Assignment 2
Experimental Stress Analysis

Submitted as Part of Course Requirement of Experimental Stress Analysis Course

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September 2018
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1 Solutions

1.1 Question 1:

We have the Following Data in our Hands

- Weight of the Crucible = 47.6504gms
- weight of the Composite material = 50.1817gms
- weight after Burnoff = 49.4776gms
- Density of the glass fiber = $2.5\frac{gm}{cm^3}$
- Density of the Resin Matrix = $1.2\frac{gm}{cm^3}$

From the above Weight of the Matrix is $50.1817 - 49.4776 = 0.7041$gms
Weight of the Glass Fiber is $49.4776 - 47.6504 = 1.8272$gms
Weight of Composite is $50.1817 - 47.6504 = 2.5313$gms

Weight Fraction of the Matrix is given by $\frac{0.7041}{2.5313} = 0.2781$
Weight Fraction of the Fiber is given by $\frac{1.8272}{2.5313} = 0.7218$

Volume of the Matrix is given by $\frac{0.7041}{1.2} = 0.58675cm^3$
Volume of the Fiber is given by $\frac{1.8272}{2.5} = 0.7309cm^3$

Volume Fraction of the Matrix is given by $\frac{0.58675}{1.31763} = 0.445307104cm^3$
Volume Fraction of the Fiber is given by $\frac{0.7309}{1.31763} = 0.554708074cm^3$

1.2 Question 2:

We are asked to Derive the Equation for the Formula of void

- ρ_{ct} Density of Coomposite Theoratical
- ρ_{ce} Density of Coomposite Experimental
- V_v Volume Fraction of the Void

Volume of Void = $v_{experimental}$ - $v_{theoratical}$

$$v_v = \frac{W_{ce}}{\rho_{ce}} - \frac{W_{ct}}{\rho_{ct}}$$
$$v_v = W \times \frac{\rho_{ct} - \rho_{ce}}{\rho_{ct}\rho_{ce}}$$
$$v_v = \frac{W}{\rho_{ce}} \times \frac{\rho_{ct} - \rho_{ce}}{\rho_{ct}}$$
$$v_v = v_{ce} \times \frac{\rho_{ct} - \rho_{ce}}{\rho_{ct}}$$
$$\frac{v_v}{v_{ce}} = \frac{\rho_{ct} - \rho_{ce}}{\rho_{ct}}$$
$$V_v = \frac{\rho_{ct} - \rho_{ce}}{\rho_{ct}}$$

1.3 Question 3

From Question 1.1 we are asked to find the void Fraction in the Solid.

$$\rho_{ct} \times v_{ct} = W_m + W_f$$
$$v_{ct} = v_m + v_f$$
$$v_{ct} = 0.58675 + 0.7309 = 1.31763 cm^3$$
$$W = 2.5313 gms$$
$$\rho_{ct} = 1.9211 \frac{gms}{cm^3}$$

Given that expenimental Density of the composite is given as $1.86\frac{gm}{cm^3}$Now From the formula in 1.2 we can calculate the void Fraction **0.0318**.

1.4 Question 4

Given the Youngs modulus of the fiber and the Matrix resin are $E_f = 400GPa$, $E_m = 3.2GPa$.When the Composite is loaded in the matrix direction we model that hte Strains in both fiber and matrix are given by we have to calculate the ration of the stresses for the $V_f = 0.1, 0.25, 0.5, 0.75$.

$$\varepsilon_c = \varepsilon_m = \varepsilon_f$$
$$\frac{\sigma_f}{E_f} = \frac{\sigma_m}{E_m} = \frac{\sigma_c}{E_c}$$
$$\frac{\sigma_f}{\sigma_c} = \frac{E_f}{E_c}$$
$$\frac{\sigma_m}{\sigma_c} = \frac{E_m}{E_c}$$

We can calculate the Composite Youngs modulus as following

$$F_c = F_f + F_m$$
$$\sigma_c \times A_c = \sigma_f \times A_f + \sigma_m \times A_m$$
$$\sigma_c = \sigma_f \times V_f + \sigma_m \times V_m$$
$$\frac{\sigma_f}{\sigma_m} = \frac{E_m}{E_f}$$
$$\frac{\sigma_f}{\sigma_c} = \frac{E_f}{(E_f \times V_f + E_m \times V_m)}$$
$$\boldsymbol{\frac{\sigma_f}{\sigma_c} = \frac{E_f \times E_m}{\left(\frac{E_f}{E_m} \times V_f + V_m\right)}}$$

The fiber to matrix Stress Ratio is 125. Thus in composite material fibers bear majority of the stress in fiber direction.Now the ratio of stress in fiber to the composite material for the Fiber fractions are as follows

- $V_f = 0.10$ then $\frac{\sigma_f}{\sigma_c} = 9.32835$
- $V_f = 0.25$ then $\frac{\sigma_f}{\sigma_c} = 3.90625$
- $V_f = 0.5$ then $\frac{\sigma_f}{\sigma_c} = 1.98412$
- $V_f = 0.75$ then $\frac{\sigma_f}{\sigma_c} = 1.32978$

1.5 Question 5

We need to find E_L,E_T,G_{LT} and ν_{LT} for the Following Material for the $V_f = 0.25, 0.5, 0.75$.

- Glass-epoxy
- Graphite-Epoxy
- Kevlar-Epoxy
- boron-aluminium

We have the Following Formulas for the Required Equations

Table 1: Material Properties

Material	**E(GPa)**	ν	G=$\frac{E}{2(1+\nu)}$ (GPa)
Glass Fibers	70	0.2	29.166
Epoxy	3.5	0.35	1.2962
Graphite Fibers	250	0.2	104.166
Kevlar Fibers	140	0.2	58.333
Boron Fibers	350	0.2	145.833
Aluminium	70	0.33	26.315

$$E_L = E_m \times V_m + E_f \times V_f$$
$$\frac{1}{E_T} = \frac{V_f}{E_f} + \frac{V_m}{E_m}$$
$$\frac{1}{G_{LT}} = \frac{V_f}{G_f} + \frac{V_m}{G_m}$$
$$\nu_{LT} = \nu_f \times V_f + \nu_m \times V_m$$

1.5.1 Glass-Epoxy

For $V_f = 0.25$

- Value of E_L is 20.125
- Value of E_T is 12.1739
- Value of G_{LT} is 4.5625
- Value of ν_{LT} is 0.3125

For $V_f = 0.50$

- Value of E_L is 36.75
- Value of E_T is 6.6667
- Value of G_{LT} is 2.474
- Value of ν_{LT} is 0.275

For $V_f = 0.75$

- Value of E_L is 53.375
- Value of E_T is 4.5901
- Value of G_{LT} is 1.6976
- Value of ν_{LT} is 0.2375

1.5.2 Graphite-Epoxy

For $V_f = 0.25$

- Value of E_L is 65.125
- Value of E_T is 13.4357
- Value of G_{LT} is 4.9828
- Value of ν_{LT} is 0.3125

For $V_f = 0.50$

- Value of E_L is 126.75
- Value of E_T is 6.9033
- Value of G_{LT} is 2.55239
- Value of ν_{LT} is 0.275

For $V_f = 0.75$

- Value of E_L is 188.375
- Value of E_T is 4.6444
- Value of G_{LT} is 1.7155
- Value of ν_{LT} is 0.2375

1.5.3 Kevlar-Epoxy

For $V_f = 0.25$

- Value of E_L is 37.625
- Value of E_T is 13.023
- Value of G_{LT} is 4.8464
- Value of ν_{LT} is 0.3125

For $V_f = 0.50$

- Value of E_L is 71.75
- Value of E_T is 6.829
- Value of G_{LT} is 2.528
- Value of ν_{LT} is 0.275

For $V_f = 0.75$

- Value of E_L is 105.875
- Value of E_T is 4.628
- Value of G_{LT} is 1.71006
- Value of ν_{LT} is 0.2375

1.5.4 Boron-Aluminium

For $V_f = 0.25$

- Value of E_L is 140
- Value of E_T is 175
- Value of G_{LT} is 68.3821
- Value of v_{LT} is 0.2975

For $V_f = 0.50$

- Value of E_L is 210
- Value of E_T is 116.6667
- Value of G_{LT} is 44.6471
- Value of v_{LT} is 0.265

For $V_f = 0.75$

- Value of E_L is 280
- Value of E_T is 87.5
- Value of G_{LT} is 33.1400
- Value of v_{LT} is 0.2325

1.6 Question 6

The Given Data Contains that A composite Rod Contains Two Types of Filamentous Reinforcements. $A_0 = 10cm^2$

Material	Density ($\frac{g}{cm^3}$)	Weight %	E(GPa)	σ_u
Binder	1.3	35	3.5	0.06
Material A	2.5	45	70	1.4
Material B	1.6	20	6	0.45

(a) Now calculating the density of the composite material we have the weight percentages in our hand. Assuming no voids in the Material We can write that

$$v_c = v_m + v_{fA} + v_{fB}$$
$$\rho_c = \frac{1}{\sum \frac{W_i}{\rho_i}}$$

Substituting Values we get the values that $\rho_c = 1.74\frac{gm}{cm^3}$. We Get Volume Fractions as

$$V_i = \frac{\rho_c}{\rho_i} \times W_i$$
$$V_m = 0.47$$
$$V_{fA} = 0.31$$
$$V_{fB} = 0.22$$

Similarly We can Write that Fracture Strain Percentages are

$$\varepsilon_i = \frac{\sigma_u}{E_i} \times 100$$
$$\varepsilon_m = 1.71\%$$
$$\varepsilon_{fA} = 2.0\%$$
$$\varepsilon_{fB} = 7.5\%$$

As we have seen in the above strain percent breakdown matrix fails at 1.71% strain and fibre A breaks at 2.0% strain and fiber B breaks at 7.5% strain thus fracture sequence would be in the order of **binder,fibre A , fiber B**.
Now Composite Stress at Fracture of Certain Components is given by following. Here The E_jV_j is the Remaining Materials after fracture.

$$F_i = \varepsilon_i \times \sum_j (E_jV_j) \times Area$$

The Force for Fracture are given by

- For Matrix Fracture Force is 0.422MN
- For Fiber A Fracture Force is 0.46MN
- For Fiber B Fracture Force is 0.099MN

The maximum load carried by the rod is 0.46MN

(b) Fiber B will Rupture Last in the Entire Composite material.

(c)

Load Maintied Test we know that from above calculations as we increase the load Strain in the Composite to mainitain the Load of 0.422 MN. After the failure of Binder strain required to maintain that load is given by

$$\frac{0.422 * 100}{70 * 0.31 + 6 * 0.22} = 1.83\%$$

Now for to maintain Load of 0.46 MN after Fiber A Failure is given by

$$\frac{0.46 * 100}{6 * 0.22} = 34.05\%$$

But we cannot achieve the strain because the Failure will takes place at 7.5% for failure of Fiber B.

Elongation Maintained Test

- Load on the Rod at strain of 1.71% after failure of the binder = 0.394MN
- Load on the Rod at strain of 2.0% after failure of the Fiber A = 0.0264MN

1.7 Question 8

There are Two variants of Graphite Listed in the Table 1.1

- High Modulus Fiber $E_f = 390GPa$, $\sigma_{fu} = 2100MPa$
- High tesnsile Strength Fiber $E_f = 240GPa$, $\sigma_{fu} = 2400MPa$

High Modulus Fiber

$$70 = 390V_f + 3.5(1 - V_f)$$
$$V_f = 0.172$$
$$\rho_c = 0.172 \times 1.90 + 0.828 \times 1.2 = 1.3204$$
$$weightsaving = \frac{2.7 - 1.3204}{2.7} \times 100 = 51.1\%$$

High tesnsile Strength Fiber

$$70 = 240V_f + 3.5(1 - V_f)$$
$$V_f = 0.281$$
$$\rho_c = 0.281 \times 1.90 + 0.719 \times 1.2 = 1.3967$$
$$weightsaving = \frac{2.7 - 1.3967}{2.7} \times 100 = 48.3\%$$

1.8 Question 9

Now For above Question Load carried by the Fiber without Epoxy is Carried is calculated as follows

High modulus Fiber

$$\sigma_{cu} = V_f \sigma_{fcu} = 0.172 * 2100 = 361.2MPa$$

High tesnsile Strength Fiber

$$\sigma_{cu} = V_f \sigma_{fcu} = 0.281 * 2500 = 702.5MPa$$

Here we can see that the Using High Tensile Strength variant have higher fraction of load carried

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1.9 Question 10

Given Data

- Maxium Load Target 2000N
- Volume Fraction of the Composite is $V_f = 0.65$
- Properties of Steel
  - Elastic Modulus = 210 GPa
  - Poissons ration = 0.3
  - Tensile Strength = 450MPa
  - Specific Gravity = 7.8
- Properties of graphite
  - Tensile Strength = 2200MPa
  - Specific Gravity = 1.8
- Properties of Resin
  - Specific Gravity = 1.2

Area of Steel Rod = $\frac{2000}{450} = 4.44mm^2$ Now We Make certain Assumptions To Simplify Calculations

- Load Carried By epoxy is neglected
- Cost of Resin is not considered

The Assumptions are justified as the Majority of the tensile load is carried by the Fiber. Now the Density of the Composite are given by

$$\sigma_{cu} = 0.65 \times 2200 = 1430MPa$$
$$\rho_c = 0.65 \times 1.8 + 0.35 \times 1.2 = 1.59$$

The cross section area is given by value

$$A_c = \frac{2000}{1430} = 1.4mm^2$$

Weight and Cost ratios are

$$WeightRatio = \frac{7.8 \times 4.44}{1.59 \times 1.4} = 15.56$$
$$CostRatio = \frac{7.8 \times 4.44}{5 * 1.59 \times 1.4} = 3.11$$

Thus in both cases Composite material is better option

1.10 Question 11

We are Given the Composite material of Graphite Epoxy with following Volume Ratio $V_f = 0.4$. Now calculting the tensile Strength of the composite material.We assume that the composite failure occurs due to failure of the fiber which is true for most cases. There is Case where the even after failure of fiber matrix can take up the load when the fiber volume ratio is very less.

$$\sigma_{cu} = \sigma_{fu} V_f + (\sigma_m)_{\varepsilon_f^*}(1 - V_f)$$
$$\sigma_{cu} = \sigma_{mu}(1 - V_f)$$

when above two are equal the matrix can take up load thus there must be a certain volume fraction of the fiber.The Critical Volume Ratio is given by

$$V_{crit} = \frac{\sigma_{mu} - (\sigma_m)_{\varepsilon_f^*}}{\sigma_{fu} - (\sigma_m)_{\varepsilon_f^*}}$$

Now for strengthing we add fiber and the Previous composite act as matrix

$$\sigma_{cu} = 700 \times 0.4 + \frac{700}{70 \times 10^3} \times 3.5 \times 10^3 \times 0.6 = 301MPa$$

Now for New failure strain assuming the carbon fiber is the first to fail in strengthed composite material

$$\varepsilon_f^* = \frac{700 \times 10^3}{350} = 2 \times 10^{-3}$$
$$\sigma_{fu} = 700MPa$$
$$\sigma_{mu} = 301MPa$$
$$(\sigma_m)_{\varepsilon_f^*} = E_{\frac{galss}{epoxy}} \times \varepsilon_f^* = 60.2MPa$$
$$V_{crit} = \frac{301 - 60.2}{700 - 60.2} = 30.7\%$$

1.11 Question 12

Longitudinal Modulus of the current Composite = $E = 3.5 \times 0.3 + 70 \times 0.7 = 50.5GPa$. Now We have to add the Carbon fibers to double the Longitudianl Modulus

$$2 \times 50.5 = 3.5 \times 0.3 + 70 \times (0.7 - V_{cf}) + 350V_{cf}$$
$$V_{cf} = \frac{50.5}{350 - 70} = 17.9\%$$

Now The LOngitudinal Stress of Two Composites are to be calculated. For The Composite Before Mixing Carbon Fibers We have that

$$\sigma_{cu} = 700 \times 0.7 + \frac{700}{70 \times 10^3} \times 3.5 \times 10^3 \times 0.3 = 500.5MPa$$

For Enhanced Composite With Carbon Fiber The Longitudinal Stresses are Calculated as follows.For Carbon Fiber Failure At $\varepsilon = 2 \times 10^{-3}$

$$\sigma_{cu} = 700 \times 0.179 + 70 \times 10^3 \times 2 \times 10^{-3}(0.7 - 0.179) + 3.5 \times 10^3 \times 2 \times 10^{-3} \times 0.3 = 200.34MPa$$

Composite Stress at Facture of glass Fiber $\varepsilon = 10^{-2}$

$$\sigma_{cu} = 70 \times 10^3 \times 10^{-2} \times (0.7 - 0.179) + 3.5 \times 10^3 \times 10^{-2} \times 0.3 = 375.2MPa$$

Now Thus enhanced composte longitudian stress is 375.2 MPa. As we can see that the longitudinal strength is decresed because the Carbon fiber has larger Young's Modulus thus Fracture Strain is very less thus After Fracture of Carbon Fiber the longitudinal tensile Stress is decresed.

1.12 Question 13

$$\varepsilon^{|}_{LU} = \frac{0.0036}{0.25} = 0.0144$$
$$\sigma^{|} = 40 \times 10^3 \times 0.0144 = 576MPa$$

1.13 Question 14

Given to Determine the Change of $\frac{\sigma_{TU}}{\sigma_{mU}}$ with change in the Volume Ratio

$$\frac{\sigma_{TU}}{\sigma_{mU}} = \frac{1}{SCF} = \frac{1 - \left(\frac{4V_f}{\pi}\right)^{\frac{1}{2}}\left(1 - \frac{E_f}{E_m}\right)}{1 - (V_f)\left(1 - \frac{E_f}{E_m}\right)}$$
$$\frac{\sigma_{TU}}{\sigma_{mU}} = \frac{1}{SMF} = 1 - \left(\frac{4V_f}{\pi}\right)^{\frac{1}{2}}\left(1 - \frac{E_f}{E_m}\right)$$

We are Given the $\frac{E_m}{E_f}$ ration Now Substituting in the Equation We get the Values and then We can plot values

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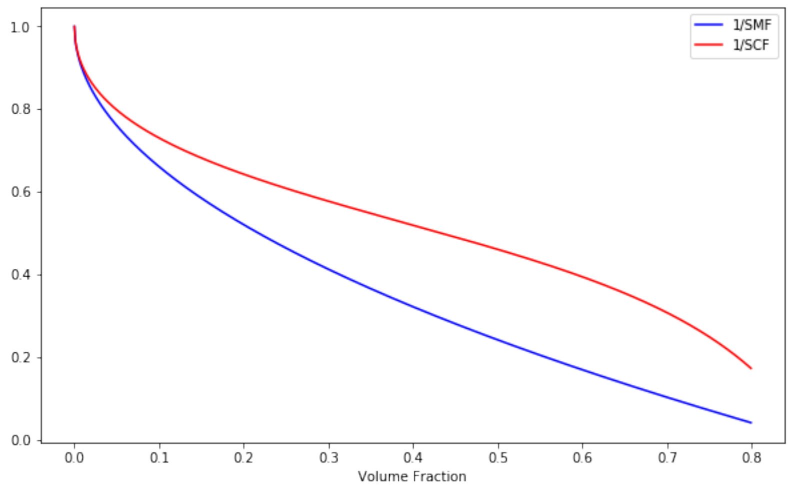

Figure 1: Graph of $\frac{\sigma_{TU}}{\sigma_{mU}}$ vs Volume Fraction

1.14 Question 15

We are Given the following Data to calculate the cofficient of Thermal Expansion of the Fibers.

$$E_f = 294GPa$$
$$E_m = 3GPa$$
$$\alpha_m = 54 \times 10^{-6}$$
$$V_f = 55\%$$
$$\alpha_L = -0.61 \times 10^{-6}$$

We have the Following Relation

$$\alpha_L = \frac{1}{E_L}\left(\alpha_f E_f V_f + \alpha_m E_m V_m\right)$$
$$E_L = 294 \times 0.55 + 3.5 \times 0.45 = 163.3$$
$$\alpha_f = -1.14 \times 10^{-6}$$

1.15 Question 16

Given that Only matrix Absorbs water thus

$$W_m = \frac{\rho_m}{\rho_c} V_m = \frac{1.2}{1.6} \times 0.3 = 0.225$$

Maximum Mositure in Composite = $0.225 \times 6 = 1.35\%$

1.16 Question 17

For Continuos Fiber Composite Calculating the Elastic Modulus. We can write that

$$E_c = E_f V_f + E_m V_m = 30.1Gpa$$

For Matrix Yield Stress in Shear is Equal to half of Yield Stress in Tension thus $\tau_y = 14MPa$.

(a) Now We are asked to find out the Load transfer lenght of the fiber for given loads on the Composite

$$\left(\sigma_f\right)_{max} = \frac{E_f}{E_c}\sigma_c$$
$$\frac{l_t}{d} = \frac{\left(\sigma_f\right)_{max}}{2 \times \tau_y}$$

Now Calculating the values for the $\sigma_c = 70, 210$

For $\sigma_c = 70GPa$

$$\left(\sigma_f\right)_{max} = 162.38MPa$$
$$\frac{l_t}{d} = 0.1744mm$$

Now Calculating the values for the $\sigma_c = 70, 210$

For $\sigma_c = 210GPa$

$$\left(\sigma_f\right)_{max} = 488.4MPa$$
$$\frac{l_t}{d} = 0.5323mm$$

(b) Now we can calculte the Average Stress using the following Equations

$$\overline{\sigma_f} = \frac{1}{2}(\sigma_f)_{max} = \frac{\tau_y l}{d} \qquad for \qquad l < l_t$$
$$\overline{\sigma_f} = (\sigma_f)_{max}\left(1 - \frac{l_t}{2l}\right) \qquad for \qquad l > l_t$$

Using These Equations We can calculate the Average stress for following

For $\sigma_c = 70GPa$

$$l = \frac{l_t}{2d} = 2.907$$
$$\overline{\sigma_f} = 40.7MPa$$

$$l = 4l_t$$
$$\overline{\sigma_f} = 142.45MPa$$

For $\sigma_c = 210GPa$

$$l = \frac{l_t}{2d} = 8.72$$
$$\overline{\sigma_f} = 122.1MPa$$

$$l = 4l_t$$
$$\overline{\sigma_f} = 427.35MPa$$

(c) Now the Strain varies Linearly and reaching the maximum value at the moddle of the fiber Since Fiber length is less than or equal to load transfer length maximum value of the strains are calculated as follows:

For $\sigma_c = 70GPa$

$$l = l_t$$
$$(\varepsilon_f)_{max} = \frac{\sigma_{max}}{E_f} = 0.00233$$
$$l = \frac{l_t}{2}$$
$$(\varepsilon_f)_{max} = \frac{\sigma_{max}}{E_f} = 0.001165$$

For $\sigma_c = 210GPa$

$$l = l_t$$
$$(\varepsilon_f)_{max} = \frac{\sigma_{max}}{E_f} = 0.00698$$
$$l = \frac{l_t}{2}$$
$$(\varepsilon_f)_{max} = \frac{\sigma_{max}}{E_f} = 0.00349$$

1.17 Question 18

Now We can calculate The Value of the Critical Length

$$\frac{l_c}{d} = \frac{\sigma_{fu}}{2\tau_y}$$
$$\frac{l_c}{d} = 50$$

for Case where l is less than l_c

$$\sigma_{cu} = \frac{(\sigma_f)_{max}}{2} V_f + V_m \sigma_{mu}$$
$$\sigma_{cu} = 16.8 + 280\frac{l}{l_c}$$

For Case When lenght is grater than the critical length we can consider that as a infinite fiber

$$\sigma_{cu} = (\sigma_f)_{max}\left(1 - \frac{l}{2d}\right) V_f + V_m \sigma_{mu}$$
$$\sigma_{cu} = 576.8 - 280\frac{l_c}{l}$$

1.18 Question 19

$$\rho_c = \frac{1}{\frac{W_f}{\rho_f} + \frac{W_m}{\rho_m}} = 1.535\frac{g}{cm^3}$$
$$V_f = \frac{\rho_c}{\rho_f} W_f = 0.1228$$
$$\frac{E_T}{E_m} = \frac{1 + 2\eta_T V_f}{1 - \eta_T V_f}$$
$$\eta_T = \frac{\frac{E_f}{E_m} - 1}{\frac{E_f}{E_m} + 2} = \frac{19}{22}$$
$$E_T = 4.746GPa$$

Now For Randomly oriented Fibers we have the Equation that

$$E_R = \frac{3}{8}E_L + \frac{5}{8}E_T$$
$$E_L = 10.76GPa$$

Now We have the Relation of the Equation we can write that

$$\frac{E_L}{E_m} = \frac{1+2\frac{l}{d}\eta_L V_f}{1-\eta_L V_f}$$
$$\eta_T = \frac{\frac{E_f}{E_m}-1}{\frac{E_f}{E_m}+2\frac{l}{d}} = \frac{20-1}{20+2\frac{l}{d}}$$

Solving the Above we can get the

$$a = \frac{2l}{d} = 132.5$$
$$l = 1325\mu m$$

1.19 Question 20

For length grater than critical fiber length composite strength

$$\sigma_{cu} = \overline{\sigma_f}V_f + (\sigma_m)_{\varepsilon_f^*}(1-V_f)$$

For V_f less than V_{min} strength is controlled by the matrix and thus for limiting case we can write that

$$\overline{\sigma_f}V_{min} + (\sigma_m)_{\varepsilon_f^*}(1-V_{min}) = \sigma_{mu}(1-V_{min})$$
$$V_{min} = \frac{\sigma_{mu} - (\sigma_m)_{\varepsilon_f^*}}{\sigma_f + \sigma_{mu} - (\sigma_m)_{\varepsilon_f^*}}$$

At $V = V_{crit}$ we will get $\sigma_{cu} = \sigma_{mu}$ Thus

$$\overline{\sigma_f}V_{crit} + (\sigma_m)_{\varepsilon_f^*}(1-V_{crit}) = \sigma_{mu}$$
$$V_{crit} = \frac{\sigma_{mu} - (\sigma_m)_{\varepsilon_f^*}}{\overline{\sigma_f} - (\sigma_m)_{\varepsilon_f^*}}$$

1.20 Question 21

Writing the Ratio of the Fiber Strength in discontinuous and Continous fiber composite as follows

$$\frac{(\sigma_{cu})_{disc}}{(\sigma_{cu})_{cont}} = \frac{\sigma_{fu}\left(1-\frac{l_c}{2l}\right)V_f + (\sigma_m)_{\varepsilon_f^*}(1-V_f)}{\sigma_{fu}V_f + (\sigma_m)_{\varepsilon_f^*}(1-V_f)}$$
$$\frac{(\sigma_{cu})_{disc}}{(\sigma_{cu})_{cont}} = 1 - \frac{\frac{l_c}{2l}V_f\sigma_{fu}}{\sigma_{fu}V_f + (\sigma_m)_{\varepsilon_f^*}(1-V_f)}$$
$$\frac{(\sigma_{cu})_{disc}}{(\sigma_{cu})_{cont}} = 1 - \frac{1}{\frac{2l}{l_c}\left(1+\frac{(\sigma_m)_{\varepsilon_f^*}}{\sigma_{fu}}(\frac{1}{V_f}-1)\right)}$$

For Limiting Case of $V_f = 1$ we get

$$\frac{(\sigma_{cu})_{disc}}{(\sigma_{cu})_{cont}} = 1 - \frac{l_c}{2l}$$

We can Plot This Value and Get The Corresponding Curve

1.21 Question 22

We have the Following Relation

$$V_r = \frac{1}{2(1-\frac{B}{W_r})(1+\frac{t_m}{t_r})}$$
$$V_r = \frac{1}{1.1\left(1+\frac{t_r}{2W_r}\right)}$$

We can Plot the Above Equation to Get the Required Relation

1.22 Question 23

For Square Array We can Write That

$$V_f = \frac{\frac{\pi}{4}d^2}{(d+0.1d)^2} = 0.649$$

For Hexagonal Array We have

$$V_f = \frac{\frac{1}{2}\frac{\pi}{4}d^2}{(d+0.1d)^2}$$

1.23 Question 24

when σ_y is the only non-zero stress we have

$$\sigma_L = \sigma_y \sin^2\theta$$
$$\sigma_T = \sigma_y \cos^2\theta$$
$$\tau_{LT} = \sigma_y \sin\theta\cos\theta$$
$$\varepsilon_L = \sigma_y\left(\frac{\sin^2\theta}{E_L} - \nu_{TL}\frac{\cos^2\theta}{E_T}\right)$$
$$\varepsilon_T = \sigma_y\left(\frac{\cos^2\theta}{E_T} - \nu_{TL}\frac{\sin^2\theta}{E_L}\right)$$
$$\gamma_{LT} = \frac{\sigma_y\sin\theta\cos\theta}{G_{LT}}$$

By transformation of the Strain We can get that

$$\varepsilon_x = -\sigma_y\left(\frac{\nu_{LT}}{E_L} - \frac{1}{4}\left(\frac{1}{E_L} + \frac{2\nu_{LT}}{E_L} + \frac{1}{E_T} - \frac{1}{G_{LT}}\right)\sin^2 2\theta\right)$$
$$\varepsilon_y = \sigma_y\left(\frac{\sin^4\theta}{E_L} + \frac{\cos^4\theta}{E_T} + \frac{1}{4}\left(\frac{1}{G_{LT}} - \frac{2\nu_{LT}}{E_L}\right)\sin^2 2\theta\right)$$
$$\gamma_{xy} = -\sigma_y\sin^2 2\theta\left(\frac{\nu_{LT}}{E_L} + \frac{1}{E_T} - \frac{1}{2G_{LT}} - \sin^2\theta\left(\frac{1}{E_L} + \frac{2\nu_{LT}}{E_L} + \frac{1}{E_T} - \frac{1}{G_{LT}}\right)\right)$$

From the Above Equations we can get

$$\frac{1}{E_L} = \frac{\varepsilon_y}{\sigma_y} = \frac{\sin^4\theta}{E_L} + \frac{\cos^4\theta}{E_T} + \frac{1}{4}\left(\frac{1}{G_{LT}} - \frac{2\nu_{LT}}{E_L}\right)\sin^2 2\theta$$
$$\frac{\nu_{yx}}{E_Y} = -\frac{\varepsilon_x}{\varepsilon_y E_Y} = -\frac{\varepsilon_x}{\sigma_y} = \frac{\nu_{LT}}{E_L} - \frac{1}{4}\left(\frac{1}{E_L} + \frac{2\nu_{LT}}{E_L} + \frac{1}{E_T} - \frac{1}{G_{LT}}\right)\sin^2 2\theta$$
$$m_y = \frac{\gamma_{xy}}{\frac{\sigma_y}{E_L}} = \sin 2\theta\left(\nu_{LT} + \frac{E_L}{E_T} - \frac{E_L}{2G_{LT}} - \sin^2\theta\left(1 + 2\nu_{LT} + \frac{E_L}{E_T} - \frac{E_L}{2G_{LT}}\right)\right)$$

1.24 Question 25

When τ_{xy} is the only Non -zero Stress Strains are Given by following

$$\varepsilon_x = \tau_{xy}\sin 2\theta\left(\frac{\nu_{LT}}{E_L} + \frac{1}{E_T} - \frac{1}{2G_{LT}} - \cos^2\theta\left(\frac{1}{E_L} + \frac{2\nu_{LT}}{E_L} + \frac{1}{E_T} - \frac{1}{G_{LT}}\right)\right)$$
$$\varepsilon_y = \tau_{xy}\sin 2\theta\left(\frac{\nu_{LT}}{E_L} + \frac{1}{E_T} - \frac{1}{2G_{LT}} - \cos^2\theta\left(\frac{1}{E_L} + \frac{2\nu_{LT}}{E_L} + \frac{1}{E_T} - \frac{1}{G_{LT}}\right)\right)$$

Thus Definition of the m_x and m_y

$$m_x = \frac{\varepsilon_x}{\frac{\tau_{xy}}{E_L}} = \sin 2\theta\left(\nu_{LT} + \frac{E_L}{E_T} - \frac{E_L}{2G_{LT}} - \cos^2\theta\left(1 + 2\nu_{LT} + \frac{E_L}{E_T} - \frac{E_L}{2G_{LT}}\right)\right)$$
$$m_y = \frac{\varepsilon_y}{\frac{\tau_{xy}}{E_L}} = \sin 2\theta\left(\nu_{LT} + \frac{E_L}{E_T} - \frac{E_L}{2G_{LT}} - \cos^2\theta\left(1 + 2\nu_{LT} + \frac{E_L}{E_T} - \frac{E_L}{2G_{LT}}\right)\right)$$

1.25 Question 26

For Aluminium We have that E = 70GPa and $\nu = 0.33$ and for steel E = 210GPa and $\nu = 0.3$

$$[Q]_{AL} = \begin{bmatrix} 78.55 & 25.92 & 0 \\ 25.92 & 78.55 & 0 \\ 0 & 0 & 26.32 \end{bmatrix}$$
$$[Q]_{steel} = \begin{bmatrix} 224 & 56 & 0 \\ 56 & 224 & 0 \\ 0 & 0 & 84 \end{bmatrix}$$

Now We have the Relations

$$[A] = 5(Q_{AL} + Q_{steel}) \tag{1}$$
$$[B] = \frac{25}{2}(Q_{AL} + Q_{steel}) \tag{2}$$
$$[D] = \frac{125}{3}(Q_{AL} + Q_{steel}) \tag{3}$$

Calculating values we get that

$$A = \begin{bmatrix} 1512.75 & 409.60 & 0 \\ 409.6 & 1512.75 & 0 \\ 0 & 0 & 551.60 \end{bmatrix}$$
$$B = \begin{bmatrix} 1818 & 376 & 0 \\ 376 & 1818 & 0 \\ 0 & 0 & 721 \end{bmatrix}$$
$$D = \begin{bmatrix} 12606 & 3413 & 0 \\ 3416 & 12606 & 0 \\ 0 & 0 & 4597 \end{bmatrix}$$

Non Zero value of Coupling Matrix B indicated that Bimettalic Strip of aluminum and steel coupling between Extensiona and Bending Exist.

1.26 Question 27

Conditions for Quasi Isotropic matrices are

$$A_{11} = A_{22} \tag{4}$$
$$A_{11} - A_{12} = 2A_{66} \tag{5}$$
$$A_{16} = A_{26} = 0 \tag{6}$$

Now For An Exaple We can make a laminate Quasi isotric with following steps

- Total Number of Layers must Be Three or more
- individual layers must have identical stifness matrices
- Layes must be oriented at equal angles thus for n layers we must have angle between adgecent angles as $\frac{\pi}{n}$

Now the Given laminate has three layers and meet the Requirements thus forming a quasi isotropic Lamina

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——— THE END ———