ASSIGNMENT 1
AEROELASTICITY

SUBMITTED AS PART OF COURSE REQUIREMENT OF AEROELASTICITY COURSE

SUBMITTED BY

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1 Question

Consider a One Degree of Freedom linear and Non Linear System and Calculate the Responce for Following Conditions

- Linear Aerodynamics and Linear Elastic
- Non-linear Aerodynamics (Quadrtic) and Linear Elastic
- Linear Aerodynamics and Non-Linear Elastic (Quadratic)
- Linear Aerodynamics and Non-Linear Elastic (Cubic)
- Non-Linear Aerodynamics(Quadratic) and Non-Linear Elastic (Quadratic)

Consider The Following Parameters for plotting Responces

$$\alpha_R = -5^0, -3^0, 0, +3^0, +5^0 \tag{1}$$
$$\overline{q} = 0:0.1:2 \tag{2}$$
$$l_a = 0:0.1:2 \tag{3}$$
$$r_s = 0:0.1:2 \tag{4}$$

2 Solutions

Now Assuming the one Dimensonal System, The Kinetic Energy of the System is Zero since the system is not moving.

2.1 Linear Aerodynamics and Linear Elastic

Potential Energy of the System

$$U = \frac{1}{2}K_\theta \theta^2 \tag{5}$$
$$L = \frac{1}{2}\rho v^2 c C_L \tag{6}$$
$$C_L = a(\alpha + \theta) \tag{7}$$
$$W = \int L \times e c d\theta \tag{8}$$

Now Applying the Langranges Energy Equation

$$\frac{\partial \frac{\partial T}{\partial x_j}}{\partial t} - \frac{\partial T}{\partial x_j} + \frac{\partial \tau}{\partial x_j} + \frac{\partial U}{\partial x_j} = Q_j = \frac{\partial \delta W}{\partial \delta x_j} \tag{9}$$

- c - chord length
- C_L - lift Cofficient
- τ - Work Done By Dissipitive Forces
- W - Work
- q = $\frac{1}{2}\rho V^2$ - Dynamics Pressure

Now Substituting the values WE get the Equations

$$K\theta = qec^2 a(\alpha+\theta) \tag{10}$$

$$\theta = \frac{qec^2 a\alpha}{K - qec^2 a} \tag{11}$$

$$q_D = \frac{K}{ec^2 a} \tag{12}$$

$$\overline{q} = \frac{q}{q_D} \tag{13}$$

$$\theta = \frac{\overline{q}\alpha}{1-\overline{q}} \tag{14}$$

Now We Can Plot The Responce Curve as Follows

2.2 Non-linear Aerodynamics (Quadrtic) and Linear Elastic

Now Assuming the one Dimensonal System, The Kinetic Energy of the System is Zero since the system is not moving.

Potential Energy of the System

$$U = \frac{1}{2}K_\theta \theta^2 \tag{15}$$

$$L = \frac{1}{2}\rho v^2 c C_L \tag{16}$$

$$C_L = a_1(\alpha+\theta) + a_1(\alpha+\theta)^2 \tag{17}$$

$$W = \int L \times ec d\theta \tag{18}$$

$$W = qec^2\left(a_1\int(\alpha+\theta)d\theta + a_2\int(\alpha+\theta)^2 d\theta\right) \tag{19}$$

$$W = qec^2\left(a_1\left(\alpha\theta + \frac{\theta^2}{2}\right) + a_2\left(\alpha^2\theta + \frac{\theta^3}{3} + \alpha\theta^2\right)\right) \tag{20}$$

Now Substituting this in Langranges Equation and solving for theta we get that

$$\theta = \frac{-\left(-\frac{1}{q} - 1 + 2r\alpha_R\right) \pm \sqrt{-\left(-\frac{1}{q} - 1 + 2r\alpha_R\right)^2 - 4r(r\alpha_R^2 - \alpha_R)}}{2r} \tag{21}$$

$$r = \frac{a_2}{a_1} \tag{22}$$

2.3 Linear Aerodynamics and Non-Linear Elastic

2.3.1 Quardic Elastic

Potential Energy of the System

$$M = K_{theta_0}\theta + \theta_{theta_1}\theta^2 \tag{23}$$

$$L = \frac{1}{2}\rho v^2 c C_L \tag{24}$$

$$C_L = a(\alpha+\theta) \tag{25}$$

$$W = \int L \times ec d\theta \tag{26}$$

$$W = qec^2\left(a_1\int(\alpha+\theta)d\theta\right) \tag{27}$$

Now Solving For Theta We get

$$\theta_{1,2} = \frac{-(1-\overline{q}) \pm \sqrt{(1-\overline{q})^2 + 4\overline{q}r\alpha_R}}{2r} \tag{28}$$

2.3.2 Cubic Elastic

Potential Energy of the System

$$M = K_{theta_0}\theta + \theta_{theta_1}\theta^3 \tag{29}$$
$$L = \frac{1}{2}\rho v^2 c C_L \tag{30}$$
$$C_L = a(\alpha + \theta) \tag{31}$$
$$W = \int L \times ec d\theta \tag{32}$$
$$W = qec^2\left(a_1 \int (\alpha + \theta) d\theta\right) \tag{33}$$

Now Solving For Theta We get

$$q_D = \frac{K_{\theta_0}}{ec^2 C_{L_\alpha}} \tag{34}$$
$$\overline{q} = \frac{q}{q_D} \tag{35}$$
$$r = \frac{K_{\theta_1}}{K_{\theta_0}} \tag{36}$$
$$\overline{q}(\alpha_R + \theta) = \theta + r\theta^3 \tag{37}$$

Now solving this we get hte Real solution

$$\theta = -\frac{-3\overline{q}+3}{3\sqrt[3]{\frac{27\alpha_r\overline{q}}{2} + \frac{\sqrt{729\alpha_r^2\overline{q}^2 - 4(-3\overline{q}+3)^3}}{2}}} - \frac{\sqrt[3]{\frac{27\alpha_r\overline{q}}{2} + \frac{\sqrt{729\alpha_r^2\overline{q}^2 - 4(-3\overline{q}+3)^3}}{2}}}{3} \tag{38}$$

$$\theta = -\frac{-3\overline{q}+3}{3\left(-\frac{1}{2} - \frac{\sqrt{3}i}{2}\right)\sqrt[3]{\frac{27\alpha_r\overline{q}}{2} + \frac{\sqrt{729\alpha_r^2\overline{q}^2 - 4(-3\overline{q}+3)^3}}{2}}} - \frac{\left(-\frac{1}{2} - \frac{\sqrt{3}i}{2}\right)\sqrt[3]{\frac{27\alpha_r\overline{q}}{2} + \frac{\sqrt{729\alpha_r^2\overline{q}^2 - 4(-3\overline{q}+3)^3}}{2}}}{3} \tag{39}$$

$$\theta = -\frac{-3\overline{q}+3}{3\left(-\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)\sqrt[3]{\frac{27\alpha_r\overline{q}}{2} + \frac{\sqrt{729\alpha_r^2\overline{q}^2 - 4(-3\overline{q}+3)^3}}{2}}} - \frac{\left(-\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)\sqrt[3]{\frac{27\alpha_r\overline{q}}{2} + \frac{\sqrt{729\alpha_r^2\overline{q}^2 - 4(-3\overline{q}+3)^3}}{2}}}{3} \tag{40}$$

for $\alpha = 0$ we get

$$\theta = 0 \tag{41}$$
$$\theta = \pm\frac{\overline{q} - 1}{r} \tag{42}$$

2.4 Non-Linear Aerodynamics and Non-Linear Elastic

$$M = K_{theta_0}\theta + \theta_{theta_1}\theta^2 \tag{43}$$
$$L = \frac{1}{2}\rho v^2 c C_L \tag{44}$$
$$C_L = a_1(\alpha + \theta) + a_1(\alpha + \theta)^2 \tag{45}$$
$$W = \int L \times ec d\theta \tag{46}$$
$$W = qec^2\left(a_1 \int (\alpha + \theta) d\theta + a_2 \int (\alpha + \theta)^2 d\theta\right) \tag{47}$$
$$W = qec^2\left(a_1\left(\alpha\theta + \frac{\theta^2}{2}\right) + a_2\left(\alpha^2\theta + \frac{\theta^3}{3} + \alpha\theta^2\right)\right) \tag{48}$$

Now Solving For Theta We get

$$q_D = \frac{K_{\theta_0}}{ec^2 C_{L_\alpha}} \tag{49}$$

$$\overline{q} = \frac{q}{q_D} \tag{50}$$

$$r_1 = \frac{K_{\theta_1}}{K_{\theta_0}} \tag{51}$$

$$r_2 = \frac{a_2}{a_1} \tag{52}$$

$$qec^2\left(a_1(\alpha_R + \theta) + a_2(\alpha_R + \theta)^2\right) = k_{\theta_0}\theta + k_{\theta_1}\theta^2 \tag{53}$$

$$\overline{q}((\alpha_R + \theta) + r_2(\alpha_R + \theta)^2) = \theta + r_1\theta^2 \tag{54}$$

Now Solving The Equation We get the Result

$$\theta = \frac{-2\alpha_r\overline{q}r_2 - \overline{q} - \sqrt{4\alpha_r^2\overline{q}r_1r_2 + 4\alpha_r\overline{q}r_1 - 4\alpha_r\overline{q}r_2 + \overline{q}^2 - 2\overline{q} + 1} + 1}{2(\overline{q}r_2 - r_1)} \tag{55}$$

$$\theta = \frac{-2\alpha_r\overline{q}r_2 - \overline{q} + \sqrt{4\alpha_r^2\overline{q}r_1r_2 + 4\alpha_r\overline{q}r_1 - 4\alpha_r\overline{q}r_2 + \overline{q}^2 - 2\overline{q} + 1} + 1}{2(\overline{q}r_2 - r_1)} \tag{56}$$

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