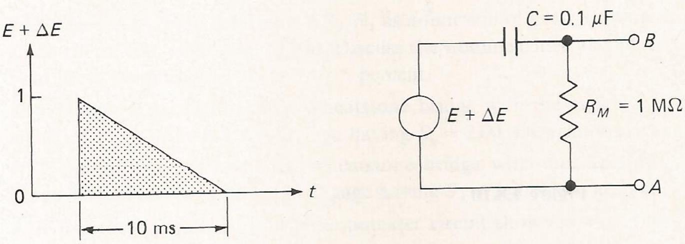

Figure E8.5

8.6 How could the simple filter shown in Fig. E8.5 be changed to improve the fidelity of the signal?

8.7 The triangular voltage pulse shown in Fig. E8.7 is fed into the filter as indicated. Resolve this voltage pulse into its Fourier components and consider the pulse distortion as the first five components pass through the filter. Discuss the results obtained and indicate how this filter design can be improved.

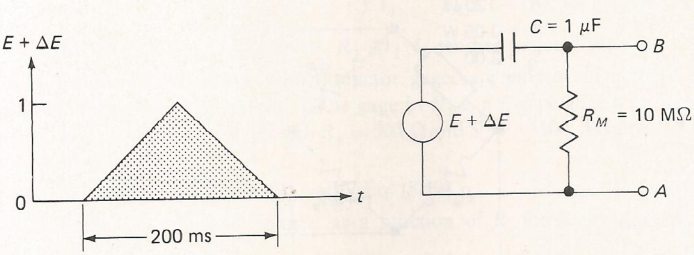

Figure E8.7

8.8 Beginning with Eqs. (8.12) and (8.13), verify Eqs. (8.14) and (8.16).

8.9 Verify Eq. (8.19) and the preceding Eqs. (*e*) and (*f*). Note that if the second-order terms are neglected, Eq. (8.19) is linear in terms of $\Delta R/R$. However, if the second-order terms are retained,

$$\Delta E = V \frac{R_1 R_2}{(R_1 + R_2)^2}\left(\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4}\right)(1 - \eta)$$