Assignment 1
Experimental Stress Analysis

Submitted as Part of Course Requirement of Experimental Stress Analysis Course

Submitted By

A.V.S.Chaitanya
14AE30002

Indian Institute of Technology
Kharagpur

AVS

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IIT KGP

1 Solutions

1.1 Q.1) Define the Following Terms

Elasticity : The ability of a body to Resist a distorting influence and return to the original shape when the influence or the force is removed is called Elasticity.

Homogenous : A material or a system that has same properties everywhere with out irregularities in the material.

Anisotropy : It is a property of being directionally dependent thus having different properties when measured in the different direction.
Exmaples of Anisotroy: In many Materials the youngs modulus is different in different direction. Some other Examples are being that the Refractive index of the Quartz crystals are different in different direction.

Surface Forces: Surface and Body Forces are the Forces that act on the surface of the material Bodies. *Examples:* Normal and Shear Forces,Pressure.

Body Forces: Body forces are the force that acts on the bulk of the body by the virtue of the mass. *Exmaples:* 1) Centeripetal Force 2.)Gravity Forces 3.) Electrical and Magnetic Forces.

Plane Stress : Plane stress is a condition in which the stress is zero across a paticular surface over and entire structure.
Exmaples: Thin Plates are said to be instate of the Plane stress condition because the Stress on the surface with smaller depth doesn't contain sigificant amount of stress and can be assumend to be zero..

Plane Strain: When the displacement of Material Particles in one direction are assumed to be much smaller in scale of dimention of any part of the body it is not uncommon to assume that the strain in that direction is zero thus the spacimen is in plane strain condition. *Example:*In case of Infinitisemal Deformation of the Continous Body in which the vector and displacement gradient is assumed to be small compared to unity

Airy Stress Function: The Two Dimentional Problems where the Stresses can be written as single function of x and y such that the substitutuion of stresses in terms of this fucntion automatically satisfies the boundary conditions and comatibility equation. The Airy stress condition has to determined such that it satifies the Boundary condiitions and Compatibility Equations.
Airy stess fucntion in Cartesian Co-ordinates can be defined as

$$\sigma_x = \frac{\partial^2 \phi}{\partial y^2} \tag{1}$$

$$\sigma_y = \frac{\partial^2 \phi}{\partial x^2} \tag{2}$$

$$\tau_{xy} = \frac{\partial^2 \phi}{\partial x \partial y} \tag{3}$$

Airy stess fucntion in Polar Co-ordinates can be defined as

$$\sigma_r = \frac{1}{r}\frac{\partial^2 \phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2} \tag{4}$$

$$\sigma_\theta = \frac{\partial^2 \phi}{\partial x^2} \tag{5}$$

$$\tau_{r\theta} = \frac{\partial}{\partial \theta}(\frac{1}{r}\frac{\partial \phi}{r}) \tag{6}$$

Strain Gauge :- Strain Gauge is used to measure the strain on a object. It Typically Works by measuring strain by measureing the change in the resistance in the strain gauge.

Sensitivity :- Typical Strain Gauge is a Restistance Bonded to the specimen . The Resistance change rate is dependent on the length cross section area and the Resistivity.

$$\frac{dR}{R} = \frac{dl}{l} - \frac{dA}{A} + \frac{d\rho}{\rho} \tag{7}$$

Now the given Material Sensitivity of Resistance can be calibrated by

$$\mathbf{S} = \frac{\frac{dR}{R}}{\epsilon_l} \tag{8}$$

$$\epsilon_l = \frac{dl}{l} \tag{9}$$

$$\mathbf{S} = 1 + 2v + \frac{\frac{d\rho}{\rho}}{\epsilon_l} \tag{10}$$

This is called the Sensitivity of the Strain Gauge.

Gage Factor Gage Factor is the ratio of relative change in Eletric Resistance to Mechanical Strain.

$$GF = \frac{\frac{\delta R}{R}}{\epsilon} = \frac{\frac{\Delta\rho}{\rho}}{\epsilon} + 1 + 2v \tag{11}$$

Gage Size The Active area of the Strain Gage is Called Gage Size.

Range of Strain: The range of Temperature in which the Strain Gage can be operated without permanent change in the Mesurement Properties.

Presicion of readout: Now Prescicion is Description of Random Errors and Statistical Varability. This is Related to Repeatability and Reproducibility of measurement. A machine is said to be precise when the Reading Produced are very close. The Presicion of machine is different than Accuracy. **Accuracy:**Accuracy is Measurement of Closness of the Readout to the Exaact or true value

Eularen and Natural Strain Now the Average Strain is Taken over a Finite Portion body while a point strain is mesured with the dimentions of measured Portio is made zero by limit process.

Fintie Strains are Obtained by exact measure of Changes in dimentions while the infinitesimal strains are obtained by linearizing the finite strain with respect to the displacement gradient. Now the Definition of Stain is Given by

$$\epsilon_{ar} = \frac{L - L_0}{L_{ref}} = \frac{\delta}{L_{ref}} \tag{12}$$

$L_{ref} = L_0$ Initial Gauge Length → Langrangian Strain.
$L_{ref} = L$ Final Gauge Length → Eularean Strain.

Eularen Strain is Used in Fluids while the Langrangian Strain is Used in Solids. If Strain gets Smaller all the Versions of Strain definitions merge into Infitesimal Strain Version. **Natural Strain** is Defined by the Equation.

$$\ln \frac{l_f}{l_i} \tag{13}$$

where l_f id final length and l_i is initial length.

Engineering and Tensorial Strain: Finite Strain Thoery Deals With the Deformations in which both Rotations and strains are arbitarily large with infinitesimal strain theory deals with only small dispalcements. Engineering strain is used for structural and material engineering for small deformations.

Engineering Strain is simple ratio of deformation to initial dimention.The tensorial strain is defined in terms of the Engineering strain as follows.

$$\epsilon = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{bmatrix} = \begin{bmatrix} \epsilon_{xx} & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{yx} & \epsilon_{yy} & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{zx} & \frac{1}{2}\gamma_{zy} & \epsilon_{zz} \end{bmatrix} \tag{14}$$

where γ is engineering strain definition. Tensorial Strain Definitions are Useful in instance where we transform stress from one plane of reference to other plane of reference which is at aan angle to the former.

Strength of Materials and Elasitcity Approach: This Thoery Deals With behaviour of solid Objects when subject to stress and Strains. This Study often Refers to Various Methods of calculating Stress and Strains in structural Members such as beams, shafts, columns. This Predicts behavious of Loaded Specimen Using Properties like Yield Strength, Ultimate Strength, Youngs Modulus and Poission Ratio.This Elasticity appraoch uses there Properties to predict the Failure using Following Phenomena.

- **Maximum Principal stress theory**: This theory assume that when the maximum principal stress in a complex stress system reaches the elastic limit stress in a simple tension, failure will occur.
- **Maximum shear stress theory**: This theory states that teh failure can be assumed to occur when the maximum shear stress in the complex stress system is equal to the value of maximum shear stress in simple tension.
- **Maximum Principal strain theory**: This Theory assumes that failure occurs when the maximum strain for a complex state of stress system becomes equals to the strain at yield point in the tensile test for the three dimensional complex state of stress system.
- **Total strain energy per unit volume theory**:The theory assumes that the failure occurs when the total strain energy for a complex state of stress system is equal to that at the yield point a tensile test.
- **Maximum shear strain energy per unit volume theory** This theory states that the failure occurs when the maximum shear strain energy component for the complex state of stress system is equal to that at the yield point in the tensile test.

Laws Of Stress and Strain Transformation in 3-d scenarios : The Stress Transformation in 2 Dimentional Scenario is easily obtainable in the scenario. Now for Transforming Stresses in 3 dimentions we can have the Transformation Equation

$$[\sigma^{|}] = [Q].[\sigma].[Q^{|}] \tag{15}$$

Where Q is the Transformation Matrix.

$$Q_{ij} = \begin{bmatrix} cos(x_1, x_1^{|}) & cos(x_1, x_2^{|}) & cos(x_1, x_3^{|}) \\ cos(x_2, x_1^{|}) & cos(x_2, x_2^{|}) & cos(x_2, x_3^{|}) \\ cos(x_3, x_1^{|}) & cos(x_3, x_2^{|}) & cos(x_3, x_3^{|}) \end{bmatrix} \tag{16}$$

Compatibilty Equations Form given displacement field three equations expressing u,v,w as functions of x,y,z we may get a unique solution but the detemined strain field may impose a impossible displacement field. For Example it may Result in a displacement field that contains voids after deformations in homogenous material. To ensure valid dispalcement field we impose compatibility conditions.

The Six Equations of Compatibility conditions are given by :

$$\frac{\partial^2 \epsilon_{yy}}{\partial z^2} + \frac{\partial^2 \epsilon_{zz}}{\partial y^2} = 2\frac{\partial^2 \epsilon_{yz}}{\partial y \partial z} \tag{17}$$

$$\frac{\partial^2 \epsilon_{zz}}{\partial x^2} + \frac{\partial^2 \epsilon_{xx}}{\partial z^2} = 2\frac{\partial^2 \epsilon_{zx}}{\partial x \partial z} \tag{18}$$

$$\frac{\partial^2 \epsilon_{xx}}{\partial y^2} + \frac{\partial^2 \epsilon_{yy}}{\partial x^2} = 2\frac{\partial^2 \epsilon_{xy}}{\partial y \partial x} \tag{19}$$

$$\frac{\partial(-\frac{\partial \epsilon_{yz}}{\partial x} + \frac{\partial \epsilon_{zx}}{\partial y} + \frac{\partial \epsilon_{xy}}{\partial z})}{\partial x} = \frac{\partial^2 \epsilon_{xx}}{\partial y \partial z} \tag{20}$$

$$\frac{\partial(\frac{\partial \epsilon_{yz}}{\partial x} - \frac{\partial \epsilon_{zx}}{\partial y} + \frac{\partial \epsilon_{xy}}{\partial z})}{\partial y} = \frac{\partial^2 \epsilon_{yy}}{\partial x \partial z} \tag{21}$$

$$\frac{\partial(\frac{\partial \epsilon_{yz}}{\partial x} + \frac{\partial \epsilon_{zx}}{\partial y} - \frac{\partial \epsilon_{xy}}{\partial z})}{\partial z} = \frac{\partial^2 \epsilon_{zz}}{\partial y \partial x} \tag{22}$$

These are the six Campatibility Equations in The Cartesian Co-ordinates.

Principal Stresses The Resultant Stress Vector at T_n at a point p is depended upon the choice of plane on which the Stress is acting. If the plane is selected such that the T_n conincides with the outer normal of the plane then *shear stress vanishes*. and T_n σ_n and n are coincident such stress is called Principal stress and corresponding plane is called principal plane. For Example Consider a Cylinder and the Stress is acted on the plane cross section that is perpendicular to the cylinder then the stress corresponding is principle stress.

10 Charecterstics Used To Judge Strain Gauge:

- The Calibration constant should be same and Should not vary with time or temperature
- The Gauge should be able to measure strains with accuracy of $\pm 1\mu\frac{inch}{inch}$ over strain range of 10 Percent.
- The gauge size (i.e Gauge length and Gauge width) should so that strain at a point is adequtly approximated.
- The Responce of the gauge largely controlled by the Interia should be sufficiet to allow for Dynamics strain Recoding.
- The Strain should support for onsite or Remote Readout.
- The Readout should be independent of the temperature and the ther environmental variables.
- The gauge and the associated equipment is economically feasible.
- The Gauge system shouldn't have complex installation or Operation Procedure.
- Gauge should Exhibit Linear Responce to Strain.
- Gauge should be suitable to use as a sensing element where other property is measured in terms of strain.

1.2 Q.2) Strain Mesurement Techniques

There are many measures Used in measurement of Strain

- Mechanical
- Optical

- Electrical
- Accoustic
- Grid Techniques

1.2.1 Mechanical

Mechanical strain gauges are also known as Extensometers used to measure static or gradually varying load conditions. These gauges are usually provided with two knife edges which are clamped firmly in contact with the test component by means of a clamping spring at a specific distance of gauge length. When the specimen under testing is strained the knife edges undergoes displacement, this displacement is amplified by a mechanical linkages and the strain is displaced on a calibrated scale.

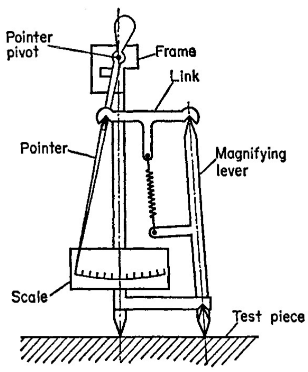

Figure 1: Huggenbeger Extensometer

Reading = Amplification Factor * Strain

Now If We know the Amplification Factor Then We can Calcualte the Strain and thus we can see the Strain.

1.2.2 Optical Method

There are Multiple Methods of Measuring Strain using optical techniques. One of Such mehtod is Using Sharp edges slits attached to the specimen. Now one edge is firmly attached to the Specimen while the other is movable with extension. Now The diffraction pattern produced by the slit by illuminating light on the slit. Now we can Measure the strain using the diffraction pattern formula thus calculating the formula.Now When we shine monochromatic light source on the slit we can measure the the formation of diffraction and then based on the pattern formed we can calculate the Strain on the specimen.

1.2.3 Electrical Method

The principle of the electrical resistance strain gauge was discovered by Lord Kelvin when he observed that the resistance of a wire alters when it is subjected to stress.

A typical strain gauge element consists of a continuous length of resistance wire of about 0.025 mm diameter wound as and cemented to a paper backing. Gauges are also available in which the conductor takes the form of a metallic foil "ribbon", 0-01 mm to 0-025 mm thick.

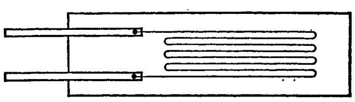

Figure 2: Resistance Based Strain gauge

As the test specimen extends or contracts under stress in the direction of the windings, the length and cross-sectional area of the conductor alter, resulting in a corresponding increase or decrease in electrical resistance. The alteration in resistance is, within wide limits, proportional to the strain in the specimen, that is:

$$\epsilon = \frac{1}{k}\frac{\delta R_0}{R_0} \tag{23}$$

where k is termed the gauge factor and R_0 is the gauge resistance. On the basis of change of length and cross-sectional area only, it can be shown that the gauge factor would be (1 + 2v), where v is Poisson's ratio. However, for most materials the specific resistance of the conductor changes with strain and the gauge factor is usually about 2.0.It will be appreciated that although a gauge of the type shown above is intended to measure strains in the direction of its length only, the small lengths of the conductor in the direction at right angles to the axis of the gauge impart some degree of sensitivity to transverse strain. fortunately, this effect is small enough to be neglected in work of normal accuracy, amounting to 2 per cent of longitudinal sensitivity in wire wound gauges and about one tenth of this in foil gauges. It will also be noted that a linear gauge measures normal strains only.A Simple Inner Circuit Working for Measuring The Resistance changes are shown in below Figure

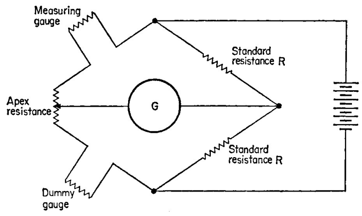

Figure 3: Resistance Change Measuring Circuit

1.2.4 Capacitance based circuits

The possibility obviously exists of arranging a condenser in such a way that the spacing of its plates, and thus its capacity, is altered by variation of the distance between two gauge points. This principle has in fact been explored and has been used for strain measurement . The simplest arrangement is suggested in below figure where two condenser plates are attached to the test piece by suitable posts mounted at some convenient distance apart; one of these plates is earthed and the other is connected to a circuit, by means of which the small change of capacity due to relative movement of the plates can be measured. The simplest arrangement is that shown below figure in which the measuring condenser C_1 and a shunt condenser C_2 are charged to about 100 V by a battery B_1.

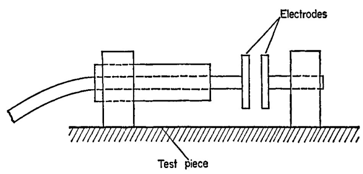

Figure 4: Capacitance Based Strain Gauge

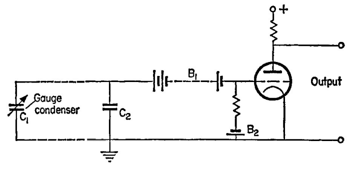

Figure 5: Capacitance Based Stain Gauge Circuit

The small grid bias battery B_1 sets the grid voltage at such a value as will ensure that the output from the valve is directly proportional to the charge in C_1, at least over a useful range. The output is fed to a cathode ray oscilloscope for observation or recording. The electrical characteristics of this circuit are such that it is unsuitable for measuring static or slowly varying strains. it is, however, satisfactory for observations of short period transient phenomena.

1.2.5 Induction Based Strain Gauges

The third electrical circuit parameter which can be altered by mechanical means is the inductance of a coil.The eddy current method depends on the fact that the impedance of a coil is affected by moving a non-magnetic conductor through its field. Finally, use may be made of the fact that the magnetic characteristics of certain materials are affected by stress so that it is possible to use the magneto striction effect to measure a force.The simplest method of detecting the change of inductance of a coil is to place an ammeter in the supply line to the coil as in below figure

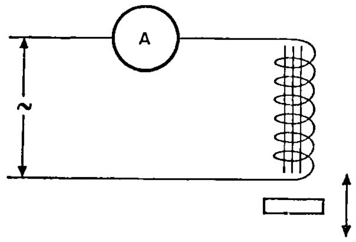

Figure 6: Inductance Based Strain Gauge

This, however, has the serious disadvantage that the alteration of the current passing through the coil due to a change in the position of the armature (or the core in the case of a variable core instrument), may be small compared with the total current passing and thus only a small part of the ammeter scale is usefully employed. It is, therefore better to use a bridge circuit or an arrangement of inductances which limits the measured signal to the change produced by the effect being observed. The basic form of bridge circuit consists of four inductances arranged as in Below.

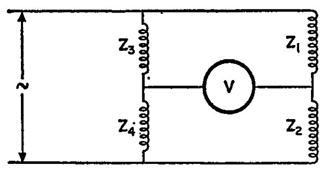

Figure 7: Inductance Based Stain Gauge Circuit

1.2.6 Accoustic Method

This method is paticularly useful for strain testing thin wires. Now the Wires are Introduced into setup and the Then we apply reduired force to produce the Strain then the Thin wires is exited to produce natural frequency ant the strain is measured from the beats phenomenag occuring dute to interference of the original and strained wires. Then From the Phenomenah we can calculate the Strain in the Wire precisely.

1.2.7 Grid Method

In this Grid Mehtod We Attach a Grid made of thread To the Testing Specimen like Circular plate and then when the stress is applied on the Circular Plate We can mesure the Strain using our original definition of strain and thus can accurately calculte the strain.

In a similar method we can calculate the Strain as above but here we can simply Make a replica of the original species with carving the grid on top of the specimen. This is Called **Replica Method**.

2 Q.4) Prove That $\sigma_{xy} = \sigma_{yx}$, $\sigma_{xz} = \sigma_{zx}$, $\sigma_{zy} = \sigma_{yz}$?

We can write the Stress Tensor of the Element with stress direction denoted as follows. Now the σ_{zx} denotes that the stress on plane Z in the x direction. Now we can apply equilibrium equations on the element.

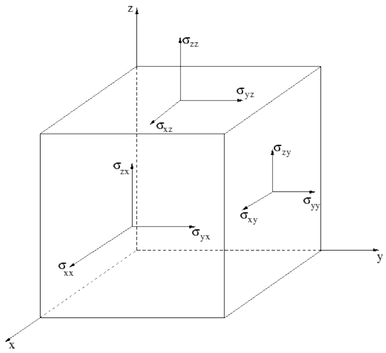

Figure 8: Stress Tensor

Applying the Equations of Equilibirum on the element we get Force balance Equations in x direction as follows:

$$(\frac{\partial \sigma_{xx}}{\partial x}dx)dydz + (\frac{\partial \tau_{yx}}{\partial y}dy)dxdz + (\frac{\partial \sigma_{zx}}{\partial z}dz)dydx + F_x = 0 \tag{24}$$

Now applying this equation in y,z directions we get the equations we can write

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \sigma_{zx}}{\partial z} + F_x = 0 \tag{25}$$

$$\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \sigma_{zy}}{\partial z} + F_y = 0 \tag{26}$$

$$\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + F_z = 0 \tag{27}$$

Now Similarly we can Write the Moment banlancing Equations we can write

$$(\tau_{zx} + \frac{\partial \tau_{zx}}{\partial z}\frac{dz}{2})dxdy\frac{dz}{2} + (\tau_{zx} - \frac{\partial \tau_{zx}}{\partial z}\frac{dz}{2})dxdy\frac{dz}{2} + (\tau_{xz} + \frac{\partial \tau_{xz}}{\partial x}\frac{dx}{2})dxdy\frac{dz}{2} + (\tau_{xz} + \frac{\partial \tau_{xz}}{\partial x}\frac{dx}{2})dxdy\frac{dx}{2} = 0 \tag{28}$$

$$\tau_{zx}dxdydz + \tau_{xz}dxdydz = 0 \tag{29}$$

$$\tau_{zx} = \tau_{xz} \tag{30}$$

Now similarly We can use the Momentum Equations in other Direction to prove that

$$\tau_{yx} = \tau_{xy} \tag{31}$$

$$\tau_{zy} = \tau_{yz} \tag{32}$$

Hence we Can prove that the Static Stress tensor is Symmetric.

3 Question 5

We have been Given the Determinant

$$\begin{bmatrix} \sigma_{xx} - \sigma_n & \tau_{yx} & \tau_{zx} \\ \tau_{xx} & \sigma_{yy} - \sigma_n & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_{zz} - \sigma_n \end{bmatrix} = 0 \tag{33}$$

Now this Signifies that if Determinant is zero then there exist non trivial solutions for directional cosines of principle palne.Expading The Determinant we get the Equation.

$$\sigma_n^3 - (\sigma_{xx} + \sigma_{yy} + \sigma_{zz})\sigma_n^2 + (\sigma_{xx}\sigma_{yy} + \sigma_{yy}\sigma_{zz} + \sigma_{zz}\sigma_{xx} - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2)\sigma_n - (\sigma_{xx}\sigma_{yy}\sigma_{zz} + 2\tau_{xy}\tau_{yz}\tau_{zx} - \sigma_{xx}\tau_{yz}^2 - \sigma_{yy}\tau_{zx}^2 - \sigma_{zz}\tau_{xz}^2) = 0 \quad (34)$$

The Roots of the Above Equation are Principal Stresses. Now We can Solve for the σ_n by substituting the Cartesian stress tensor

1. If σ_1, σ_2, σ_3 are distinct then n_1, n_2, n_3 are unique and mutually perpendicualr.
2. if $\sigma_1 = \sigma_2 \neq \sigma_3$ then n_3 is unique and every direction perpendicular to n_3 is Principle Direction.
3. if $\sigma_1 = \sigma_2 = \sigma_3$ then Hydrostatic State of Stress Exist and Every Direction is Principle Direction.

4 Question 6

Now the Cartesian stress Co ordinates are Given by

$$\sigma_{xx} = 60MPa \quad (35)$$
$$\sigma_{yy} = -30MPa \quad (36)$$
$$\sigma_{zz} = 30MPa \quad (37)$$
$$\tau_{xy} = 40MPa \quad (38)$$
$$\tau_{yz} = 0MPa \quad (39)$$
$$\tau_{zx} = 0MPa \quad (40)$$
$$\quad (41)$$

We are Asked to find the Normal Shear Strain on a plane with following angles to cartesian system

$$\cos(n,x) = \frac{6}{11} \quad (42)$$
$$\cos(n,y) = \frac{6}{11} \quad (43)$$
$$\cos(n,z) = \frac{7}{11} \quad (44)$$

The Angles are

$$(n,x) = 56.944 \quad (45)$$
$$(n,y) = 56.944 \quad (46)$$
$$(n,z) = 50.478 \quad (47)$$

Now to Calculate to the Stress on the given plane is calculated from the 3-d stress transformation.We can Write the Cauchy Transformation Law as follows.

$$T_n = [\sigma].\hat{n} \quad (48)$$

Now substitute the values in the Formulas

$$Q_{ij} = \begin{bmatrix} \frac{6}{11} \\ \frac{6}{11} \\ \frac{7}{11} \end{bmatrix} \quad (49)$$

$$T_{ij} = \begin{bmatrix} 60 & 40 & 0 \\ 40 & 30 & 0 \\ 0 & 0 & -30 \end{bmatrix} MPa \quad (50)$$

Now Calculating the Stress on the Plane

$$T^{|}_{ij} = \begin{bmatrix} 54.5454 \\ 38.1818 \\ -19.09 \end{bmatrix} MPa \tag{51}$$

Now We need to Calculate the

$$\begin{bmatrix} \sigma_{xx}-\sigma_n & \tau_{yx} & \tau_{zx} \\ \tau_{xx} & \sigma_{yy}-\sigma_n & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_{zz}-\sigma_n \end{bmatrix} = 0 \tag{52}$$

$$\sigma_n^3 - (\sigma_{xx}+\sigma_{yy}+\sigma_{zz})\sigma_n^2 + (\sigma_{xx}\sigma_{yy}+\sigma_{yy}\sigma_{zz}+\sigma_{zz}\sigma_{xx}-\tau_{xy}^2-\tau_{yz}^2-\tau_{zx}^2)\sigma_n - (\sigma_{xx}\sigma_{yy}\sigma_{zz}+2\tau_{xy}\tau_{yz}\tau_{zx}-\sigma_{xx}\tau_{yz}^2-\sigma_{yy}\tau_{zx}^2-\sigma_{zz}\tau_{xz}^2) = 0 \tag{53}$$

The Principal Stresses are $-36.55Mpa$ and $48.279 \pm i21.42Mpa$ Thae values of the Maximum shear stress are given by

$$\tau_{12} = \frac{\sigma_1-\sigma_2}{2} \tag{54}$$
$$\tau_{23} = \frac{\sigma_2-\sigma_3}{2} \tag{55}$$
$$\tau_{31} = \frac{\sigma_3-\sigma_1}{2} \tag{56}$$
$$\tag{57}$$

From this the Values the maximum is $2*48.279MPa$

5 Question 7

Now the Cartesian stress Co ordinates are Given by

$$\sigma_{xx} = 70MPa \tag{58}$$
$$\sigma_{yy} = 60MPa \tag{59}$$
$$\sigma_{zz} = 50MPa \tag{60}$$
$$\tau_{xy} = 20MPa \tag{61}$$
$$\tau_{yz} = -20MPa \tag{62}$$
$$\tau_{zx} = 0MPa \tag{63}$$
$$\tag{64}$$

We are Asked to find the Normal Shear Strain on a plane with following angles to cartesian system

$$\cos(n,x) = \frac{12}{25} \tag{65}$$
$$\cos(n,y) = \frac{15}{25} \tag{66}$$
$$\cos(n,z) = \frac{16}{25} \tag{67}$$

Now to Calculate to the Stress on the given plane is calculated from the Cauchy Transformation.The Stress on the Plane is Calculated by using Following Formula

$$T_n = [\sigma].\hat{n} \tag{68}$$

Now substitute the values in the Formulas

$$Q_{ij} = \begin{bmatrix} \frac{12}{25} \\ \frac{15}{25} \\ \frac{16}{25} \end{bmatrix} \tag{69}$$

$$T_{ij} = \begin{bmatrix} 70 & 20 & 0 \\ 20 & 60 & 0 \\ 0 & 0 & 50 \end{bmatrix} MPa \tag{70}$$

Now Calculating the Stress on the Plane

$$T^{|}_{ij} = \begin{bmatrix} 45.6 \\ 45.6 \\ 32 \end{bmatrix} MPa \tag{71}$$

Now We need to Calculate the

$$\begin{bmatrix} \sigma_{xx} - \sigma_n & \tau_{yx} & \tau_{zx} \\ \tau_{xx} & \sigma_{yy} - \sigma_n & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_{zz} - \sigma_n \end{bmatrix} = 0 \tag{72}$$

$$\sigma_n^3 - (\sigma_{xx} + \sigma_{yy} + \sigma_{zz})\sigma_n^2 + (\sigma_{xx}\sigma_{yy} + \sigma_{yy}\sigma_{zz} + \sigma_{zz}\sigma_{xx} - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2)\sigma_n - (\sigma_{xx}\sigma_{yy}\sigma_{zz} + 2\tau_{xy}\tau_{yz}\tau_{zx} - \sigma_{xx}\tau_{yz}^2 - \sigma_{yy}\tau_{zx}^2 - \sigma_{zz}\tau_{xz}^2) = 0 \tag{73}$$

The Principal Stresses are $90Mpa$ and $30Mpa$ and $60Mpa$. Thae values of the Maximum shear stress are given by

$$\tau_{12} = \frac{\sigma_1 - \sigma_2}{2} \tag{74}$$

$$\tau_{23} = \frac{\sigma_2 - \sigma_3}{2} \tag{75}$$

$$\tau_{31} = \frac{\sigma_3 - \sigma_1}{2} \tag{76}$$

$$\tag{77}$$

From this the Values are $30MPa$,$-15MPa$,$-15MPa$ so the maximum is $30MPa$

6 Question 8

Given to us to solve the Case of the Simply supported beam that has uniformly distributed load q on the length l heigh t and unit width and compare strength of material solutions.Now for the given situations we have to calculate stresses and strains. The Airty stress function assumed as.

$$\phi = \frac{a}{2}x^2 + \frac{b}{2}x^2y + \frac{c}{6}y^3 + \frac{d}{6}x^2y^3 - \frac{d}{30}y^5 \tag{78}$$

Now By Definition we have that

$$\sigma_x = \frac{\partial phi^2}{\partial y^2} = cy + d(x^2y - \frac{2y^3}{3} \tag{79}$$

$$\sigma_y = \frac{\partial phi^2}{\partial x^2} = a + by + \frac{dy^3}{3} \tag{80}$$

$$\tau_{xy} = \frac{\partial phi^2}{\partial y \partial x} = -bx - dxy^2 \tag{81}$$

Now for Simply Supported Beam Following Conditions are imposed

$$(\tau_{xy})_{y=\pm\frac{h}{2}} = 0 \tag{82}$$
$$(\tau_y)_{y=\frac{h}{2}} = 0 \tag{83}$$
$$(\tau_y)_{y=-\frac{h}{2}} = -q \tag{84}$$

Now at $x = \pm\frac{L}{2}$

$$\int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x dy = 0 \tag{85}$$
$$\int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x y dy = -q \tag{86}$$
$$\int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{xy} dy = 0 \tag{87}$$

Now We can apply the boundary conditions to get the following

$$\sigma_x = \frac{q}{2I}(L^2 - x^2)y + \frac{q}{I}(\frac{y^3}{3} - \frac{h^2 y}{5}) + \tag{88}$$
$$\sigma_y = \frac{q}{2I}(y^3 - y^2 h) + \frac{2}{3}h^3 \tag{89}$$
$$\tau_{xy} = \frac{q}{2I}x(h^2 - y^2) \tag{90}$$

7 Question 9

Given to us to solve the Case of the End Loaded Cantilever beam that End Loaded load q on the length l heigh t and unit width and compare strength of material solutions.Now for the given situations we have to calculate stresses and strains. The Airty stress function assumed as.

$$\phi = Ax^2 + Bx^2 y + Cy^3 + 5Dx^2 y^3 - Dy^5 \tag{91}$$

Now By Definition we have that

$$\sigma_x = \frac{\partial phi^2}{\partial y^2} = 6Cy + 30Dx^2 y - 20Dy^3 \tag{92}$$
$$\sigma_y = \frac{\partial phi^2}{\partial x^2} = 2A + 2By + 10Dy^3 \tag{93}$$
$$\tau_{xy} = \frac{\partial phi^2}{\partial y \partial x} = -2Bx - 30Dxy^2 \tag{94}$$

Now for Simply Supported Beam Following Conditions are imposed

$$(\tau_{xy})_{y=\pm\frac{h}{2}} = 0 \tag{95}$$
$$(\tau_y)_{y=\frac{h}{2}} = 0 \tag{96}$$
$$(\tau_y)_{y=-\frac{h}{2}} = -q \tag{97}$$

Now at $x = 0$

$$\int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x y dy = 0 \tag{98}$$

Now We can apply the boundary conditions to get the following

$$A = -\frac{q}{4} \tag{99}$$

$$B = \frac{3q}{4h} \tag{100}$$

$$C = -\frac{q}{10h} \tag{101}$$

$$D = -\frac{q}{5h^3} \tag{102}$$

——— THE END ———