Flight Vehicle Terminology

1.0 Axes Systems

There are 3 axes systems which can be used in Aeronautics, Aerodynamics & Flight Mechanics:

- Ground Axes – $G(x_0, y_0, z_0)$
- Body Axes – $G(x, y, z)$
- Aerodynamic Axes – $G(x_a, y_a, z_a)$

1.1 Ground Axes

(x_0, y_0, z_0) are an orthogonal set of forces obeying the right hand rule.
z_0 is in the vertical plane of symmetry normal to the datum axis where positive is down.

1.2 Body Axes

(x, y, z) are an orthogonal set of forces obeying the right hand rule.

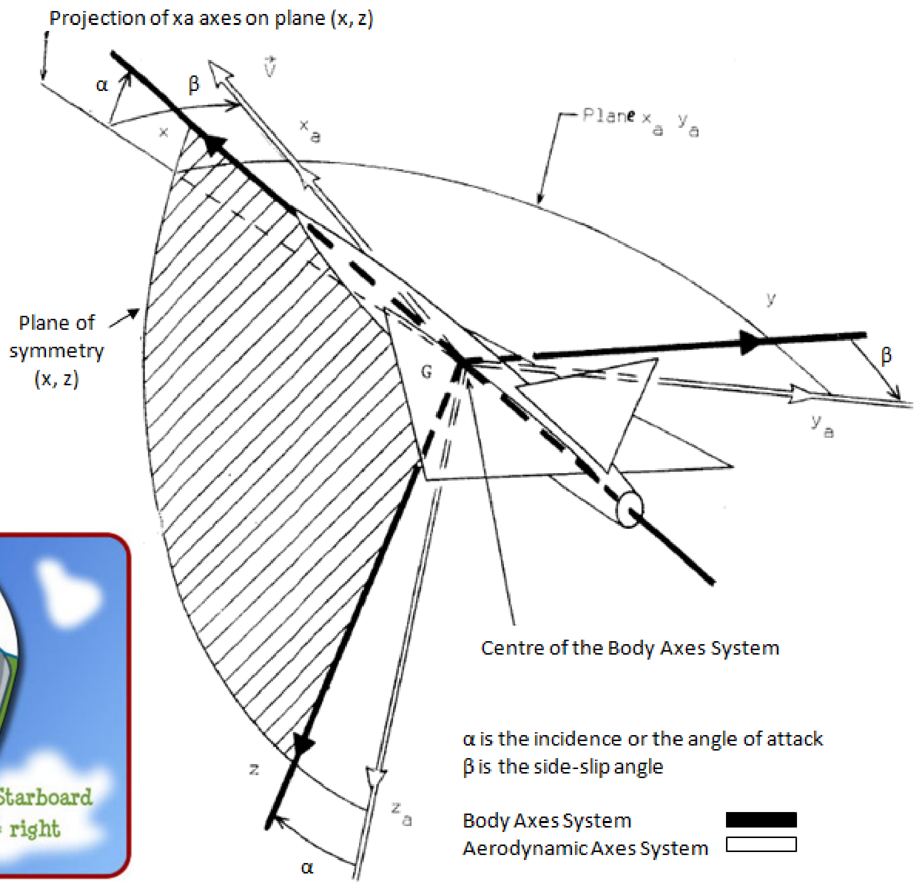
- **x** is along the model datum axis positive forward
- **y** side force is normal to the vertical plane of symmetry positive to starboard
- **z** is in the vertical plane of symmetry normal to the datum axis positive down

1.3 Aerodynamic Axes

- $\mathbf{x_a}$ is along the velocity vector **v** with the same direction
- $\mathbf{y_a}$ is the same as body axes
- $\mathbf{z_a}$ is in the vertical plane of symmetry normal to the aerodynamic axis positive down

2.0 Angles

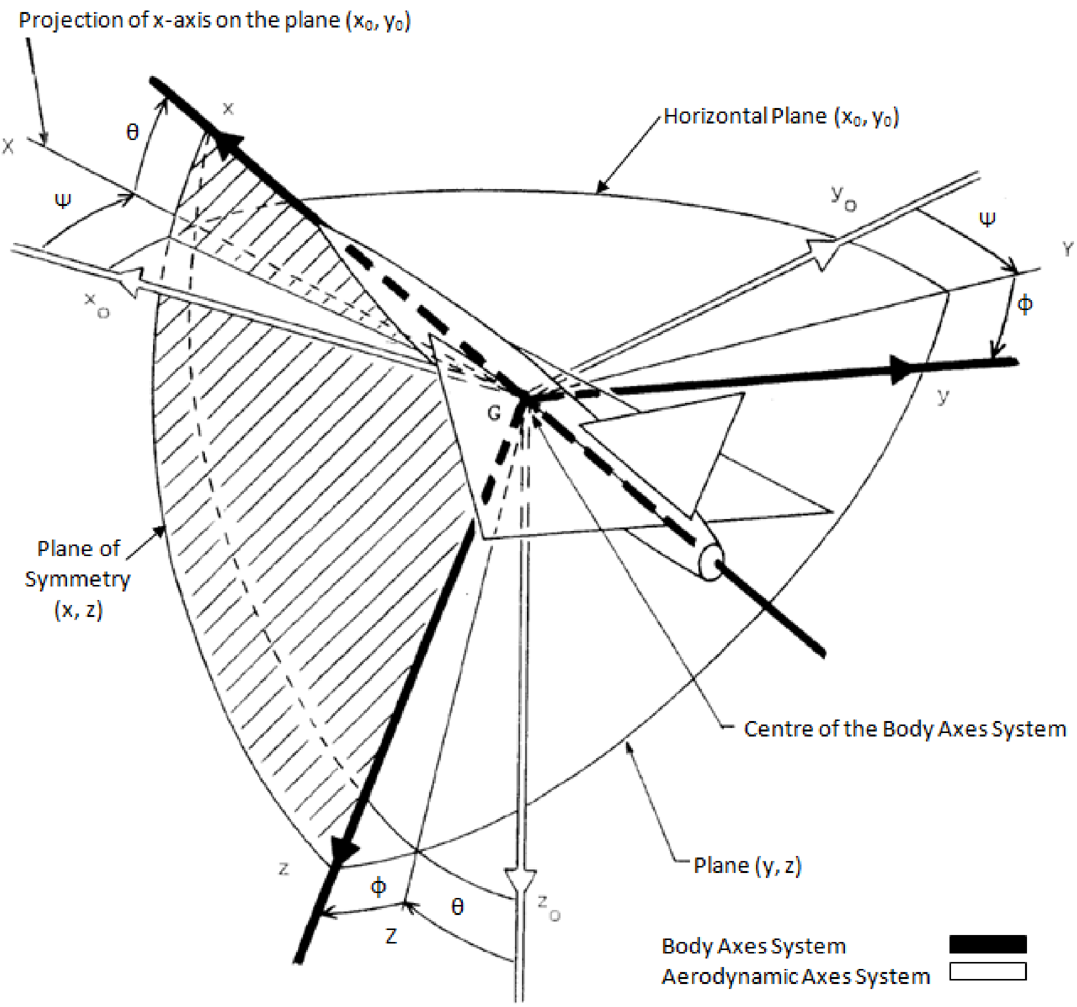
2.1 From Aerodynamics Axes to Body Axes Systems



2.2 From Ground Axes to Body Axes Systems

Three rotations are required in the following order:

- Ψ rotation around Gz_0 ; (Gx_0, Gy_0) => (GX, GY)
  Ψ is called the azimuth angle
- θ rotation around GY ; (Gz, Gz_0) => (Gx, Gz)
  θ is called the pitch angle
- ϕ rotation around Gx ; (GY, GZ) => (Gy, Gz)
  ϕ is called the roll angle



3.0 Forces & Moments

3.1 Body Axes

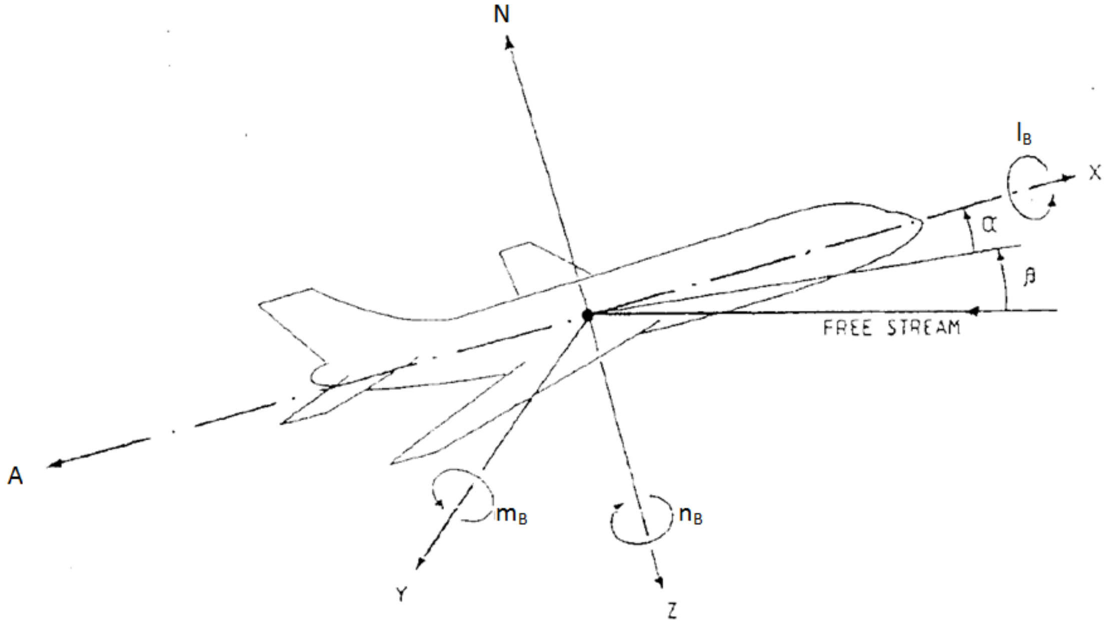
(X, Y, Z) are an orthogonal set of forces obeying the right hand rule.

- **X** is along the model datum axis positive forward
- **Y** side force is normal to the vertical plane of symmetry positive to starboard
- **Z** is in the vertical plane of symmetry normal to the datum axis positive down

Q, m, n are the moments about each of these axes defined as positive clockwise looking along the positive direction of the force.

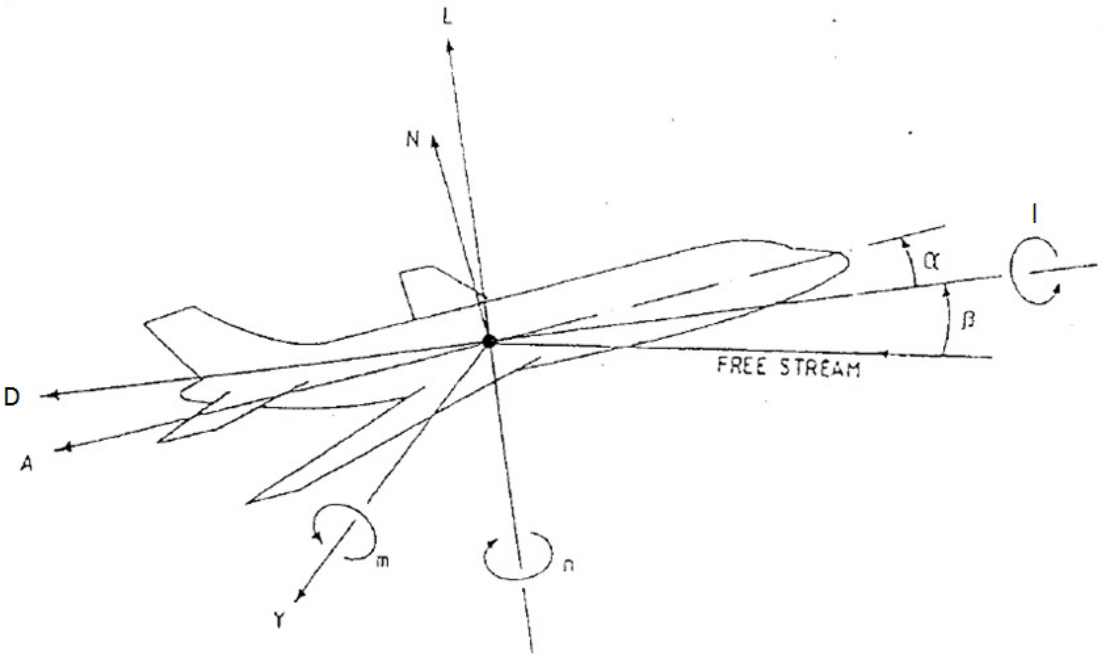
- $\mathbf{l_B}$ rolling moment, positive starboard (RH) side down.
- $\mathbf{m_B}$ pitching moment, positive nose up
- $\mathbf{n_B}$ yawing moment, positive nose to starboard

However, it is more convenient to use:
A (= -X), axial force positive rearwards
N (= -Z), normal force positive upwards



3.2 Aerodynamic Axes

- **Y** is the same as body axes
- **N** is resolved into **L** (lift)
- **A** is resolved into **D** (drag)
- **L** is in the vertical plane of symmetry normal to the free stream
- **D** is normal to the (L, Y) plane
- **l** is the moment about the D axis
- **m** is the same as body axes (pitching moment, positive nose up)
- **n** is the moment about the L axis



Flight Vehicle Terminology

4.0 Aircraft

Main components of the aircraft:

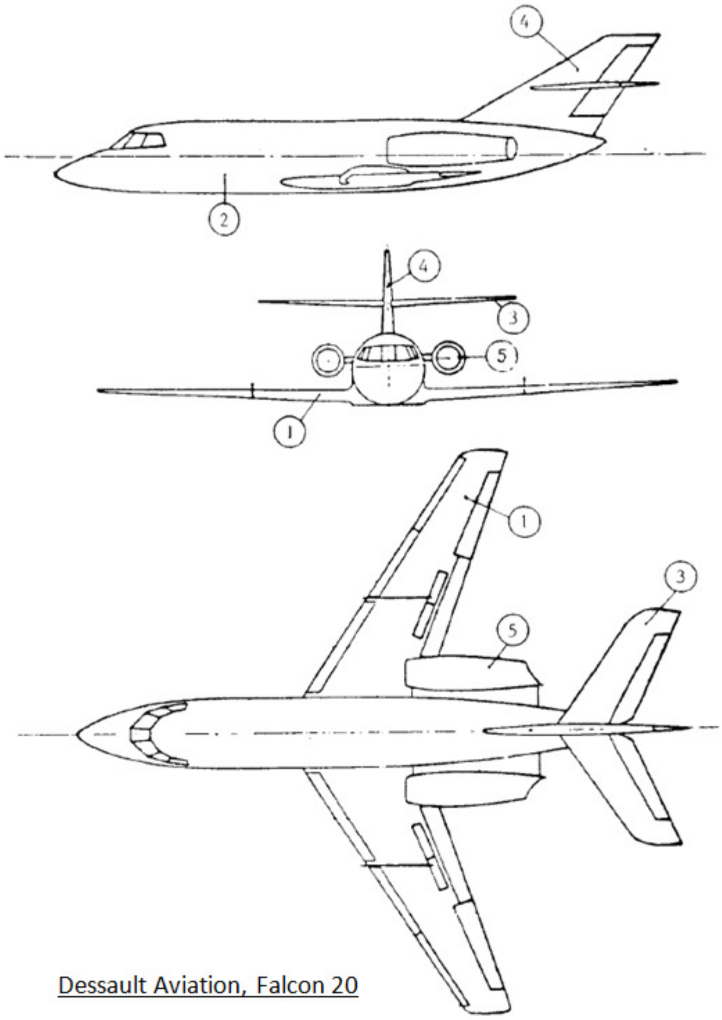
(1) Wing

(2) Fuselage

(3) HTP
(Horizontal Tail Plan)

(4) VTP
(Vertical Tail Plan)

(5) Engine (Duct)



Dessault Aviation, Falcon 20

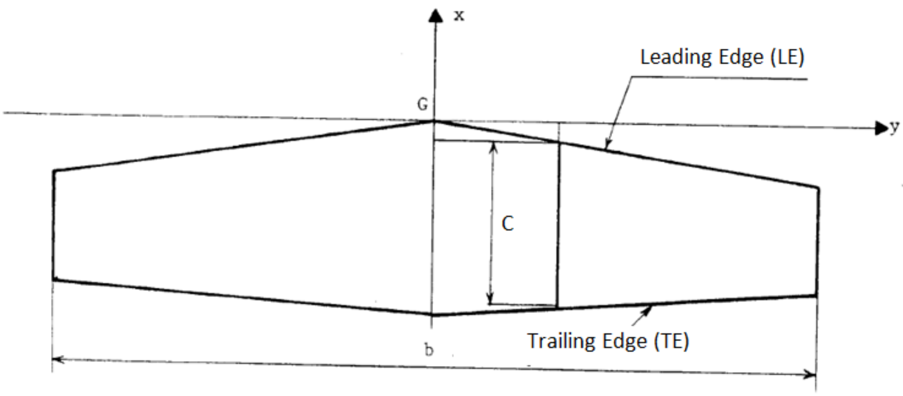
4.1 Wings

While the fuselage may be the part of the airplane of greatest concern to the passengers, the wing is certainly the most important to the aerodynamics of the airplane.

Aerodynamically, it is the heart of the airplane. Most of the aerodynamic behaviour of the aircraft will depend on how the designer configures the wing.

4.1.1 Geometrical Parameters

b = span
c = chord



Flight Vehicle Terminology

Wing Area, S

This is the gross projected area of the wing, including any fuselage area (in projected plan) cut off by the leading edge and the trailing edge, continued to the fuselage centreline.

Aspect Ratio, AR

$$\boxed{AR = \frac{b^2}{S}}$$

Mean Chord, $\bar{c}$

If the chord varies across the span, due to taper or curved leading & trailing edges, the mean chord is often used.

$$\boxed{\bar{c} = \frac{S}{b}}$$

$\bar{c}$ could also be calculated by the following formulas:

$$\boxed{\bar{c} = \frac{1}{b}\int_{-b/2}^{b/2} c(y).\,dy} \quad \text{OR} \quad \boxed{\bar{c} = \frac{1}{S}\int_{-b/2}^{b/2} c^2(y).\,dy}$$

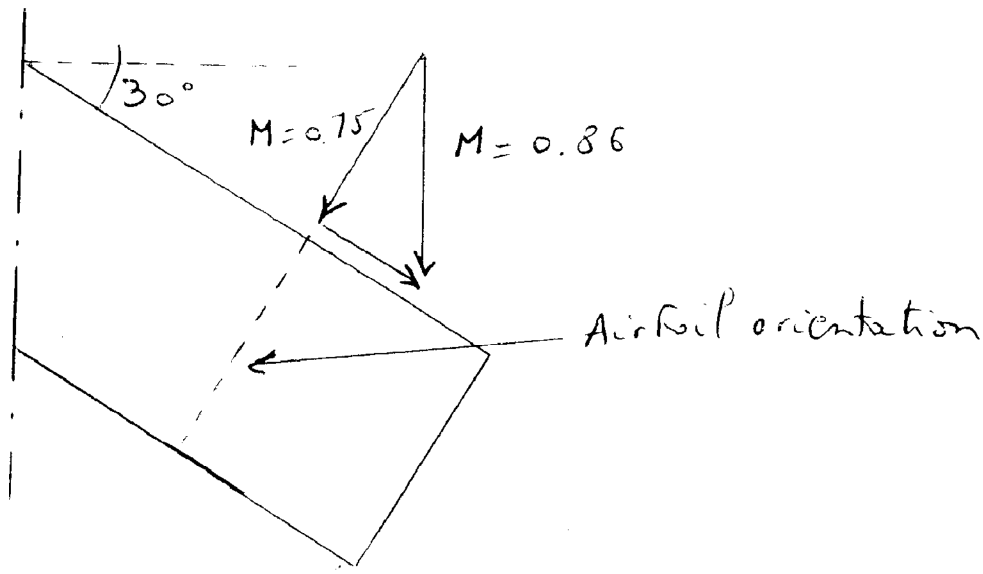
Taper Ratio, λ

This is the ratio between the tip chord of the wing and the chord at the root, taken on the fuselage centreline.

$$\boxed{\lambda = \frac{c_{tip}}{c_{root}}}$$

Sweep Angle, Λ or φ

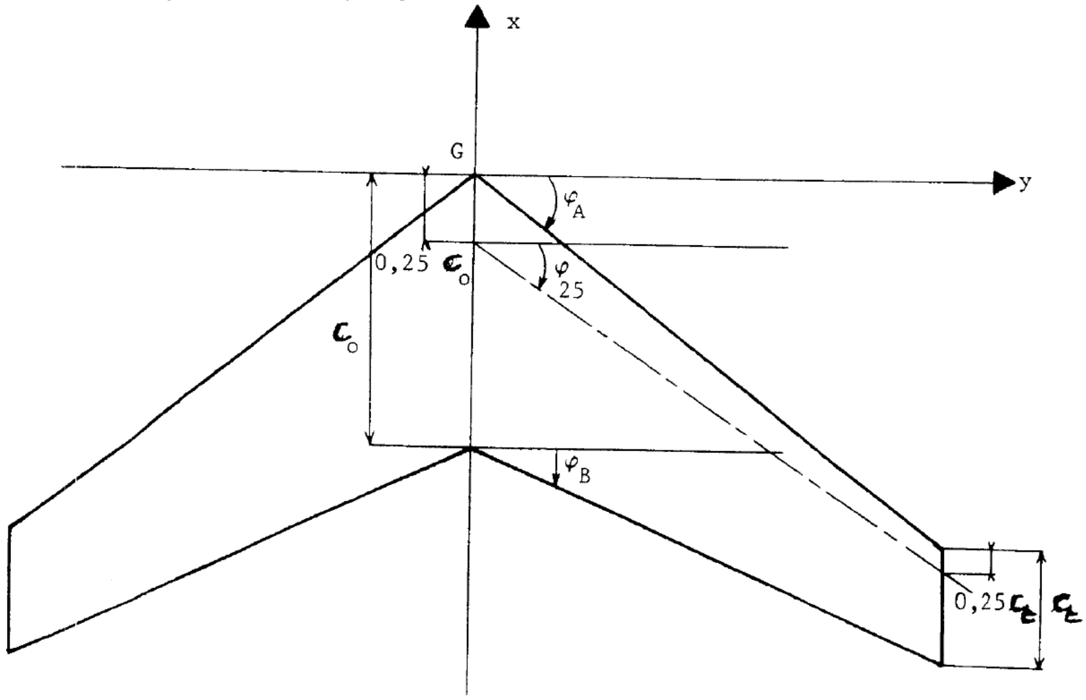
One of the first breakthrough's that allowed for high critical mach numbers was the idea of wing sweep. If a wing is swept aft (towards rear), only a component of the velocity of the air will flow over it chord wise. Another component will flow span wise along the wing. This allows the airplane to fly at a higher mach number, while the wing's airfoil only "sees" a portion of this speed.



Note: The wing's low-speed performance is degraded by sweep. Remember that a significant part of the air velocity is now flowing span wise and not contributing to lift. This will raise the stall speed and the resulting take-off and landing distance over an equivalent straight wing.

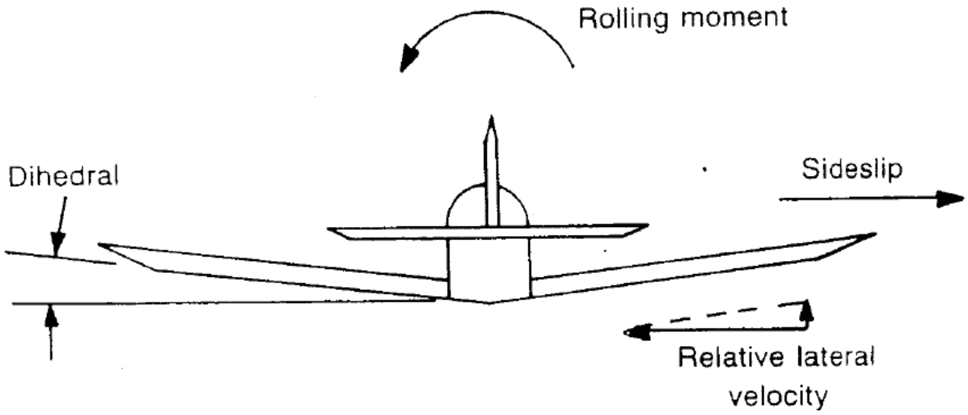
There are 3 different types of sweep angles:

- **φ_A** is φ at the Leading Edge (LE)
- **φ_B** is φ at the Trailing Edge (TE)
- **φ_{25}** is the sweep angle at 25% of the chord



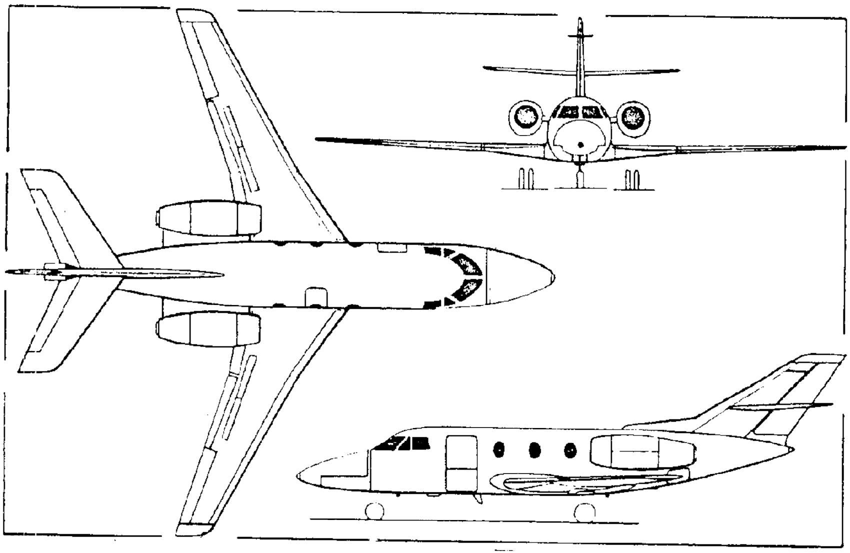
Dihedral Angle, δ

This is the angle at which each wing is set relative to the line at right angles to the fin, in the front view of the aircraft. For dihedral angle to be positive, the wing tip is higher than the wing root. If the tip is below the root, the wing is said to be 'Anhedral'



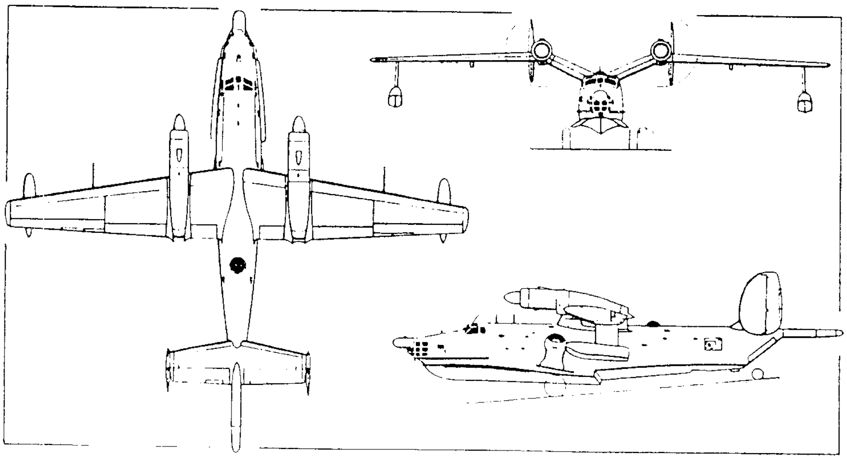
Dihedral wings provide lateral stability from the upward component of the relative lateral velocity resulting from the sideslip.

The figure above shows an airplane with dihedral wings. If it were side slipping to the right, as shown, a component of the relative wind would be acting inbound against the right wing. A component of this velocity would be acting against the bottom of the wing, tending to roll it to the left. Thus a roll to the right tends to slip the airplane to the right, but with dihedral, an opposite moment is created to level the wings and arrest the slip.



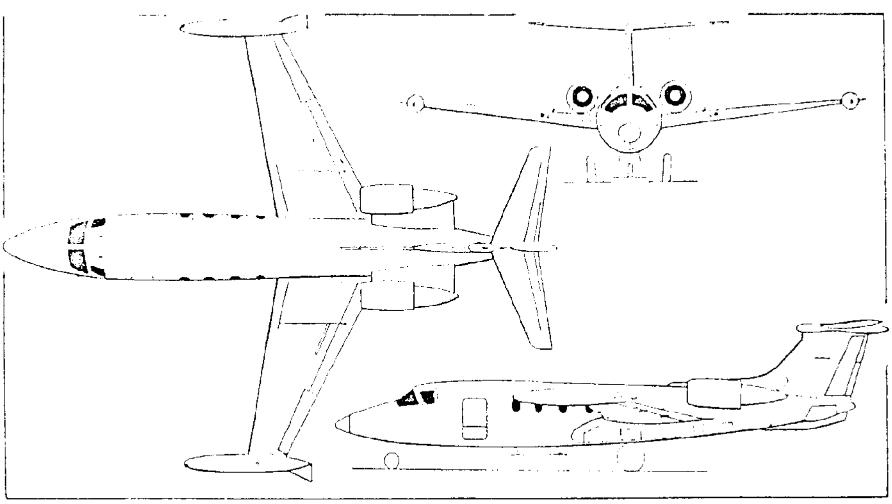
AMDBA FALCON 10 (France)

b = 13,08 m S = 24,1 AR = 7,1
M = 0,84



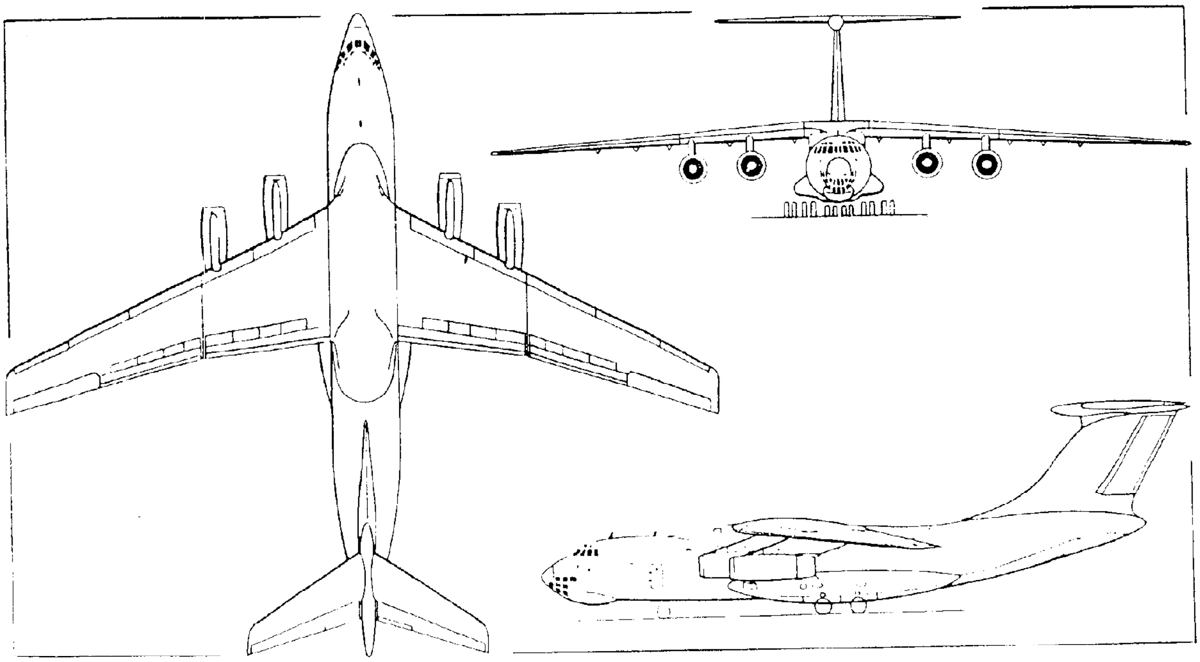
BERIEV M-12 TCHAIKA (URSS)

b = 29,70 m



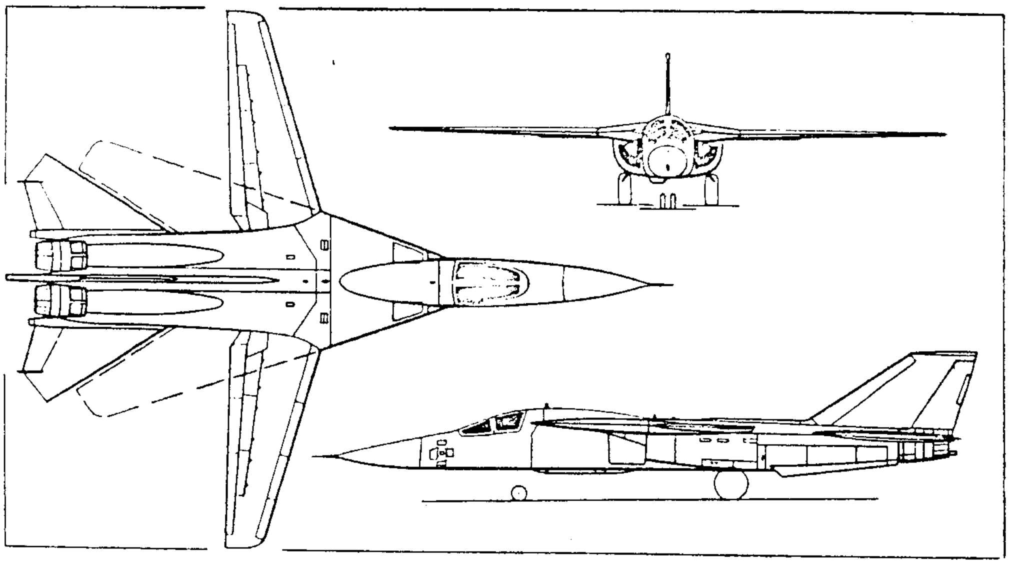
MBB HFB 330 HANSA (RFA)

b = 17,31 m S = 30,14 m^2 AR = 9,94
φ = - 25° δ = 6° M_{max} = 0,85



ILYUSHIN II-76 (URSS)

$b = 50{,}50$ m $\varphi_A = 28°$ $V = 850$ km/h



GENERAL DYNAMICS F-111 A (USA)

$b = 19{,}20$ m $\varphi_A = 16°$

$b = 9{,}74$ m $\varphi_A = 72{,}5°$ $M = 2{,}5$

4.1.2 Wing Geometry

Wings can be classified into 3 categories according to the sweep angle and the aspect ratio (AR):

- **a** High to Medium AR with a low sweep angle
- **b** Medium AR with a medium sweep angle
- **c** Low AR with a High sweep angle

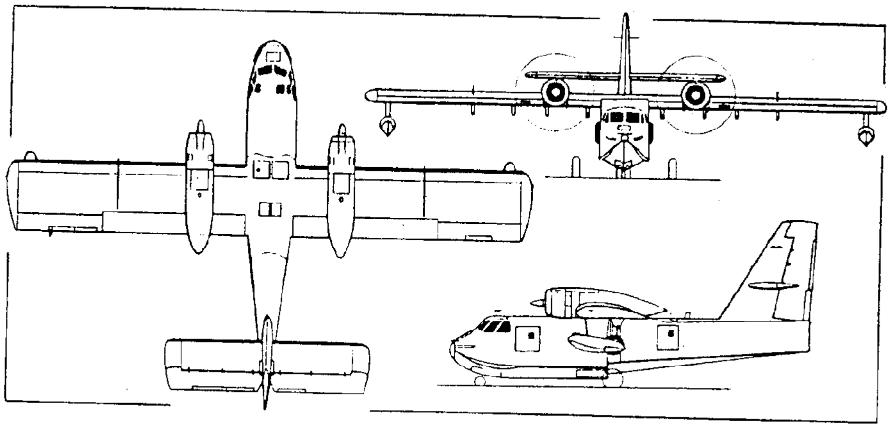
These 3 categories correspond to a Mach number (M) range for airplanes, i.e.

- Subsonic Airplanes $M < 0.6$
- Transonic Airplanes $0.7 < M < 0.9$
- Supersonic Airplanes $M > 1.2$

Consider the following examples for the 3 categories of wings:

a – High to Medium AR with low sweep angle

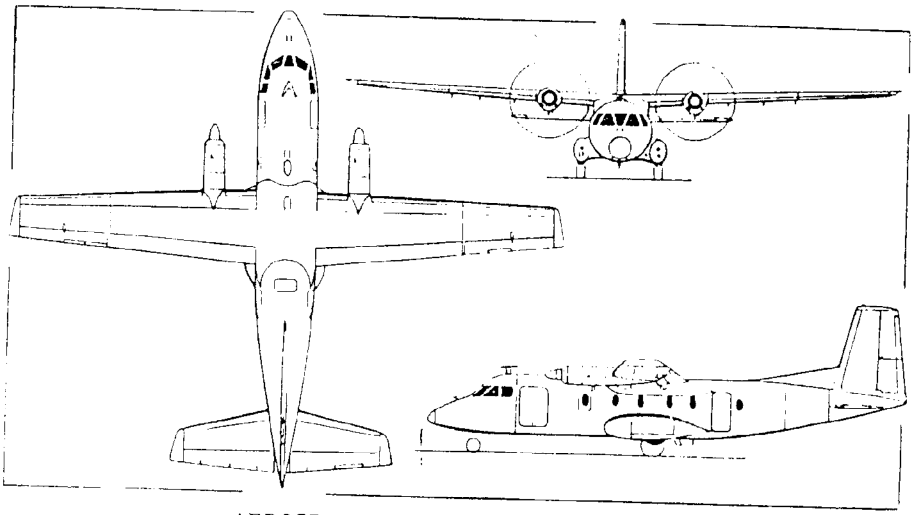
Rectangular Wing:



CANADAIR CL-215 (Canada)

$b = 28{,}60$ m $S = 100{,}33$ m^2 $AR = 8{,}15$
$V_{max} = 293$ km/h

Trapezoid Wing:

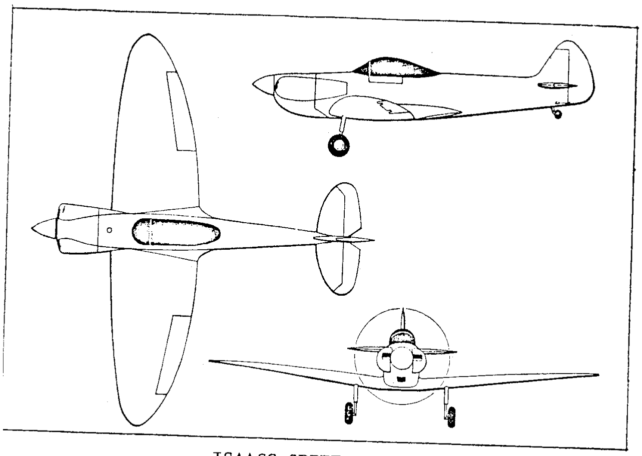


AEROSPATIALE FREGATE (France)

$b = 22{,}60$ m $S = 55{,}79$ m^2 $AR = 9{,}15$
$V_{max} = 408$ km/h

Flight Vehicle Terminology

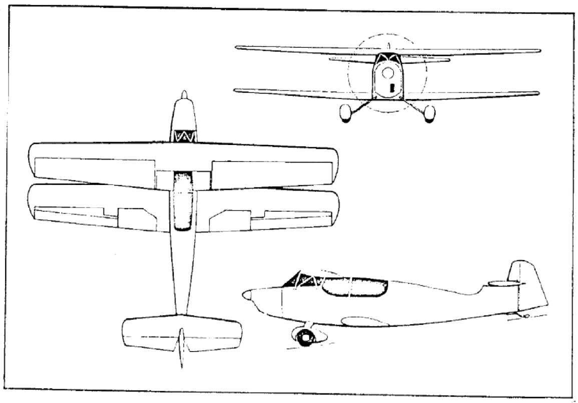
Elliptic Wing:



ISAACS SPITFIRE (RU)

b = 6,75 m S = 8,08 m^2 **AR** = 5,63
V_{max} = 240 km/h

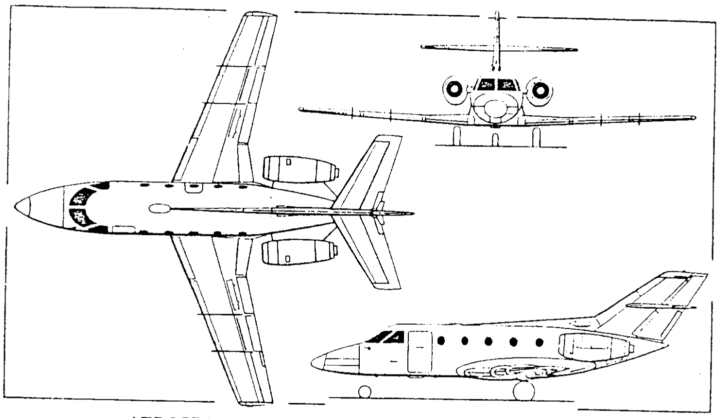
Biplane:



LEMBERGER LD 20 b (RFA)

chaque aile b = 7,28 m S = 7,00 m^2 **AR** = 7,57
V_{max} = 178 km/h

b – Medium AR with a medium sweep angle

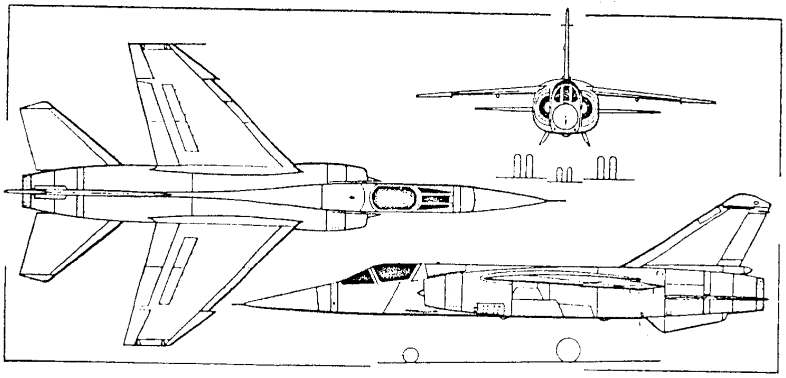


AEROSPATIALE SN 600 CORVETTE (France)

b = 12,80 m S = 22,00 m^2 **AR** = 7,45
φ_A = 22°32' δ = 3°6'
M_{max} = 0,7 750 km/h

c – Low AR with a high sweep angle

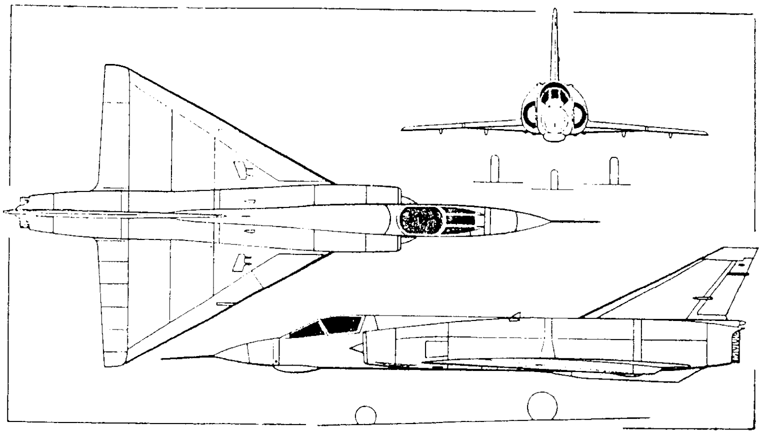
High Sweep Angle:



AMDBA MIRAGE F1 (France)

b = 8,40 m S = 25,00 m^2 **AR** = 2,82
φ_A = 50°
M_{max} = 2,2

Δ (Delta) Wing:



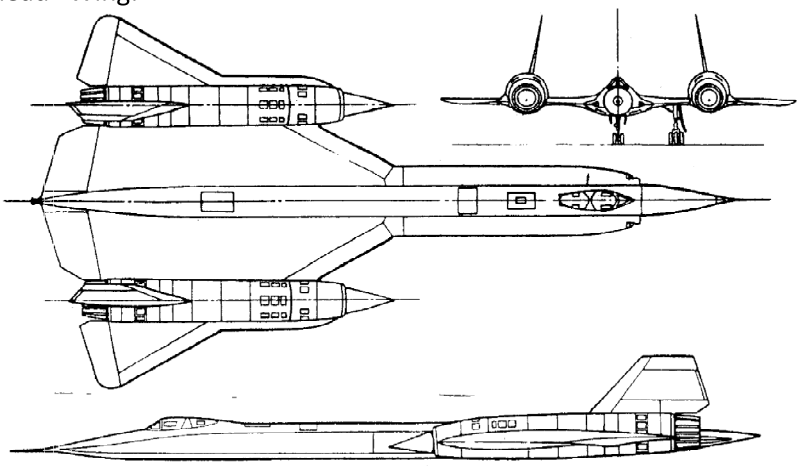
AMDBA MIRAGE III.E (France)

b = 8,22 m S = 34,85 m^2 AR = 1,88

φ_A = 60°34'

M_{max} = 2,2

“Spearhead” Wing:

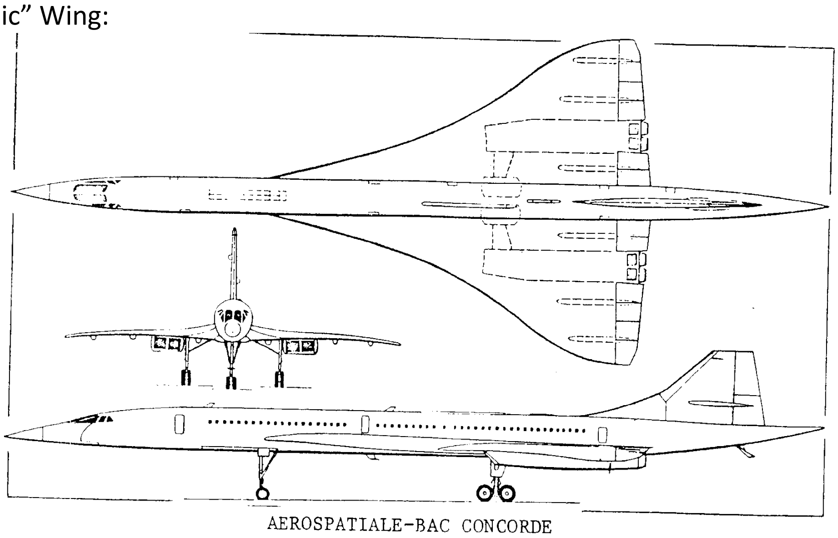


LOCKHEED SR-71 (USA)

b = 16,95 m

φ = 60° M = 3 (H = 30 000 m)

“Gothic” Wing:



AEROSPATIALE-BAC CONCORDE

b = 25,60 m S = 358,25 m^2 AR = 1,7

M = 2,2

4.1.3 Devices On A Wing

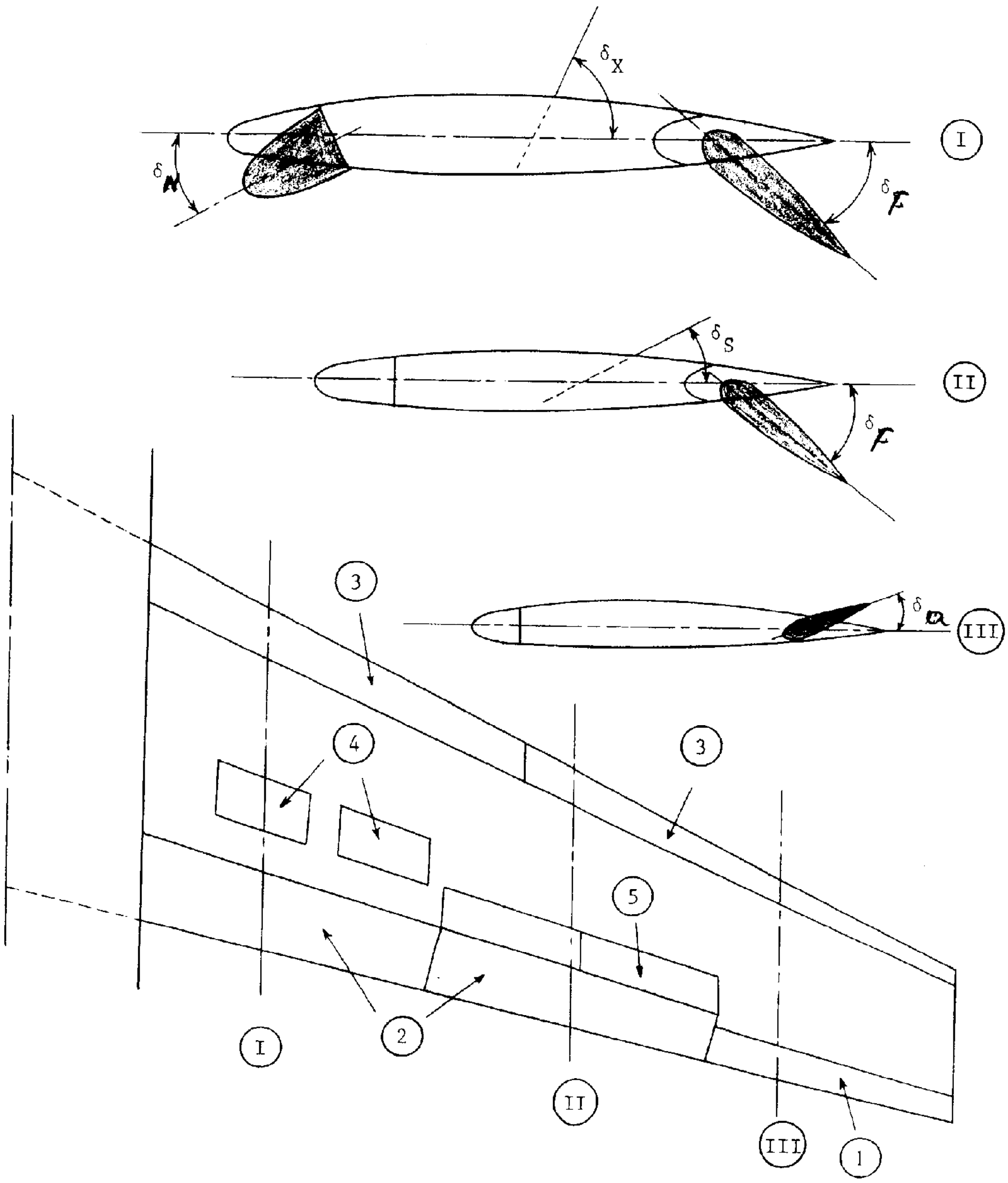
Slots & Slats

A more common device found in the leading edge is the slot. This device allows air to flow from the lower surface to the upper surface at high angles of attack. The higher pressure air from the lower surface has more energy, which will delay the separation of the airflow on the top surface and thus, the onset of stall. It is another way of achieving higher lift at low speed.

The disadvantage of the slot is that it creates excessive drag at lower angles of attack which are associated with normal cruise speeds. A way of avoiding this situation is to have a leading edge section that will open into a slot at low speed, but close at high speed. Such a device is called a slat.

Flaps

The flap is a high lift device. The flap is a movable portion of the airfoil which is deflected through some angle from the original chord position to yield a higher camber.



(1) Ailerons, deflection angle δ_a
  ⇨ Produces a rolling moment

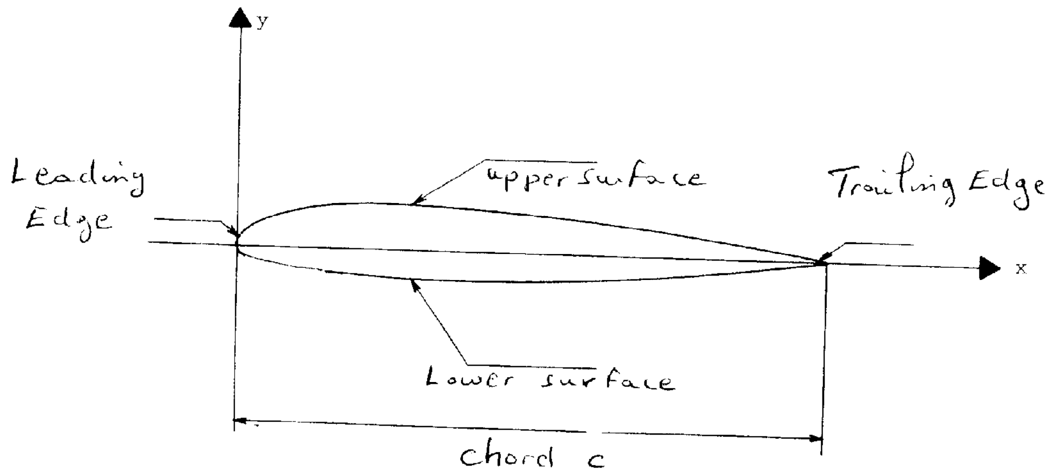
(2) Lifting Flaps, deflection angle δ_F
  ⇨ High lift device

(3) Nose Flaps, deflection angle δ_N
  ⇨ High lift device

(4) Airbrakes, deflection angle δ_x

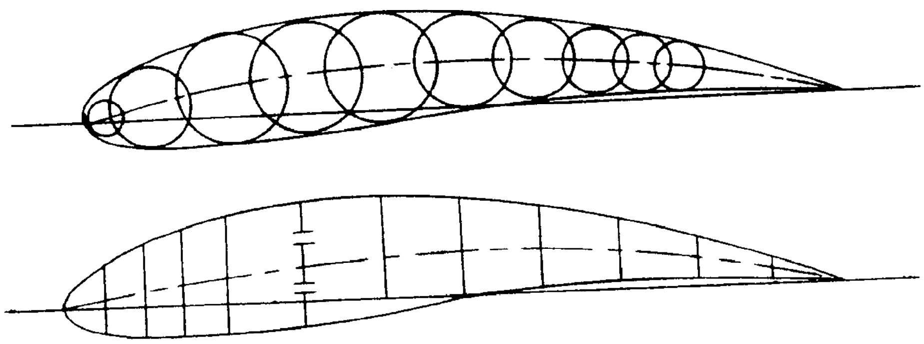
(5) Spoilers, deflection angle δ_s
  ⇨ Control of the lift
  ⇨ Roll control

4.2 Airfoil or Wing Section



Chord, c
The chord is the length of the chord line cut off or enclosed by the section. It is obviously equal to the distance between the leading and trailing edges.

Camberline or Mean Line
This is the line, each point of which is an equal distance from the upper & lower surfaces
This can be shown in the following geometric definitions:

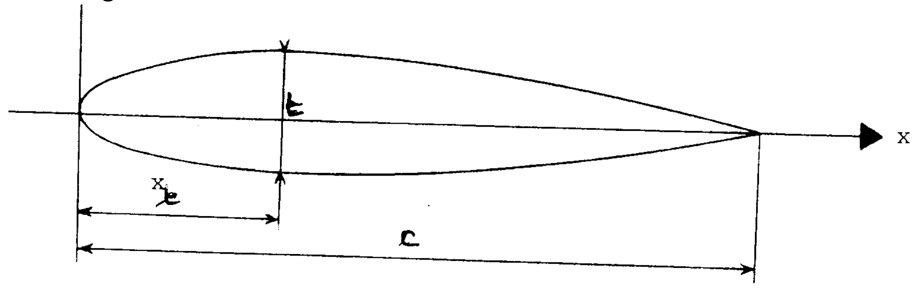


Thickness, t

This is the maximum length of a line measured perpendicularly (at a right angle) to the camberline. It is the maximum distance between the upper & lower surfaces.

Thickness/ Chord Ratio, t/c

This is an important parameter to describe the shape of the aerofoil. It is given as a percentage.



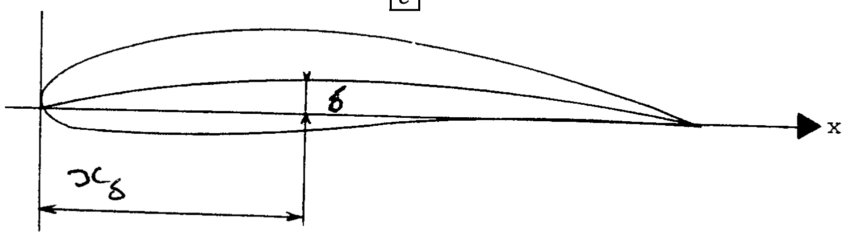
$t/c = 18\%$; $x_t/c = 30\%$

NACA 23018

Camber

This is the maximum distance of the camber line from the chord line. If the distance is δ then the camber is usually the ratio:

$\boxed{\frac{\delta}{c}}$ as a percentage



$\delta/c = 6\%$; $x_\delta/c = 40\%$

NACA 6415

5.0 Flow Types

5.1 Continuous Flow

In order to predict the flow regime which is a function of altitude & velocity, a similarity parameter called the Knudsen number (K_n) is often used. This governing parameter is the ratio of the average mean free path, λ, which can be defined as the average distance that a molecule travels between 2 successive collisions and a characteristic length, L, of the flow field. $\boxed{K_n = \frac{\lambda}{L}}$

When K_n is very small the fluid is assumed to be continuous, even though it consists of discrete molecules. It is in continuous flow.

$$\boxed{K_n \ll 1 \quad , \quad Z \leq 80Km}$$

5.2 Dependant Flows

Time Dependence

Steady Flows

A steady flow is one in which the conditions (velocity, pressure & cross-section) may differ from point to point but do not change with time.

Unsteady Flows

If at any point in the fluid, the conditions change with time, the flow is described as unsteady.

In practice there are always slight variations in the velocity & pressure, but if the average values are constant the flow is considered as steady.

Quasi-Steady Flows

In quasi-steady flows the time scale $t < \infty$ but the changes are so slow that any inertia effects maybe neglected.

Space Dependence

Uniform Flow

If the flow velocity is the same magnitude & direction at every point in the fluid it is said to be uniform.

Non-Uniform Flow

If at a given instant, the velocity is not the same at every point the flow is non-uniform.

In practice, by this definition, every fluid that flows near a solid boundary will be non-uniform as the fluid at the boundary must take the speed of the boundary (usually zero). However if the size & shape of the cross-section of the stream of fluid is constant the flow is considered uniform.

Combinations

Steady Uniform Flow

Conditions do not change with position in the stream or with time

Steady Non-Uniform Flow

Conditions change from point to point in the stream but do not change in time

Unsteady Uniform Flow

At a given instant in time, the conditions of every point are the same, but will change with time

Unsteady Non-Uniform Flow

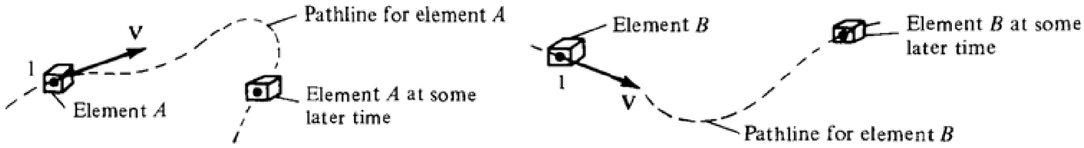
Every condition of the flow may change from point to point and with time at every point.

5.3 Axis Symmetric Flows

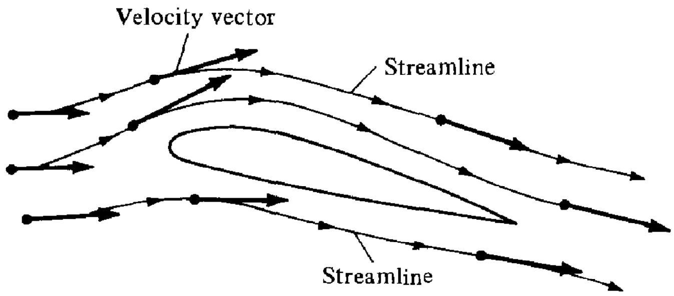
An axis symmetric flow has 2 independent variables.

Because of that, this flow is sometimes labelled as “2D” flow. However, it is actually quite different from 2D flow. In reality, axis symmetric flow is a degenerate 3D flow, and it is somewhat misleading to refer to it as 2D.

5.4 Pathlines, Streamlines & Streaklines Of A Flow



Pathlines for 2 different fluid elements passing through the same point in space: unsteady flow.



By definition, a streamline is a curve whose tangent at any point is in the direction of the velocity at that point

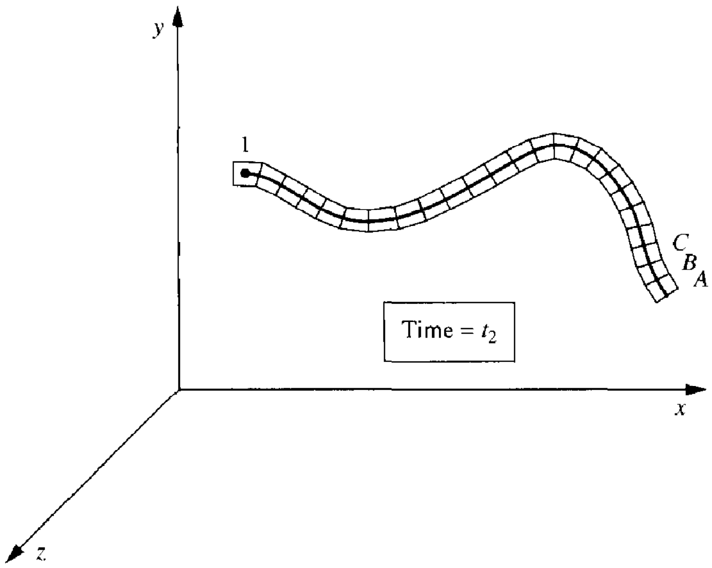


Figure 2.29 Illustration of a streakline through point 1.

Consider a fixed point in a flow field, such as point 1. Consider all the individual fluid elements that have passed through point 1 over a given time interval of $t_2 - t_1$. These fluid elements are connected with each other. Element A is the fluid element that passed through point 1 at t_1. Element B is the next element that passed through point 1 just behind element A. The figure above is an illustration made at time t_2, which shows all the fluid elements that have earlier passed through point 1 over the time interval $(t_2 – t_1)$. The line that connects all these fluid elements is, by definition, a streakline.

Aerodynamic Forces & Moments

The aerodynamic forces and moments on the body are due to only 2 basic sources:

1. Pressure distribution over the body surface
2. Shear stress distribution over the body surface

The figure below illustrates the pressure & shear distribution on an aerodynamic surface.

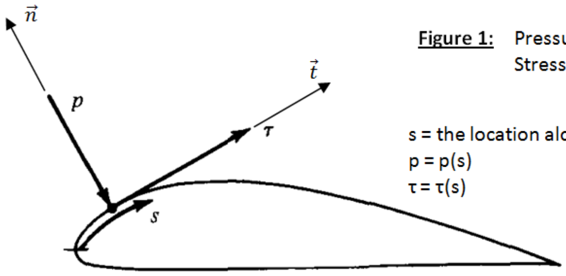


Figure 1: Pressure and Shear Stress on a surface

s = the location along the surface
p = p(s)
τ = τ(s)

Shear stress is due to the tugging action on the surface, which is caused by friction between the body & the air.

The net effects of p & τ distributions integrated over the complete body surface have a resultant aerodynamic force $\vec{R}$ & moment $\vec{M}$ over the body as shown in figure 2.

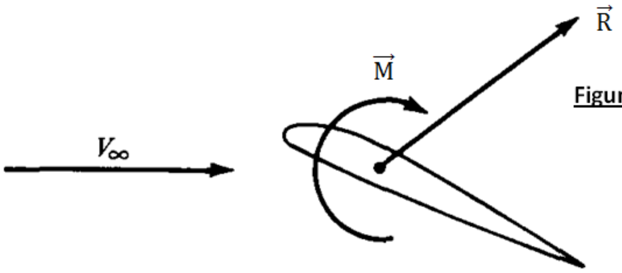


Figure 2: Resultant Aerodynamic Force & Moment on the Body

V_∞ = the free stream velocity

1. Analytical Expressions

There are 3 axes-systems:

- Ground
- Body
- Aerodynamic

Therefore the resultant aerodynamic force $\vec{R}$ could be split into components in the body or aerodynamic axes systems, as shown in figure 3.

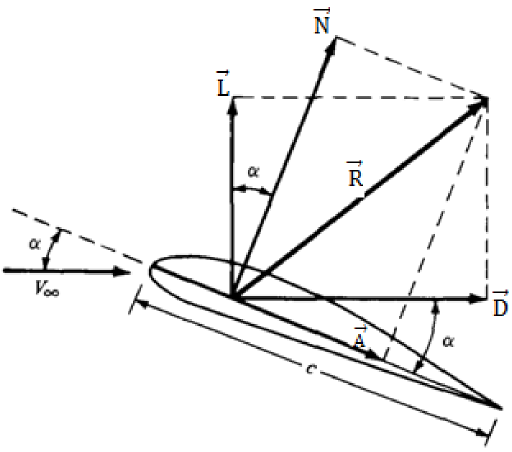


Figure 3: Resultant Aerodynamic Force

Where:
$\vec{N}$ & $\vec{A}$ are defined by body axis system
$\vec{L}$ & $\vec{D}$ are defined by aerodynamic axis system

$\vec{N}$ = normal force
$\vec{A}$ = axial force
$\vec{L}$ = lift force
$\vec{D}$ = drag force

The geometrical relations between these 2 sets of components are:

$$L = N\cos(\alpha) - A\sin(\alpha)$$
$$D = N\sin(\alpha) + A\cos(\alpha)$$

2. Calculations of Forces & Moments

Consider the 2-dimensional body defined by figure 4

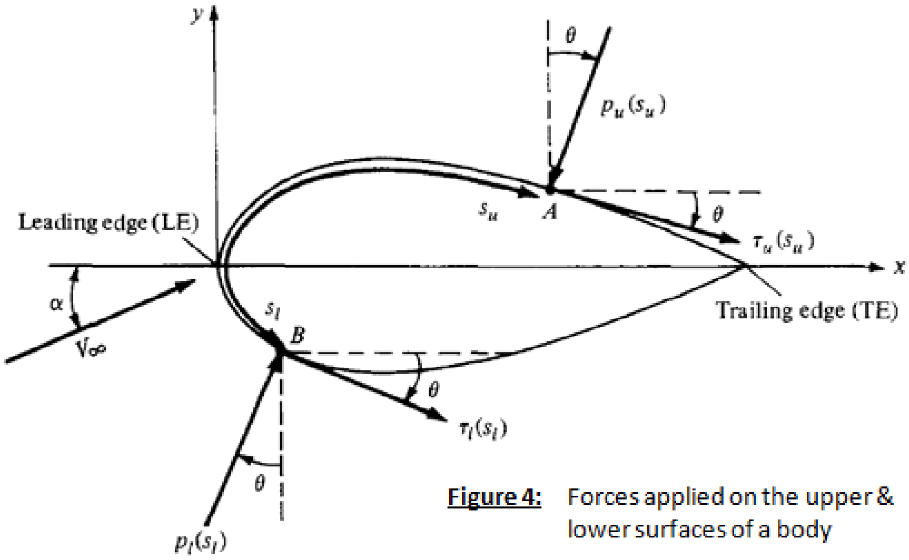


Figure 4: Forces applied on the upper & lower surfaces of a body

Now consider the 2D shape in figure 4 as a cross-section of an infinitely long cylinder of uniform section (***a unit span***) such a cylinder is shown in figure 5. Consider an elemental surface area (ds) of this cylinder, where ds = (ds) x 1

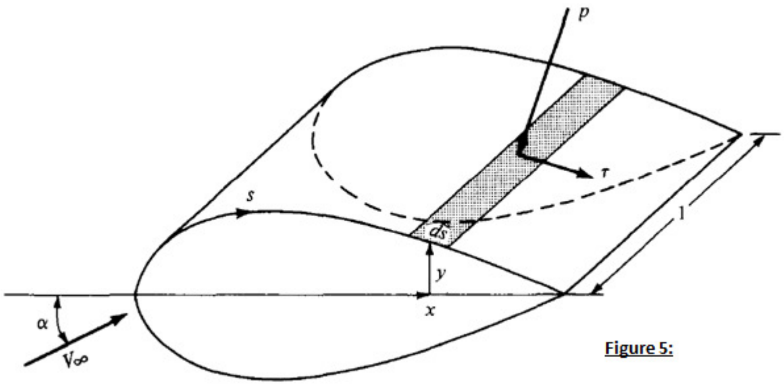


Figure 5:

The elemental normal force dN and axial force dA acting on the elemental surface ds on the upper body surface are:

$$(1) \Rightarrow \boxed{dN_u = -p_u ds_u \cos(\theta) - \tau_u ds_u \sin(\theta)}$$

$$(2) \Rightarrow \boxed{dA_u = -p_u ds_u \sin(\theta) + \tau_u ds_u \cos(\theta)}$$

In the same way on the lower surface we have:

$$(3) \Rightarrow \boxed{dN_l = p_l ds_l \cos(\theta) - \tau_l ds_l \sin(\theta)}$$

$$(4) \Rightarrow \boxed{dA_l = p_l ds_l \sin(\theta) + \tau_l ds_l \cos(\theta)}$$

To determine the normal force N & the axial force N, equations (1) to (4) must be integrated from the leading edge (LE) to the trailing edge (TE)

$$N = \int (dN + dA) \quad \text{per unit area}$$

Where:

$$dN = dN_u + dN_l$$
$$dA = dA_u + dA_l$$

Aerodynamic Forces & Moments

Therefore:

$$(5) \Rightarrow \boxed{N = -\int_{LE}^{TE} (p_u\cos(\theta) + \tau_u\sin(\theta))ds_u + \int_{LE}^{TE} (p_l\cos(\theta) + \tau_l\sin(\theta))ds_l}$$

&

$$(6) \Rightarrow \boxed{A = \int_{LE}^{TE} (-p_u\sin(\theta) + \tau_u\cos(\theta))ds_u + \int_{LE}^{TE} (p_l\sin(\theta) + \tau_l\cos(\theta))ds_l}$$

Because of the relations:

$$L = N\cos(\alpha) - A\sin(\alpha)$$
$$D = N\sin(\alpha) + A\cos(\alpha)$$

The lift & drag can be calculated using (5) & (6)

Aerodynamic Moment Exerted on the Body

It depends on the point about which moments are taken.

Sign convention for aerodynamic moments.

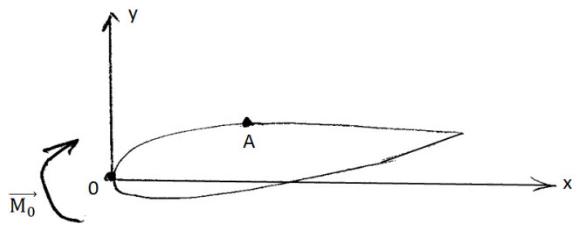
Consider the aerodynamic moment calculated about the leading edge (LE). The moment per unit span about the leading edge due to p & τ (figures 4 & 5) on the elemental area (ds) on the upper & lower surfaces are:

$$(7) \Rightarrow \boxed{dM_u = [p_u\cos(\theta) + \tau_u\sin(\theta)]x.ds_u + [-p_u\sin(\theta) + \tau_u\cos(\theta)]y.ds_u}$$

&

$$(8) \Rightarrow \boxed{dM_l = [-p_l\cos(\theta) + \tau_l\sin(\theta)]x.ds_l + [p_l\sin(\theta) + \tau_l\cos(\theta)]y.ds_l}$$

Note:



$$\overrightarrow{M_0} = \int \left(d\vec{N} + d\vec{A}\right) \Lambda\, \overrightarrow{0A}$$

$$(9) \Rightarrow \boxed{M_{LE} = \int_{LE}^{TE} \left[dM_{u,LE} + dM_{l,LE}\right]}$$

To define the dynamic pressure that arises when the fluid is in motion:

$$q_\infty = \frac{1}{2}\rho_\infty {v_\infty}^2 \quad \text{(freestream conditions)}$$

Lift Coefficient: $C_L = \frac{l}{q_\infty s}$

Drag Coefficient: $C_D = \frac{D}{q_\infty s}$

Normal Coefficient: $C_N = \frac{N}{q_\infty s}$

Axial Coefficient: $C_A = \frac{A}{q_\infty s}$

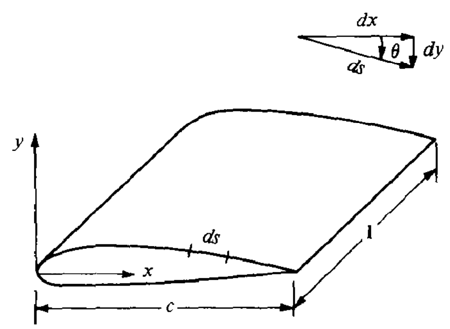
Moment Coefficient: $C_M = \frac{M}{q_\infty s l}$

s = a reference area
l = a reference length

Let's consider 2 additional non-dimensional coefficients:

Pressure Coefficient: $C_p = \frac{P - P_\infty}{q_\infty}$

Skin Friction Coefficient: $C_F = \frac{\tau}{q_\infty}$



Using the above geometry we can write:

$$dx = ds.\cos(\theta) \quad \& \quad dy = -ds.\sin(\theta)$$
$$\& \quad S = c(l)$$

$$\boxed{C_N = \frac{1}{c}\left[\int_0^c (C_{p,l} - C_{p,u})dx + \int_{LE}^{TE} (C_{F,u} - C_{F,l})dy\right]}$$

$$\boxed{C_A = \frac{1}{c}\left[\int_{LE}^{TE} (C_{p,u} - C_{p,l})dy + \int_0^c (C_{F,u} - C_{F,l})dx\right]}$$

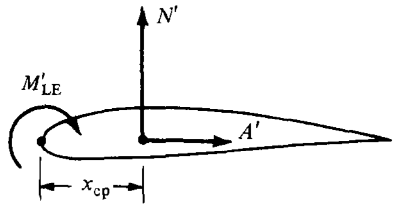
$$\boxed{C_{M,LE} = \frac{1}{c^2}\begin{bmatrix} \int_0^c (C_{p,u} - C_{p,l})x.dx - \int_{LE}^{TE} (C_{F,u} - C_{F,l})x.dy \\ + \int_{LE}^{TE} (C_{p,u} - C_{p,l})y.dy + \int_0^c (C_{F,u} - C_{F,l})y.dx \end{bmatrix}}$$

3. Centre of Pressure

The centre of pressure is the location where the resultant of a distributed load effectively acts on the body. If the moments were taken about the centre of pressure, the integrated effect of the distributed loads would be zero.

⇨ An alternate definition for the centre of pressure is the point on the body at which the aerodynamic moment is zero.

$$\boxed{x_{cp} = \frac{-M_{LE}}{N}}$$



If α is small then:

$$\sin(\alpha) = 0 \quad \& \quad \cos(\alpha) = 1$$

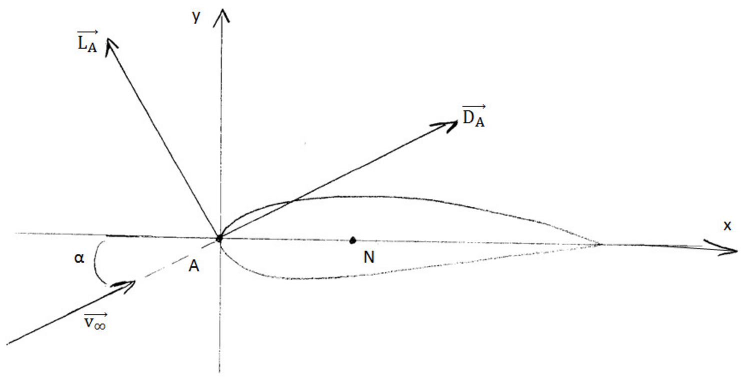
In this case, L = N, therefore:

$$x_{cp} = \frac{-M_{LE}}{L}$$

So as N & L decrease, x_{cp} increases
As the force approaches 0, $x_{cp} \to \infty$

Note: the centre of pressure is not always a convenient concept in aerodynamics

4. Change of Centre for an Airfoil



$$L_A = q_\infty . l . C_{L,A}$$

$$D_A = q_\infty . l . C_{D,A}$$

$$M_N = M_A + \overrightarrow{R_A} \wedge \overrightarrow{AN}$$

With: $\overrightarrow{R_A} = \overrightarrow{L_A} + \overrightarrow{D_A}$

$$M_N = M_A - L_A x_N \cos(\alpha) - D_A x_N \sin(\alpha)$$

$$\therefore M_N = M_A - x_N [L_A \cos(\alpha) + D_A \sin(\alpha)]$$

Dividing by $q_\infty l^2$

$$\frac{M_N}{q_\infty l^2} = \frac{M_A}{q_\infty l^2} - \frac{x_N}{l}\left[\frac{L_A}{q_\infty l}\cos(\alpha) + \frac{D_A}{q_\infty l}\sin(\alpha)\right]$$

Aerodynamic Forces & Moments

$$\boxed{C_{M,N} = C_{M,A} - \frac{x_N}{l}\left[C_{L,A}\cos(\alpha) + C_{D,A}\sin(\alpha)\right]}$$

When $\alpha \to 0$:

$$\sin(\alpha) = 0 \quad \& \quad \cos(\alpha) = 1$$

Therefore:

$$\boxed{C_{M,N} = C_{M,A} - \frac{x_N}{l} C_{L,A}}$$

$C_{M,0} = C_{M,A}$ when $C_{L,A} = 0$

Therefore:

$$\boxed{C_{M,A} = C_{M,0} + \frac{\partial C_{M,A}}{\partial C_{L,A}} \times C_{L,A}}$$

$$\boxed{C_{M,N} = C_{M,0} + C_{L,A}\left(\frac{\partial C_{M,A}}{\partial C_{L,A}} - \frac{x_N}{l}\right)}$$

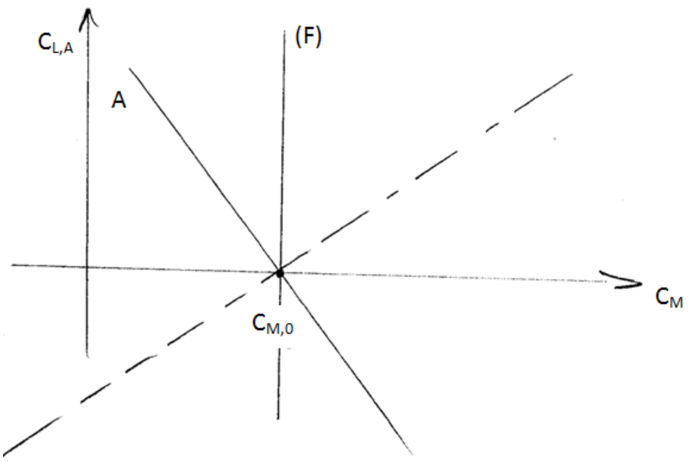
Consider a point F along the chord where

$$C_{M,F} = C_{M,0} \quad \forall\, C_{L,A}$$

$$\therefore \frac{\partial C_{M,A}}{\partial C_{L,A}} - \frac{x_F}{l} = 0$$

$$\therefore \frac{x_F}{l} = \frac{\partial C_{M,A}}{\partial C_{L,A}}$$

F is called the aerodynamic centre and does not depend on the incidence.

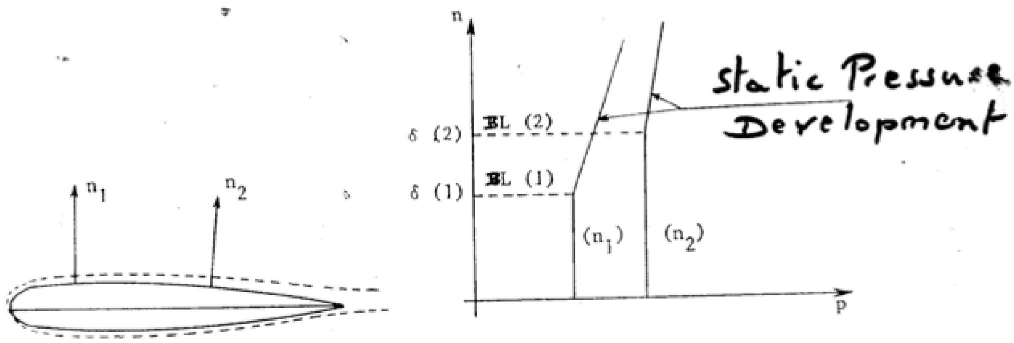


Pressure Coefficient, C_p

How to quantify the Velocity Distribution around an Airfoil

- Static Pressure Distribution

$$\frac{1}{\rho}\overrightarrow{grad\,p} + \overrightarrow{grad\,\frac{v^2}{2}} = 0$$



By Definition:

$$C_p = \frac{P - P_0}{\frac{1}{2}\rho {v_0}^2}$$

Where:

P - Static pressure at the point of interest
P_0 - Free stream static pressure
v_0 - Free stream velocity
ρ - Free stream density

$$q_0 = \frac{1}{2}\rho {v_0}^2$$

$$\therefore \boxed{C_p = \frac{P - P_0}{q_0}}$$

Also:

$$P_0 = R\rho_0 T_0$$

$$v_0 = M_0 a_0 \quad \& \quad {a_0}^2 = \gamma R T$$

$$\therefore q_0 = \frac{1}{2}\rho {v_0}^2 = \frac{1}{2}\rho (M_0 a_0)^2 = \frac{1}{2}{M_0}^2 \gamma R \rho T = \frac{1}{2}{M_0}^2 \gamma P_0$$

$$\therefore C_p = \frac{P - P_0}{\frac{1}{2}{M_0}^2 \gamma P_0}$$

$$\therefore \boxed{C_p = \frac{\frac{P}{P_0} - 1}{\frac{1}{2}\gamma {M_0}^2}}$$

Incompressible Flow

$\forall M_0$, Bernoulli's Equation is given by:

$$\frac{1}{\rho}dp + v.dv = 0$$

Pressure Coefficient, C_p

If Incompressible Subsonic Flows: $\rho = \rho_0 = \text{constant}$

$$\therefore P + \frac{1}{2}\rho_0 v^2 = P_0 + \frac{1}{2}\rho_0 {v_0}^2$$

$$\therefore P - P_0 = \frac{1}{2}\rho_0 ({v_0}^2 - v^2)$$

Therefore:

$$C_p = \frac{P - P_0}{\frac{1}{2}\rho_0 {v_0}^2} = \frac{{v_0}^2 - v^2}{{v_0}^2}$$

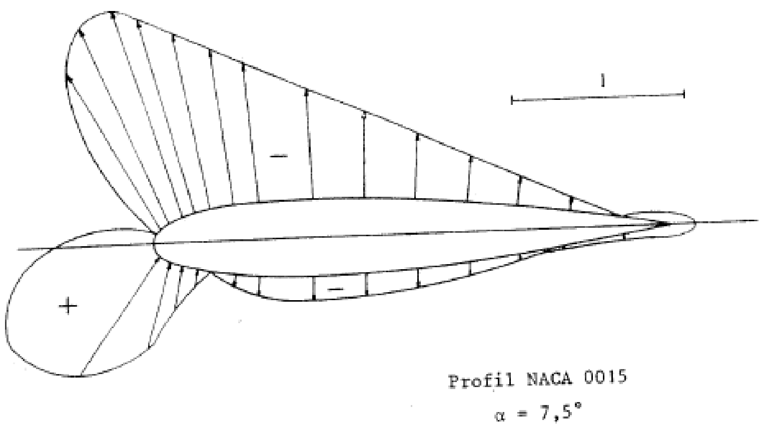
$$\therefore\ C_p = 1 - \left(\frac{v}{v_0}\right)^2$$

This creates 2 main results:

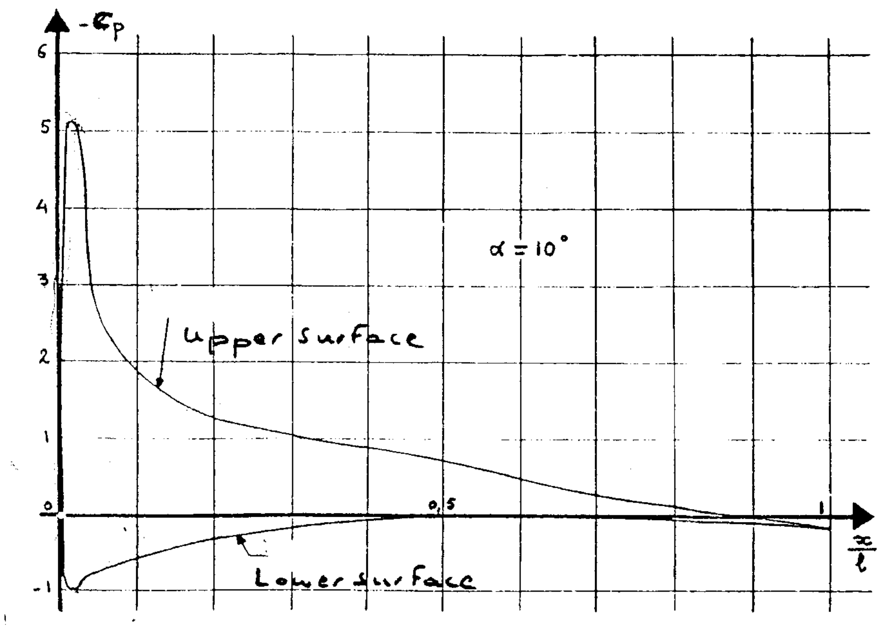
- At the Stagnation Point
  v = 0 => C_p = 1

- v = v_0 => C_p = 0

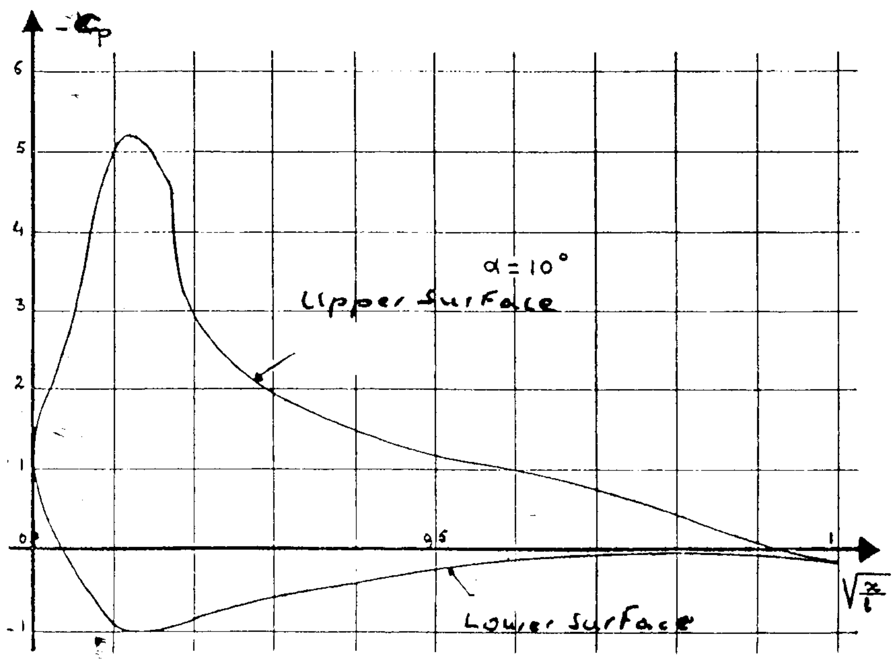
Pressure Distribution Around A Profile



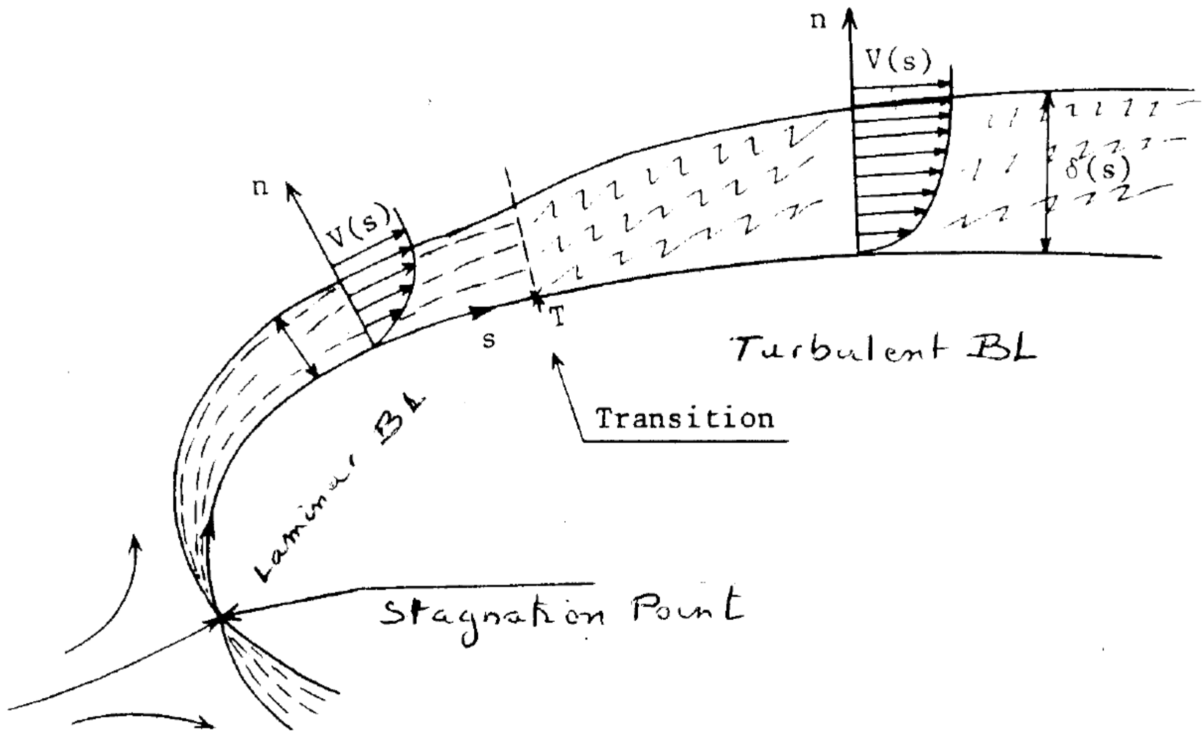
C_p Distributions Along A Profile



Pressure Coefficient, C_p



Boundary Layer Development Along A Profile

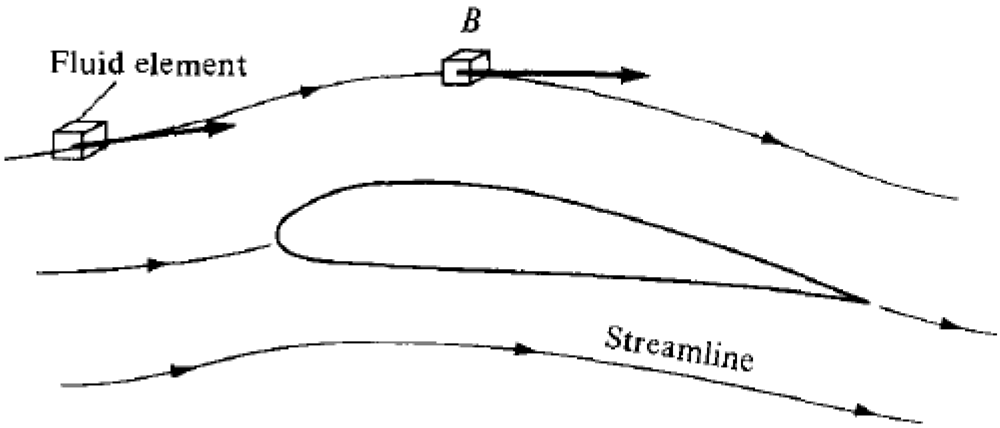


Atmospheric Models

1.0 Aerodynamic Variables

Four of the most frequently used words in aerodynamics:
- Pressure (p)
- Density (ρ)
- Temperature (T)
- Flow Velocity (**V**)

1.1 Pressure



The pressure is defined at <u>a point</u> in the fluid or a point on a solid surface. The pressure can vary from one point to another.

Definition:
"Pressure is the normal force per unit area exerted on a surface due to the time rate of change of momentum of the gas molecules impacting on (or crossing) that surface" (J. Anderson, Fundamentals of Aerodynamics, 2001, pg. 13)

$$(1) \Rightarrow \boxed{p = \lim_{dA\to 0}\left(\frac{dF}{dA}\right)}\ N/m^2$$

dA = Elemental Area at B (m^2)
dF = Force on 1 side of dA due to pressure (N)

1.2 Density

The density of a material is a measure of the amount of material contained in a given volume.

$$(2) \Rightarrow \boxed{\rho = \lim_{dv\to 0}\left(\frac{dm}{dv}\right)}\ kg/m^3$$

dv = Elemental volume around B (m^3)
dm = Mass of the fluid inside dv (kg)

Atmospheric Models

<<<<< Note: 1.3 & 1.4 (up to viscosity) were missing from original handout! >>>>>

1.3 Temperature

"The temperature *T* of a gas is directly proportional to the average kinetic energy of the molecules of the fluid. In fact, if KE is the mean molecular kinetic energy, then temperature is given by":

$$KE = \frac{3}{2}kT$$

Where *k* is the Boltzmann constant (k = 1.38 x 10^{-23} J/°K)

Temperature is "a point property, which can vary from point to point in the gas" & has "an important role in high-speed aerodynamics"

Source: J. Anderson, Fundamentals of Aerodynamics, 2001, pg. 14

1.4 Flow Velocity

A velocity is a vector value & as such must contain both scalar value & direction. In a flowing fluid at each region (or point) in the fluid there is not necessarily the same velocity. Hence this is also a point property which can vary from point to point in the flow.

"The velocity of a flowing gas at any fixed point B in space is the velocity of an infinitesimally small fluid element as it sweeps through B." (J. Anderson, Fundamentals of Aerodynamics, 2001, pg. 14)

Viscosity of oil > Viscosity of air

Sutherland's Law:

$$(3) \Rightarrow \boxed{\frac{\mu}{\mu_0} = \left(\frac{T}{T_0}\right)^{3/2} \frac{T_0 + 110}{T + 110}}$$

Where: $_0$ is reference data
T_0 = 288.16 °K
μ_0 = 1.7894 x 10^{-5} kg/ms

Dynamic Viscosity:

$$(4) \Rightarrow \boxed{\nu = \frac{\mu}{\rho}}$$

Newtonian Fluids

For many simple fluids, such as air and water, μ is a thermodynamic property which depends only on Temperature & Pressure, but not on the shear rate $\left(\tau = \mu \frac{\partial u}{\partial y}\right)$

Note: $\boxed{\text{Newtonian} \neq \text{Non Newtonian Fluids}}$

2.0 Equation of State

This may also be referred to as the *Ideal Gas Law* or the *Perfect Gas Law.*

$$(5) \Rightarrow \boxed{p = \rho R T}$$

Using SI units

R = Specific Gas Constant = 287.05287 J/kg.°K

$$\boxed{R = c_p - c_v} \quad \& \quad \boxed{\gamma = \frac{c_p}{c_v}}$$

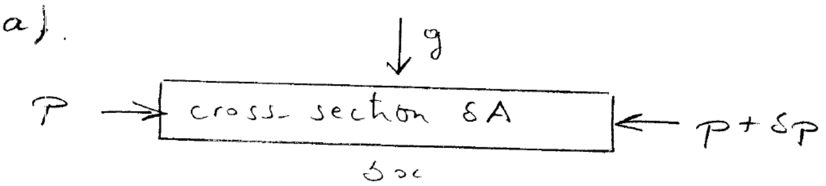
Where c_p & c_v are the specific heats

Atmospheric Models

3.0 The Hydrostatic Equation

For a fluid at rest, the pressure is constant over any horizontal surface but decreases with altitude.

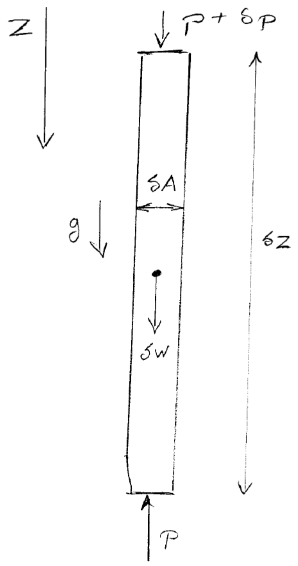
a).



If the pressure is assumed to change from 'p' at one end of the cylinder to 'p + δp' at the other end, then the net force acting on the cylinder is:

$$\boxed{p.\delta A - (p + \delta p)\delta A = -\delta p.\delta A}$$

This net force must be 0 unless the fluid cylinder is accelerating to the right (δp < 0) or to the left (δp > 0). Hence for a fluid at rest, δp = 0 & the pressure is constant along any horizontal line (or surface).



The net downwards force acting on the fluid cylinder is:

$$-p.\delta A + \delta W + (p + \delta p)\delta A = 0$$

$$\text{Where: } \delta W = \rho(\delta z.\delta A)g$$

$$\therefore -p.\delta A + \rho(\delta z.\delta A)g + p.\delta A + \delta p.\delta A = 0$$
$$\therefore \rho.\delta z.\delta A.g + \delta p.\delta A = 0$$

$$(6) \Rightarrow \boxed{\delta p = -\rho.\delta z.g}$$

OR

$$(7) \Rightarrow \boxed{\frac{dp}{dz} = -\rho.g}$$

4.0 An Energy Balance Equation

4.1 Some Definitions

- Adiabatic Process
  One in which no heat is added to or taken away from the system
- Isothermal Process
  One in which the temperature remains constant
- Reversible Process
  One in which no dissipative phenomena occur, that is, where the effects of viscosity, thermal conductivity & mass diffusion are absent
- Isentropic Process
  One that is both Adiabatic & Reversible
  OR
  If there is no heat transfer or friction in the process

4.2 Isothermal Case

If: $T = T_0$

Then the equation of state becomes:

$$p = \rho R T_0$$

$$(8) \Rightarrow \boxed{\rho = \frac{p}{RT_0}}$$

Substitute (8) into (7):

$$(9) \Rightarrow \boxed{\frac{dp}{dz} = -\frac{p}{RT_0} \times g}$$

$$\frac{dp}{p} = -\frac{g}{RT_0} \times dz$$

$$(10) \Rightarrow \boxed{p = p_0, e^{\left(-g/_{RT_0}\right)z}}$$

Where p_0 occurs at $z = 0$

Substitute (10) into (8):

$$\rho = \frac{p_0}{RT_0}, e^{\left(-g/_{RT_0}\right)z}$$

$$(11) \Rightarrow \boxed{\rho = \rho_0, e^{\left(-g/_{RT_0}\right)z}}$$

Where ρ_0 occurs at $z = 0$

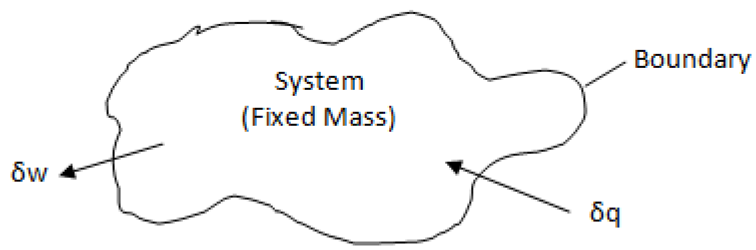
4.3 Adiabatic Process

$$(12) \Rightarrow \boxed{\delta q = 0}$$

For an isentropic process, ds = 0, where s is the entropy of the system.

Note: Entropy & the Second Law of Thermodynamics:

$$(13) \Rightarrow \boxed{ds = \frac{\delta q}{T}}$$



Thermodynamic System

First Law of Thermodynamics:

The change in energy in a system is:

$$(14) \Rightarrow \boxed{de = \delta w + \delta q}$$

For a reversible process: $\delta w = -pdv$

$$(15) \Rightarrow \boxed{de = \delta q - pdv}$$

Where dv is an elemental change in the volume due to a displacement of the boundary of the system

By deriving (13) it can be shown that $\delta q = Tds$

$$\therefore de = Tds - pdv$$

$$(16) \Rightarrow \boxed{Tds = de + p.dv}$$

The enthalpy is defined by:

$$(17) \Rightarrow \boxed{h = e + pv}$$

$$\therefore dh = de + d(pv) = de + p.dv + v.dp$$

$$\therefore de + p.dv = dh - v.dp$$

Substitute this into (16):

$$(18) \Rightarrow \boxed{Tds = dh - v.dp}$$

For a Perfect Gas:

$$de = c_v.dT \quad \& \quad dh = c_p.dT$$

Therefore by substituting into (16) & (18):

$$(19) \Rightarrow \boxed{ds = c_v\frac{dT}{T} + \frac{p.dv}{T}}$$

$$(20) \Rightarrow \boxed{ds = c_p\frac{dT}{T} - \frac{v.dp}{T}}$$

Using the Equation of State:

$$\frac{p}{\rho} = RT \quad OR \quad pv = RT$$

$$\therefore v = \frac{RT}{p} \quad OR \quad p = \frac{RT}{v}$$

Substitute these into (19) & (20):

$$\boxed{ds = c_v\frac{dT}{T} + R\frac{dv}{v}} \Leftarrow (21) \Rightarrow \boxed{ds = c_p\frac{dT}{T} - R\frac{dp}{p}}$$

$$\boxed{s_2 - s_1 = c_v.\ln\left(\frac{T_2}{T_1}\right) + R.\ln\left(\frac{v_2}{v_1}\right)} \Leftarrow (22) \Rightarrow \boxed{s_2 - s_1 = c_p.\ln\left(\frac{T_2}{T_1}\right) + R.\ln\left(\frac{p_2}{p_1}\right)}$$

For an isentropic process, δs = 0, this can be substituted into (22)

$$\boxed{0 = c_v.\ln\left(\frac{T_2}{T_1}\right) + R.\ln\left(\frac{v_2}{v_1}\right)} \quad \& \quad \boxed{0 = c_p.\ln\left(\frac{T_2}{T_1}\right) + R.\ln\left(\frac{p_2}{p_1}\right)}$$

Therefore:

$$\boxed{\ln\left(\frac{v_2}{v_1}\right) = \frac{-c_v}{R}.\ln\left(\frac{T_2}{T_1}\right)} \quad \& \quad \boxed{\ln\left(\frac{p_2}{p_1}\right) = \frac{c_p}{R}.\ln\left(\frac{T_2}{T_1}\right)}$$

Therefore:

$$\boxed{\frac{v_2}{v_1} = \left(\frac{T_2}{T_1}\right)^{\frac{-c_v}{R}}} \Leftarrow (23) \Rightarrow \boxed{\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{c_p}{R}}}$$

Where:

$$\boxed{R = c_p - c_v} \ \& \ \boxed{\gamma = \frac{c_p}{c_v}}$$

Therefore:

$$\boxed{c_v = \frac{R}{\gamma - 1}} \Leftarrow (24) \Rightarrow \boxed{c_p = \frac{\gamma R}{\gamma - 1}}$$

Substitute (24) into (23):

$$\boxed{\frac{v_2}{v_1} = \left(\frac{T_2}{T_1}\right)^{\frac{-1}{(\gamma-1)}}} \Leftarrow (25) \Rightarrow \boxed{\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{(\gamma-1)}}}$$

Where: $v = \frac{1}{\rho}$ $\therefore \frac{v_2}{v_1} = \frac{\rho_1}{\rho_2}$

Substitute this into (25):

$$(26) \Rightarrow \boxed{\frac{\rho_1}{\rho_2} = \left(\frac{T_2}{T_1}\right)^{\frac{-1}{(\gamma-1)}}}$$

Therefore:

$$(27) \Rightarrow \boxed{\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{(\gamma-1)}}}$$

An adiabatic process is characterised by:

$$(28) \Rightarrow \boxed{\frac{p}{\rho^{\gamma}} = cst}$$

4.4 Constant Lapse Rate

In the troposphere, the temperature decreases with altitude z, according to the relation:

$$(29) \Rightarrow \boxed{T = T_o - \gamma z}$$

Where T_0 = 288.16 °K at z = 0

& γ is the lapse rate

$$\therefore dT = -\gamma . dz$$

$$(30) \Rightarrow \boxed{\gamma = -\frac{dT}{dz}}$$

Note: In isothermal conditions T = T_0 = Constant. Therefore γ = 0

Using the hydrostatic equation derived from (9):

$$\frac{dp}{p} = \frac{-g}{RT} dz$$

By substituting equation (29) into it we find:

$$(31) \Rightarrow \boxed{\frac{dp}{p} = \frac{-g}{R(T_0 - \gamma z)} dz}$$

$$\therefore \frac{dp}{p} = \frac{\gamma g}{-\gamma R} \frac{dz}{(T_0 - \gamma z)}$$

$$\therefore \int \frac{dp}{p} = \int \frac{g}{\gamma R} \left(\frac{-\gamma . dz}{(T_0 - \gamma z)}\right)$$

$$(32) \Rightarrow \boxed{\frac{p}{p_0} = \left(\frac{T}{T_0}\right)^{g/\gamma R}}$$

Using the Equation of State:

$$(33) \Rightarrow \boxed{\frac{\rho}{\rho_0} = \left(\frac{T}{T_0}\right)^{(g/\gamma R).1}}$$

Unsure of character on notes!