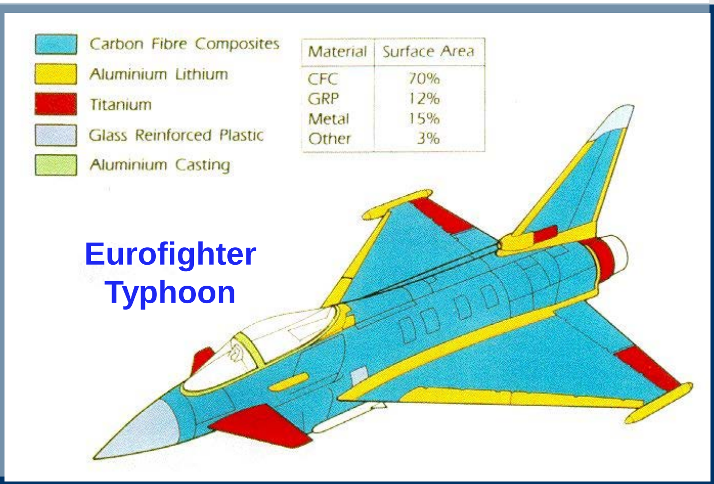
Lecture Note #4-1

IIT Kharagpur

2019 1st Semester Short Course

Design of Aircraft Components using Composite Materials



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Part 4-1

Calculation Example

A. Laminate calculations

- Using : Laminate analysis
  Netting rule
  10% rule
  Carpet plots

B. Panel buckling calculations

- Using : Laminate analysis/ESDU

C. Thin wall section calculations

- Using : Laminate analysis / thin-wall analysis

Appendix

A. Laminate solutions

B. Panel buckling solutions

C. Thin wall section solutions

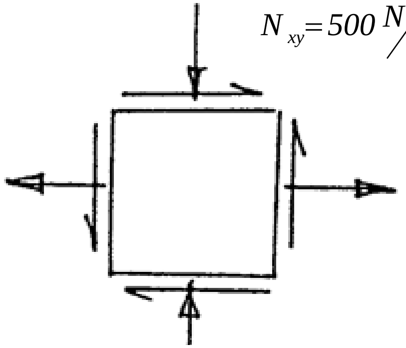
D. Data

A4.1 Laminate design for given loads

$$N_x = 2000\ N/mm$$

$$N_y = -1000\ N/mm$$

$$N_{xy} = 500\ N/mm$$



Givens :

- Membrane ultimate loading intensities from a preliminary analysis as illustrated below
- Material UD-HSCFEP/ stress free temperature 120℃
- Ply thickness 0.125mm

Design:

- Neglect thermal effect

1. Perform a netting laminate analysis for preliminary sizing
2. Check and refine by running a classical laminate analysis

(ⅰ)Using a quasi-isotropic laminate design (QI)

(0,±45,90 standard angles)

(ⅱ)Using a principal stress design (PS)

($\pm\theta$ in the direction of the calculated principal stresses)

Check:

. Effective limit strength

. Effective ultimate strength

. Stiffness coupling terms

. Engineer elastic constants

Compare : QI and PS designs

*Note:

. QI design : Use laminate netting rule to make an initial estimate of the required **.** 0,90,±45 laminate to carry the given $N_x\ N_y\ N_{xy}$ loading

. Principal Stress Design : Using laminate netting rule to make an initial estimate of the required $\pm\theta$ **l**aminate to carry the $N_1\ N_{11}$ principal loading

. For given $N_x\ N_y\ N_{xy}$ loading the principal load intensities are given by:

$$N_1 = \frac{1}{2}(N_x + N_y) + \frac{1}{2}\sqrt{(N_x - N_y)^2 + 4N^2{}_{xy}}$$

$$N_{11} = \frac{1}{2}(N_x + N_y) - \frac{1}{2}\sqrt{(N_x - N_y)^2 + 4N^2{}_{xy}}$$

$$N_{xymax} = \frac{1}{2}(N_I - N_{II})$$

where $\theta_p = \frac{1}{2}\tan^{-1}\frac{2N_{xy}}{N_x - N_y}$

*Note by definition: shear stress will be zero on the $\pm\theta_p$ principal axis

Advantages:

. More efficient(less plies needed)

Disadvantages:

. Significant $A_{16} A_{26}$ shear coupling terms due to large Poission ratio

Failure analysis

. Effective design limit strength $\equiv$ first 2-2or 1-2matrix failure

i.e. : First ply failure "FPF" considered as a partial ply failure with arbitrary stiffness degradation factors applied to matrix dominated properties

$$\left.\begin{aligned} E_{22} &\rightarrow 0.1E_{22} \\ G_{12} &\rightarrow 0.1G_{12} \end{aligned}\right\}$$ Reasonable first estimate

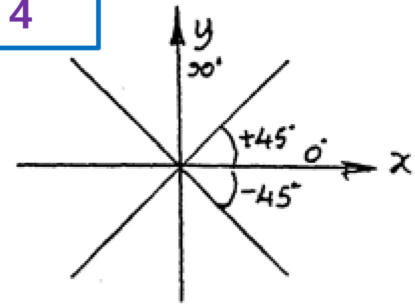
. Effective design ultimate strength $\equiv$ first 1-1 firbre failure

i.e. : Last ply failure "LPF"

HW # 4

Laminate Calculation Example A4.1

A 4.1 (i) QI Laminate Design 0,±45,90°



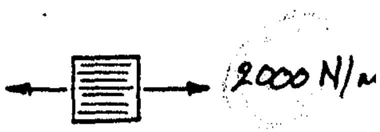
$N_x = 2000\ N/mm$

$N_y = -1000\ N/mm$

$N_{xy} = 500\ N/mm$

Netting Analysis:

For N_x , Design for $\sigma_x = \dfrac{N_x}{t} \le \sigma_1{}^*$



$\rightarrow t_{\circ} \ge \dfrac{N_x}{\sigma_1{}^*} = \dfrac{2000}{1500} = 1.33mm$

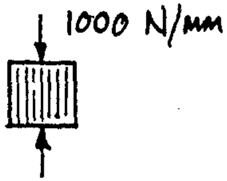
No. of plies $\ge \dfrac{t_{\circ}}{t_p} = \dfrac{1.33}{0.125} = 10.64$: Try 12 x 0°

$\sigma_x = \dfrac{N_y}{t_{90^\circ}} \le \sigma_1{}^*$

For Ny ,

$\rightarrow t_{90^\circ} \ge \dfrac{N_y}{\sigma_1{}^*} = \dfrac{-1000}{-1200} = 0.83$: → 8 x 90°

No. of plies ≥ $\dfrac{t_{90}}{t_p} = \dfrac{0.83}{0.125} =$ 6.64 : $try\ 8\ x\ 90^o$

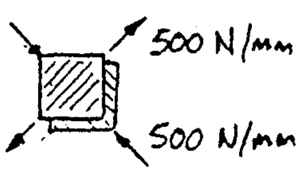


$\sigma_{xy} = \dfrac{N_{xy}}{t_{45^\circ}} \le \sigma_1{}^*$

For Nxy →

$\rightarrow t_{45^\circ} \ge \dfrac{N_{xy}}{\sigma_1{}^*} = \dfrac{500}{-1200} = 0.42$: → 4 x +45°

No. of plies ≥ $\dfrac{t_{45}}{t_p} = \dfrac{0.42}{0.125} =$ 3.36 : $try\ 4$ x +45° for ten. and 4 x -45° for comp.



or 4 x -45° Use Lower of ten. or comp. strength values

→ Initial QI laminate: 12 × 0° , 8 × 90°, 4 × (+45°), 4 × (-45°)

i.e. $\sum$ = 28 ply laminate = 3.5mm total thickness

For stacking sequence consider scheme layup guide lines;

E.g. (+45, -45, 0, 90, 0, 90 , 0 , +45 ,-45 , 0, 90, 0 , 90 , 0)$_s$

i.e. : $\left[\left\{\pm 45,(0,90)_2,0\right\}_2\right]_s$

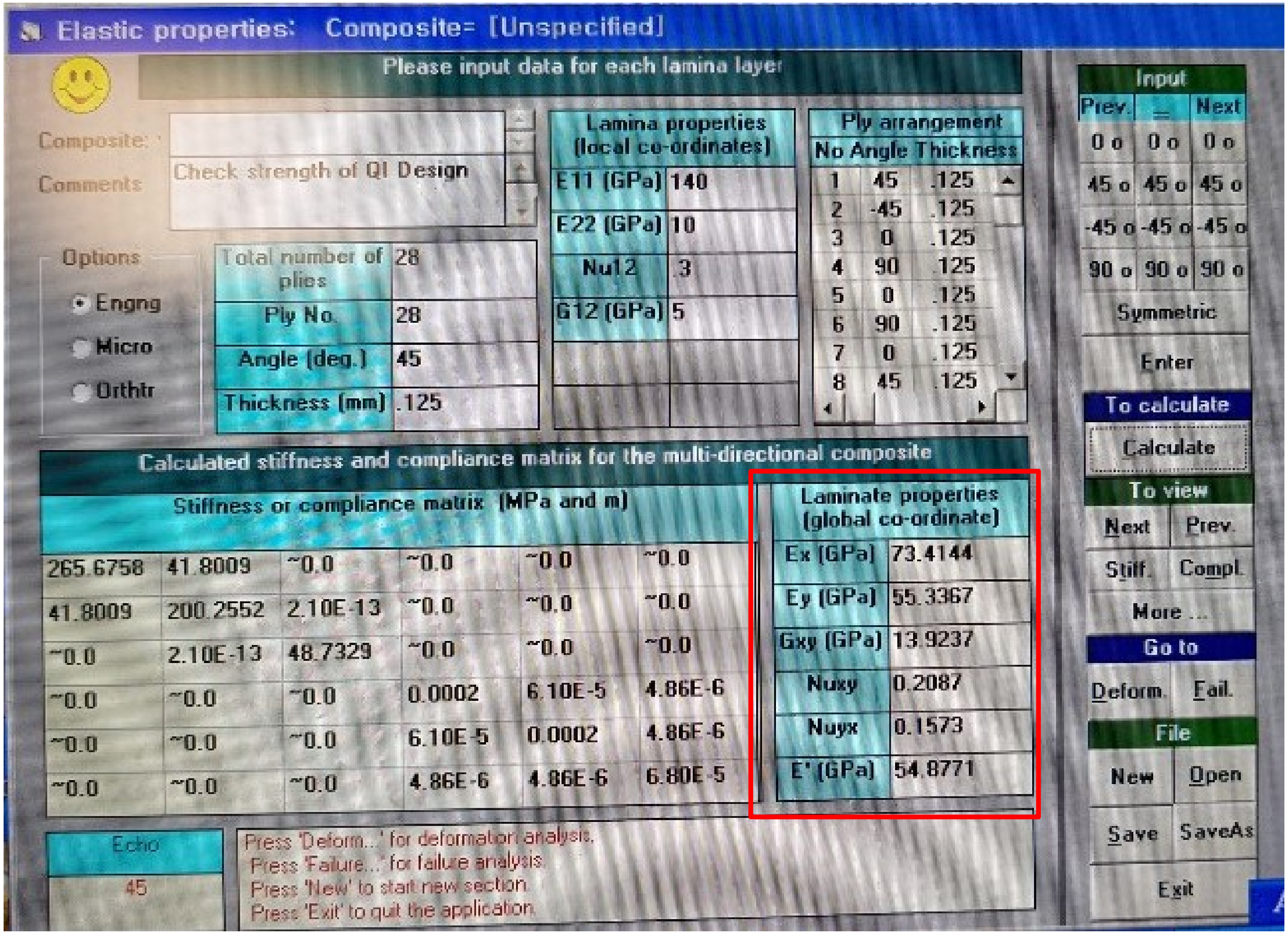
Check strength by CLT (CCD program)

$$N_x = 2000\ N/mm$$

$$N_y = -1000\ N/mm$$

$$N_{xy} = 500\ N/mm$$

Check strength of QI design by CLT (CCD program)



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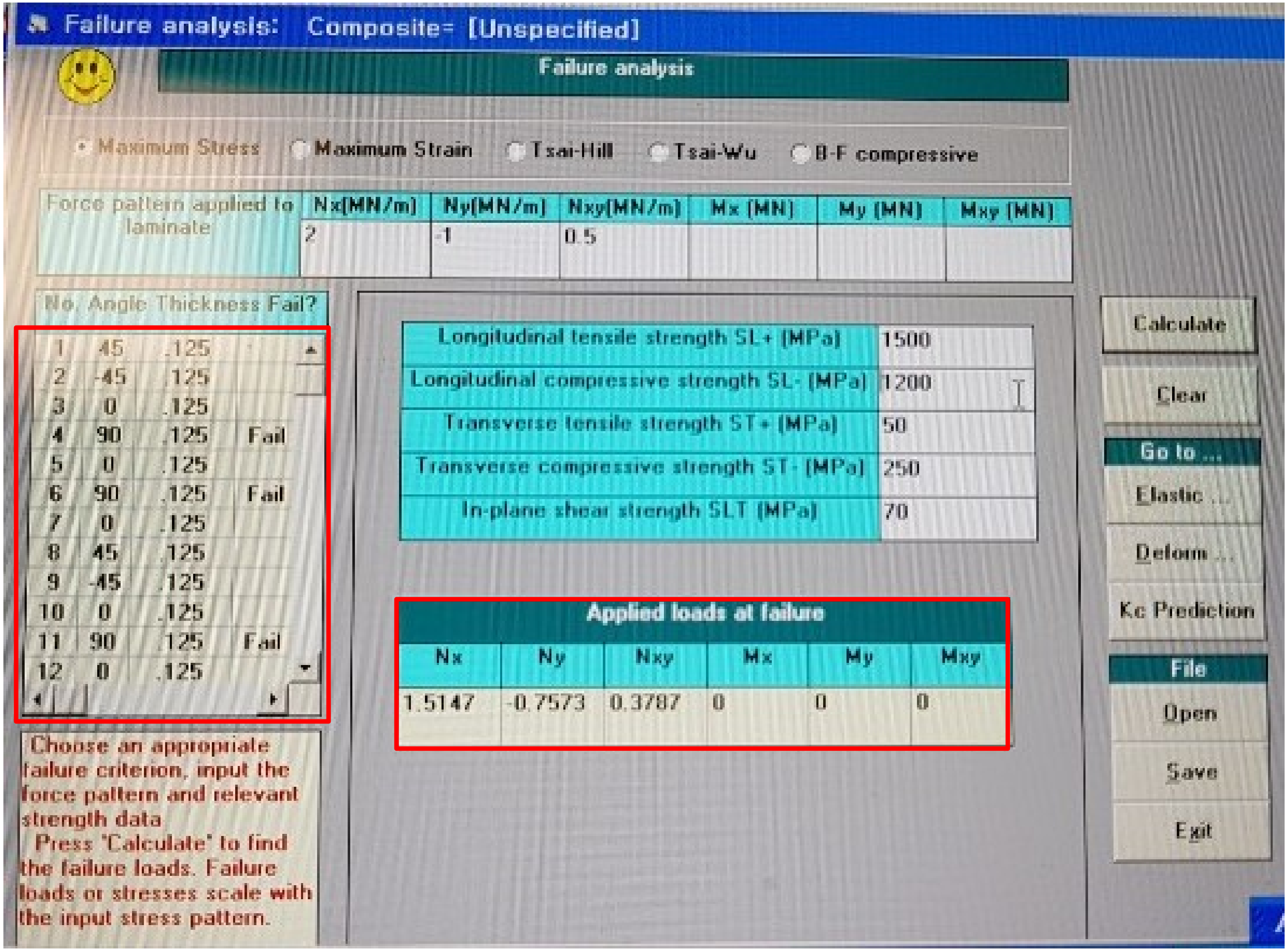
Check strength of QI design by CLT (CCD program)



A 4.1(ⅱ) PS Laminate Design

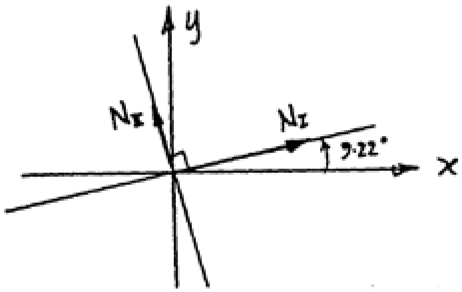
$$N_I = \frac{1}{2}(2000-1000)+\frac{1}{2}\sqrt{(2000+1000)^2+4\times500^2}$$

= 500 +1581

= 2081 N/mm

N_{II} = 500 − 1581

= -1081N/mm



$$N_{XY_{MAX}} = \frac{1}{2}(2081+1081)$$

=1581N/mm

$$\theta_p = \frac{1}{2}\tan^{-1}\left(\frac{2\times500}{2000+1000}\right)$$

= 9.22°

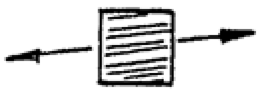
i.e. N_I **@9.22°,** N_{II} **@99.2°**

Netting Analysis

For N_I, Design for $\sigma_I = \frac{N_I}{t_{9.2^\circ}} \le \sigma_1{}^*$

$$\rightarrow\ t_{9.2^\circ} \ge \frac{N_I}{\sigma_1{}^*} = \frac{2081}{1500} = 1.39\text{ mm}$$: Try 12plies.

For N_{II}, Design for $\sigma_{II} = \dfrac{N_{II}}{t_{99.2^\circ}} \le \sigma_1 *$

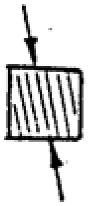


@99.2°

$$\rightarrow \ t_{99.2^\circ} \ge \frac{N_{II}}{\sigma_1 *} = \frac{-1081}{-1200} = 0.90 \text{ mm}$$: Try 8plies.

i.e. 12plies @ 9.2°, 8plies @99.2°

$\sum$ 20plies = 2.5mm total laminate thickness

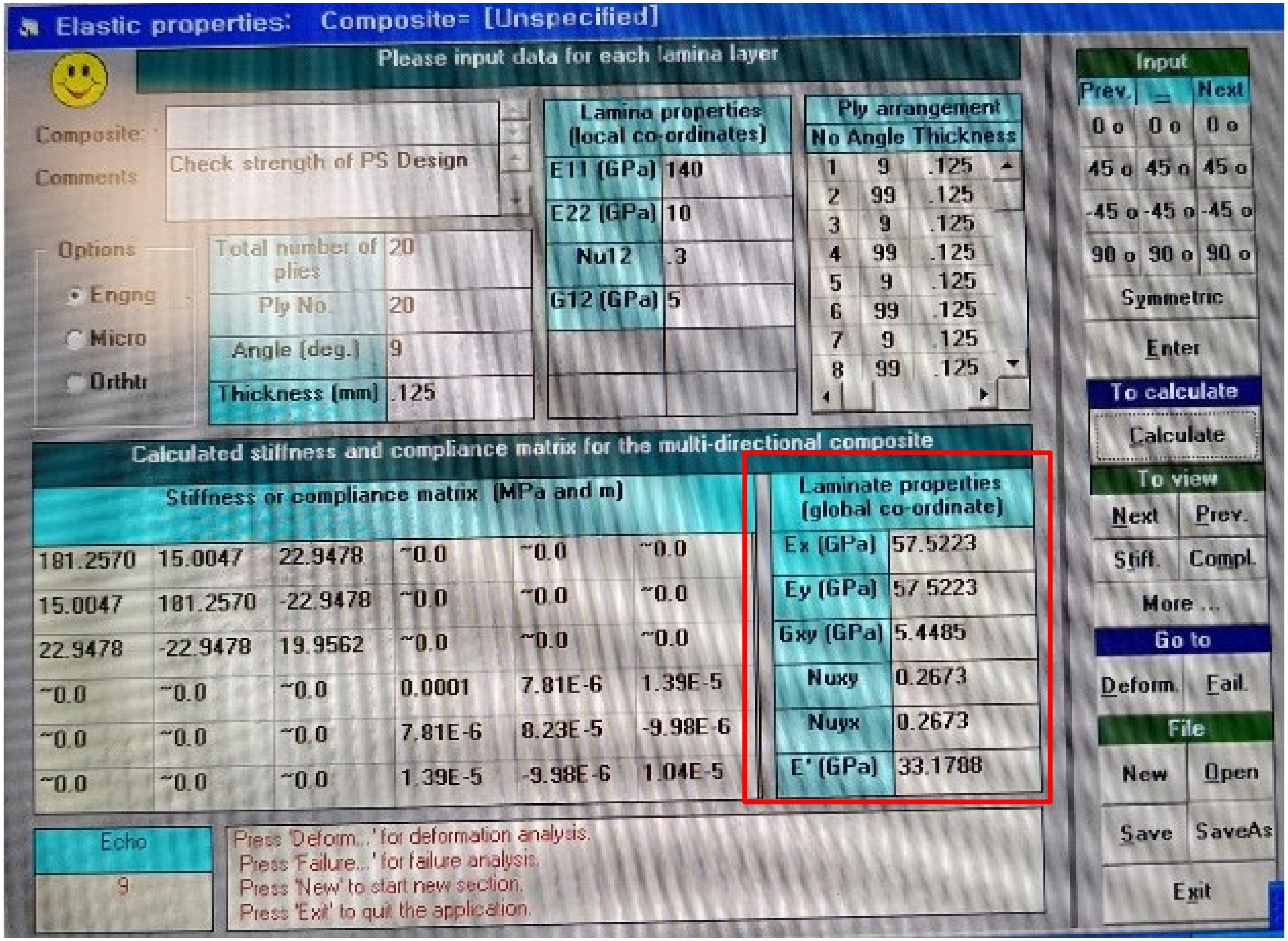


Stacking sequence ?

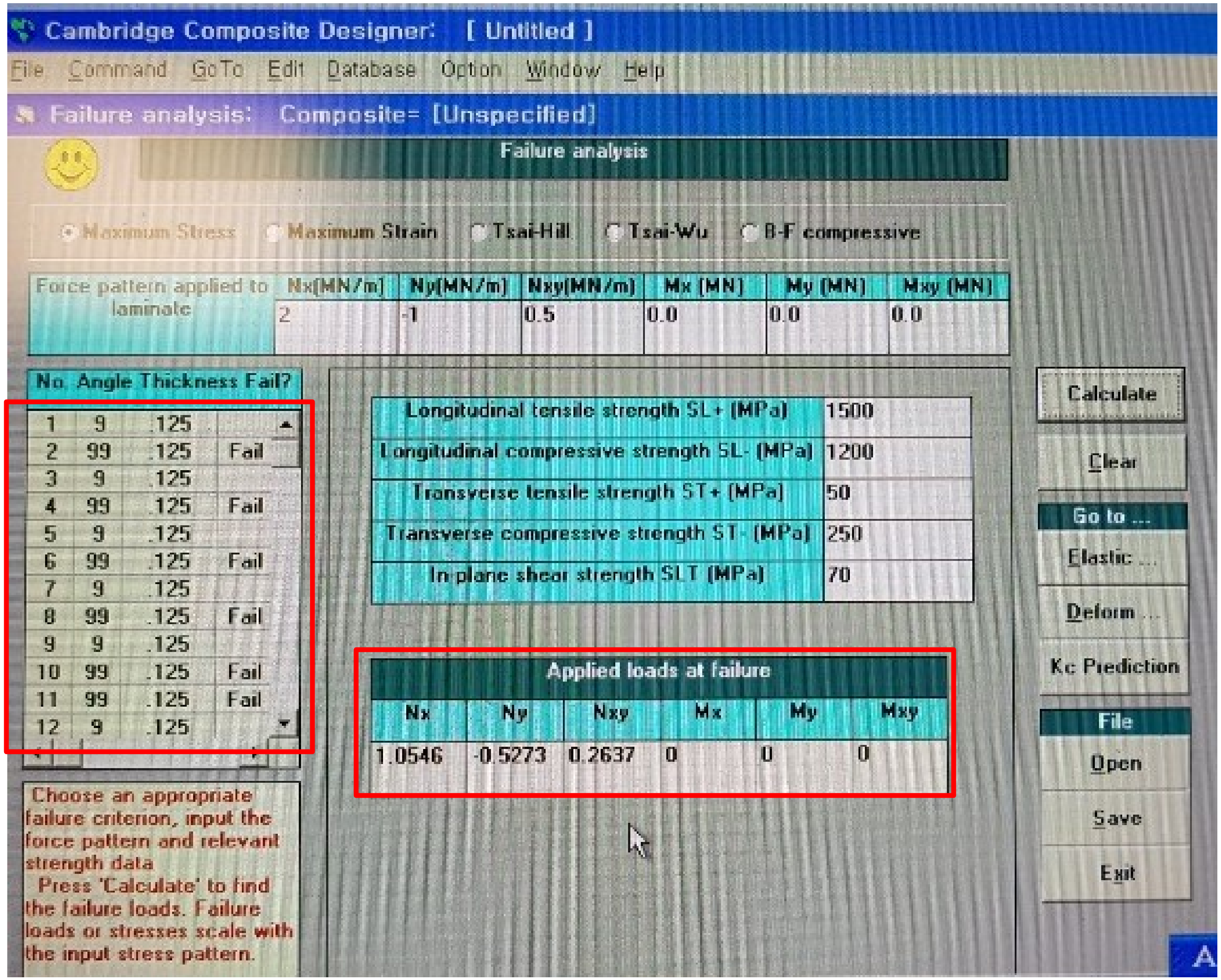
Try : (9,99,9,99,9,9,99,9,99,9)$_s$

i.e. $\left[(9,99)_2, 9_2, (99,9)_2\right]_S$

Check strength of PS design by CLT (CCD program)



Check strength of PS design by CLT (CCD program)



Comparison summary

CCD (CLT) . results :	**“QI” Laminate**	**“PS” Laminate**
* **Effective Limit Strength**		
@ “FPF” ⇒ 2-2		
	0.7573 x Applied Loading	0.5273x Applied Loading
***Effective Ultimate Strength**		
@ “LPF” ⇒ 1-1		
	1.167 x Applied Loading	1.081 x Applied Loading

“FRF” modes 2-2 tension

“LPF” modes 1-1 tension

A5.Comparison of laminate analysis methods

Compare the stiffness and strength values predicted by :

1) Laminate analysis
2) Netting rule
3) Hart-smith 10% rule
4) Carpet plots

for cross ply, angle ply and quasi-isotropic laminates

***Note:**

- For material data values use quasi-isotropic laminates 1) – 3) representing unfactored mean material values
- Use carpet plot data values for specific “HS carbon/epoxy” material

  [2] with degraded properties measured at 120℃ and 1% moisture in method 4)
- The results for carpet values are expected to be lower since degraded
- Degraded property predictions can be obtained from methods 1)-3) by using factored mean values.

E.g. : typical reduction factors for strength related design include;

1.2 for material property

1.1 for service degradation } giving a compounded factor of 1.45

1.1 for thermal effects

- Dividing input data values or predicted results by these factors provides a method of allowing for degradation effects in method 1-3

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0%(Netting) Rule for laminate stiffness or strengths

Loading	Stiffness / Strength	%ply contribution factor 0°	±45	90°	Apply to
	E_x	1.0	0	0	E_1
	E_y	0	0	1.0	E_1
	G_{xy}	No prediction of off-axis stiffness			
Uni-axial longitudinal	σ_x	1.0	0	0	σ_1 *
Uni-axial Transverse	σ_y	0	0	1.0	σ_1 *
Bi-axial Equal/opposite sign i.e. pure shear	τ_{xy}	0	1.0	0	σ_1 *

$\times$ RoM ply thickness fraction

$$\frac{t_{0^\circ}}{t} \qquad \frac{t_{45^\circ}}{t} \qquad \frac{t_{90^\circ}}{t}$$

Note $t_{45^\circ} = t_{\pm 45^\circ} / 2$

10% Rule for laminate stiffness

Loading	Stiffness	% ply contribution factor 0°	±45°	90°	Apply to
	E_x	1.0	0.1	0.1	E_1
	E_Y	0.1	0.1	1.0	E_1
	G_{xy}	0.1	0.55	0.1	$\frac{E_1}{2(1+\nu^*)}$

x RoM ply thickness fraction

$\frac{t_{o^\circ}}{t}$ $\frac{t_{\pm 45^\circ}}{t}$ $\frac{t_{90^\circ}}{t}$

....

$$\nu_{xy} \text{ for QI laminates } = \frac{1}{\left[1+4\left(\frac{\%90^\circ}{\%\pm 45^\circ}\right)\right]}$$

i.e. with plies in all 0°,±45°, 90° directions

where ν^* is the poisson ratio of the "complimentary layup" for doubly symmetric laminates, e.g.:

for ±45° $\nu^* = \nu_{\pm 45^\circ}$ =0.05

for 0°,90° $\nu^* = \nu_{0^\circ,90^\circ}$ =0.8

for 0°, ±45°,90° $\nu^* = \nu_{0^\circ,\pm 45^\circ,90^\circ}$ =0.33

10% Rule for laminates strengths[17]

*Note, here layer contribution factor also depends on loading system

Loading	Strength	% ply contribution factor 0°	±45°	90°	Apply to
Uniaxial	σ^*_x	1.0	0.1	0.1	σ_1^*
	σ^*_y	0.1	0.1	1.0	σ_1^*
Bi-axial Same sign	σ^*_x	1.0	0.55	0.1	σ_1^*
	σ^*_y	0.1	0.55	1.0	σ_1^*
Bi-axial Opposite sign i.e. Shear	τ^*_{xy}	0.1	0.55	0.1	$\sigma_1^*/2$

x RoM ply thickness fraction

$\frac{t_{0^\circ}}{t}$ $\frac{t_{\pm 45^\circ}}{t}$ $\frac{t_{90^\circ}}{t}$

LAMINATE CALCULATION

EXAMPLE A.5

(i) Cross Ply LAMINATE

- Laminate $(0,90)_S$ UDHSCFEP, 0.5mm thick Stiffness

E_x : Laminate analysis transformed layer stiffness

- **CLT** → laminate equivalent value = 75.4 GPa
- **Netting Rule :**

$$1\times\left(\frac{2\times0.125}{0.5}\right)_{0^\circ}\times140+0\times(0)etc_{\pm45^\circ}+0\times(0)_{90^\circ}=70$$

- **H.S. 10% rule:**

$$1\times\left(\frac{2\times0.125}{0.5}\right)_{0^\circ}\times140+0.1\times(0)etc_{\pm45^\circ}+0.1\times\left(\frac{2\times0.125}{0.5}\right)_{90^\circ}\times140=77$$

- **Carpet plot** **read off modulus for**

REF 2 50% 0° 0%±45° 50%90° =67*

E_y : Laminate analysis: transformed layer stiffness

CLT . → laminate equivalent value = 75.4

- **Netting Rule :**

$$0\times\left(\frac{2\times0.125}{0.5}\right)_{0^\circ}+0\times(0)etc_{\pm45^\circ}+1\times\left(\frac{2\times0.125}{0.5}\right)_{90^\circ}\times140=70$$

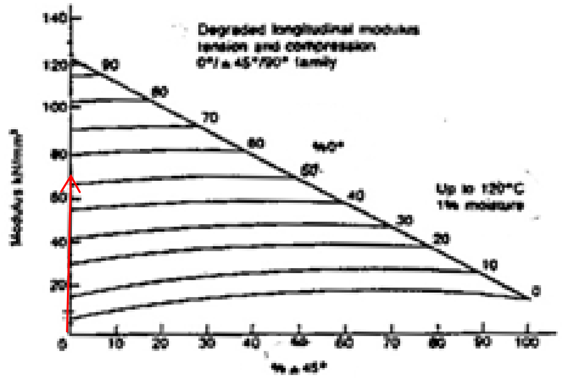
- **H.S 10%rule:**

$$0.1\times\left(\frac{2\times0.125}{0.5}\right)_{0^\circ}\times140+0.1\times(0)etc_{\pm45^\circ}+1\times\left(\frac{2\times0.125}{0.5}\right)\times140=77$$

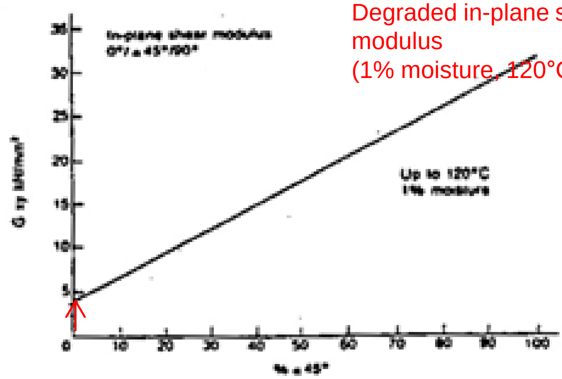
- **Carpet plot** **read off modulus for**

REF 2 50%0° 0%±45° 50%90° =67*

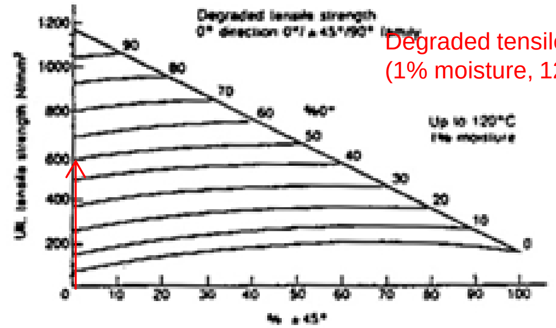
Degraded longitudinal modulus
(1% moisture, 120°C)



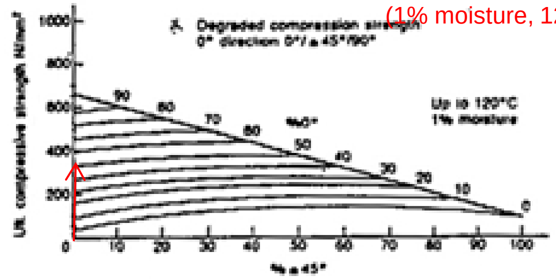
Degraded in-plane shear
modulus
(1% moisture, 120°C)



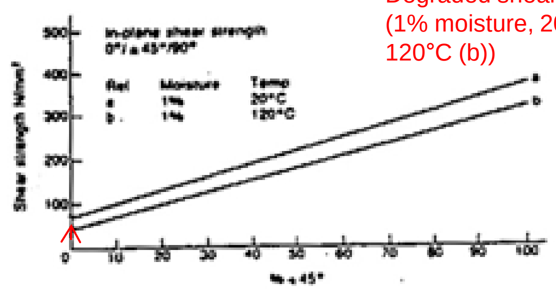
Degraded tensile strength
(1% moisture, 120°C)



Degraded compression strength
(1% moisture, 120°C)



Degraded shear strength
(1% moisture, 20°C (a) and
120°C (b))



Degraded Mechanical Properties of typical Aerospace Approved HS Carbon Epoxy

G_{xy} : Laminate analysis transformed layer stiffness

- **CLT** →Laminate Equivalent Value = 5
- Netting Rule

  No prediction for matrix dominated property

- H.S. 10%rule

$$G_{xy}=0.1\left(\frac{2\times0.125}{0.5}\right)_{0^\circ}\times\frac{140}{2(1+0.8)}+0.55\times(0)_{\pm45^\circ}etc+0.1\times\left(\frac{2\times0.125}{0.5}\right)_{90^\circ}\times\frac{140}{2(1+0.8)}=3.89$$

- Carpet plot: read off tensile strength for

REF 2 25%0° 50%±45° 25%90° =4 KN/mm^2

(ii) Cross Ply Laminate

– $(0,90)_S$ **UDHSCFEP, 0.5mm thick Strength**

σ_x *: Laminate analysis transformed layer strength

- **CLT** →1-1 Last Ply Failure @ σ_x=379.4/0.5 =759

- Netting Rule

$$1 \times \left(\frac{2 \times 0.125}{0.5}\right)_{0^\circ} \times 1500 + 0 \times (0)etc_{\pm 45^\circ} + 0 \times (0)etc_{90^\circ} = 750$$

- H.S 10% rule: $1\times\left(\frac{2\times0.125}{0.5}\right)_{0^\circ}\times1500+$ **1** $\times(0)etc_{\pm45^\circ}+0.1\times\left(\frac{2\times0.125}{0.5}\right)_{90^\circ}\times1500$

  =825

- Carpet plot READ OFF TENSILE STRENGTH FOR

  REF 2 50%O° 0%±45° 50%90° = 600 N/mm^2

σ_y *: Laminate analysis transformed layer strength

- **CLT** →1-1 Last Ply Failure @ σ_y =3.79.4/0.5 =759

- Netting Rule

$$0 \times (0)etc_{90^\circ} + 0 \times (0)etc_{\pm 45^\circ} + \mathbf{1} \times \left(\frac{2 \times 0.125}{0.5}\right)etc_{90^\circ} \times 1500 = 750$$

- H.S 10%rule

$$0.1\times\left(\frac{2\times0.125}{0.5}\right)_{0^\circ}\times1500+ \mathbf{1} \times(0)etc_{\pm45^\circ}+1\times\left(\frac{2\times0.125}{0.5}\right)_{90^\circ}\times1500 = 825$$

- Carpet plot: read off tensile strength for

  REF 2 50%O° 0%±45° 50%90° =600 N/mm^2

τ_{xy} : Laminate analysis transformed layer strength

- **CLT** →1-2 First Ply Failure @ $\tau_{xy} = 35/0.5 = 70$
- Netting Rule

  No prediction for matrix dominated properties
- H.S 10%rule

$$0.1\times\left(\frac{2\times0.125}{0.5}\right)_{0^\circ}\times\frac{1500}{2}+0.55\times(0)etc_{\pm45^\circ}+0.1\times\left(\frac{2\times0.125}{0.5}\right)_{90^\circ}\times\frac{1500}{2} = 75$$

- Carpet plot: read off tensile strength for

  REF 2 50%0° 0%±45° 50%90° =45* (70RT) N/mm^2

(i) Angle Ply Laminate

- Laminate $(\pm 45)_S$ UDHSCFEP, 0.5mm thick

E_x : Laminate analysis transformed layer stiffness

- **CLT** → Laminate Equivalent Value = 17.7
- Netting Rule

  No prediction for matrix dominated laminate
- H.S 10%rule

$$1 \times (0)etc_{0^\circ} + 0.1 \times \left(\frac{4 \times 0.125}{0.5}\right)_{\pm 45^\circ} \times 140 + 0.1 \times (0)etc_{90^\circ} = 14$$

- Carpet plot: read off tensile strength for

  REF 2 0%0° 100%±45° 0%90° =15* KN/mm^2

E_y : Laminate analysis transformed layer stiffness

- **CLT** → Laminate Equivalent Value = 17.7
- Netting Rule

  No prediction for matrix dominated laminate
- H.S 10%rule

$$0.1 \times (0)etc_{0^\circ} + 0.1 \times \left(\frac{4 \times 0.125}{0.5}\right)_{\pm 45^\circ} \times 140 + 1 \times (0)etc_{90^\circ} = 14$$

- Carpet plot: read off tensile strength for

  REF 2 0%0° 100%±45° 0%90° =15* KN/mm^2

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G_{xy} : Laminate analysis transformed layer stiffness

- **CLT** → Laminate Equivalent Value = 36.2
- Netting Rule

  No prediction for matrix dominated property
- H.S. 10%rule

$$0.1\times(0)etc_{0^\circ} + 0.55\times\left(\frac{4\times0.125}{0.5}\right)_{\pm45^\circ}\times\frac{140}{2(1+0.05)} + 0.1\times(0)etc_{0^\circ} = 36.7$$

- Carpet plot: read off tensile strength for

REF 2 0%0° 100%±45° 0%90° =32* KN/mm^2

* Degraded Carpet Plot Values

(ii) Angle Ply Laminate

– **Laminate** $(\pm45)_S$ **UDHSCFEP, 0.5mm thick**

σ_x* : Laminate analysis transformed layer strength

- **CLT** →1-2 First Ply Failure @ σ_x=70/0.5 =140
- Netting Rule

  No prediction for matrix dominated laminate
- H.S. 10%rule

$$1\times(0)etc_{0^\circ} + 0.1\times\left(\frac{4\times0.125}{0.5}\right)_{\pm45^\circ}\times1500 + 0.1\times(0)etc_{90^\circ} = 150$$

- Carpet plot: read off tensile strength for

REF 2 0%O° 100%±45° 0%90° =150* N/mm^2

σ_y*: Laminate analysis transformed layer strength

- **CLT** →1-2 First Ply Failure @ σ_x=70/0.5 =140
- Netting Rule

  No prediction for matrix dominated laminate
- H.S 10%rule

$$0.1\times(0)etc_{0^\circ}+0.1\times\left(\frac{4\times0.125}{0.5}\right)_{\pm45^\circ}\times1500+1\times(0)etc_{90^\circ}$$ =150

- Carpet plot: read off tensile strength for

  REF 2 0%O° 100%±45° 0%90° =150* N/mm^2

σ_{xy}* : Laminate analysis transformed layer strength

- ∟ **CLT** .. .→2-2 First Ply Failure @ τ_{xy}=181/0.5 =362
- Netting Rule

  No prediction for matrix dominated property
- H.S 10%rule $0.1\times(0)etc_{0^\circ}+0.55\times\left(\frac{4\times0.125}{0.5}\right)_{\pm45^\circ}\times\frac{1500}{2}+0.1\times(0)etc_{90^\circ}$ = 412.5
- Carpet plot: read off tensile strength for

  REF 2 0%0° 100%±45° 0%90° =350* N/mm^2

* Degraded Carpet Plot Values

(i) Quasi Isotropic Laminate

– **Laminate** $(0,\pm45,90)_S$ **UDHSCFEP, 1mm thick**

E_x : Laminate analysis transformed layer stiffness

- **CLT** →LAMIANTE EQUIVALENT VALUE =54.1
- Netting Rule

$$1\times\left(\frac{2\times0.125}{1}\right)_{0^\circ}\times140+0\times(0)etc_{\pm45^\circ}+0\times(0)etc_{90^\circ}$$ = 35

- H.S. 10%rule

$$1\times\left(\frac{2\times0.125}{1}\right)_{0^\circ}\times140+0.1\times\left(\frac{4\times0.125}{1}\right)\times140+0.1\times\left(\frac{2\times0.125}{1}\right)\times140$$ =45.5

- Carpet plot; read off tensile strength for

REF 2 25%0° 50%±45° 25%90° =42* KN/mm^2

E_y : Laminate analysis transformed layer stiffness

- **CLT** →Laminate equivalent value =54.1
- Netting Rule

$$0\times(0)etc_{0^\circ}+0\times(0)etc_{\pm45^\circ}+1\times\left(\frac{2\times0.125}{1}\right)_{90^\circ}\times140$$ = 35

- H.S 10%rule

$$0.1\times\left(\frac{2\times0.125}{1}\right)_{0^\circ}\times140+0.1\times\left(\frac{4\times0.125}{1}\right)\times140+1\times\left(\frac{2\times0.125}{1}\right)\times140$$ =45.5

- Carpet plot : read off tensile strength for

REF 2 25%0° 50%±45° 25%90° =42* KN/mm^2

G_{xy} : Laminate analysis transformed layer stiffness

- **CLT** →Laminate Equivalent Value =20.6
- Netting Rule

  No prediction for matrix dominated property
- H.S. 10%rule

$$0.1\times\left(\frac{2\times0.125}{1}\right)_{0^\circ}\times\frac{140}{2(1+0.33)}+0.55\times\left(\frac{4\times0.125}{1}\right)_{\pm45^\circ}\times\frac{140}{2(1+0.33)}+0.1\times\left(\frac{2\times0.125}{1}\right)_{90^\circ}\times\frac{140}{2(1+0.33)}$$

= 17.1

- Carpet plot: read off tensile strength for

  REF 2 25%0° 50%±45° 25%90° =11* KN/mm^2

(ⅱ) Quasi Isotropic Laminate

- **Laminate** $(0,\pm45,90)_S$ **UDHSCFEP, 1mm thick**

σ_x *: Laminate analysis transformed layer strength

- **CLT** →1-1 Last Ply Failure @ σ_x = 374
- Netting Rule

$$1\times\left(\frac{2\times0.125}{0.5}\right)_{0^\circ}\times1500+0\times(\,)etc_{\pm45^\circ}+0\times(\,)etc_{90^\circ} = 375$$

- H.S 10%rule $1\times\left(\frac{2\times0.125}{1}\right)_{0^\circ}\times1500+0.1\times\left(\frac{4\times0.125}{1}\right)\times1500+0.1\times\left(\frac{2\times0.125}{1}\right)\times1500$

  =487
- Carpet plot: read off tensile strength for

  REF 2 25%0° 50%±45° 25%90° =380 N/mm^2

σ_y * : Laminate analysis transformed layer strength

CLT 4A →1-1 Last Ply Failure $\sigma_y = 374$

- Netting Rule

$$0\times(0)etc_{0^\circ}+0\times(0)etc_{\pm45^\circ}+1\times\left(\frac{2\times0.125}{1}\right)_{90^\circ}\times1500 = 375$$

- H.S 10%rule

$$0.1\times\left(\frac{2\times0.125}{1}\right)_{0^\circ}\times1500+0.1\times\left(\frac{4\times0.125}{1}\right)_{\pm45^\circ}\times1500+1\times\left(\frac{2\times0.125}{1}\right)\times1500 = 487$$

- Carpet plot read off tensile strength for

REF 2 25%0° 50%±45° 25%90° =380 N/mm^2

τ_{xy} * : Laminate analysis transformed layer strength

CLT 4A → 2-2 First Ply Failure $\tau_{xy} = 289$

- Netting Rule

No prediction for matrix dominated property

- H.S 10%rule

$$0.1\times\left(\frac{2\times0.125}{1}\right)_{0^\circ}\times\frac{1500}{2}+0.55\times\left(\frac{4\times0.125}{1}\right)_{\pm45^\circ}\times\frac{1500}{2}+0.1\times\left(\frac{2\times0.125}{1}\right)_{90^\circ}\times\frac{1500}{2}$$

=243

- Carpet plot: read off tensile strength for

REF 2 25%0° 50%±45° 25%90° =110* (150 RT) N/mm^2

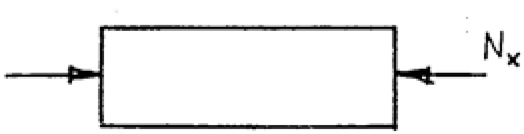
B. Panel buckling calculations

Using : **Laminate analysis / ESDU**
B1.1 Laminated plate buckling check

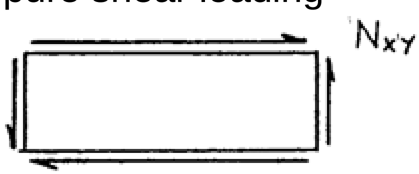
Given :

- Quasi-isotropic laminate $(\pm45,0,90)_{3/s}$
- Material UD-HSCFEP/ stress free temperature 120℃
- Ply thickness 0.125mm
- ESDU buckling curve
- Neglect thermal effect
- Panel dimensions 1000x250mm
- All edges simply supported
- Subjected to :

(ⅰ) direct compressive loading



(ⅱ) pure shear loading



Assess :

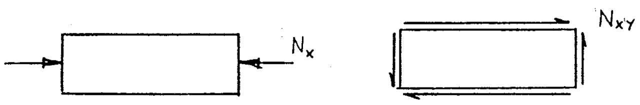
- The laminate panel buckling strength for loading cases a) and b), and compare with the laminate first ply and last ply failure strengths and also with the buckling strength of a light alloy panel of equal dimensions.

*Note:

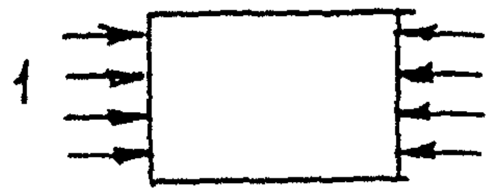
- Make use of results from a unit loading laminate analysis run
- Use derived flexural stiffness terms for buckling calculations

Appendix B. Panel buckling solutions

(i) direct compressive loading (ii) pure shear loading



B1 (i) Uniaxial Compression Loading



Laminate Analysis

- Using unit N_x load intensity
- Derived bending stiffness values from **CLT**

$$(D_{11}D_{22})^{\frac{1}{2}} = 125687 \text{ Nmm}$$

$$(D_{12} + 2D_{66}) = 161058 \text{ Nmm} \qquad = \text{“} D_{\circ} \text{”}$$

$$(D_{22}/D_{11})^{\frac{1}{4}} = 0.97 \text{ Nmm}$$

*Note laminate strength values for Compression

First Ply failure occurs @ N_x = -886 N/mm in mode 2-2/T, layer 4

“FPF” i.e. @ σ_x =-886/3=-295 N/mm^2

Last ply failure @ N_x =-1346 N/mm in mode 1-1/T, layer 3

i.e. @ σ_x =-1346/3=-449 N/mm^2

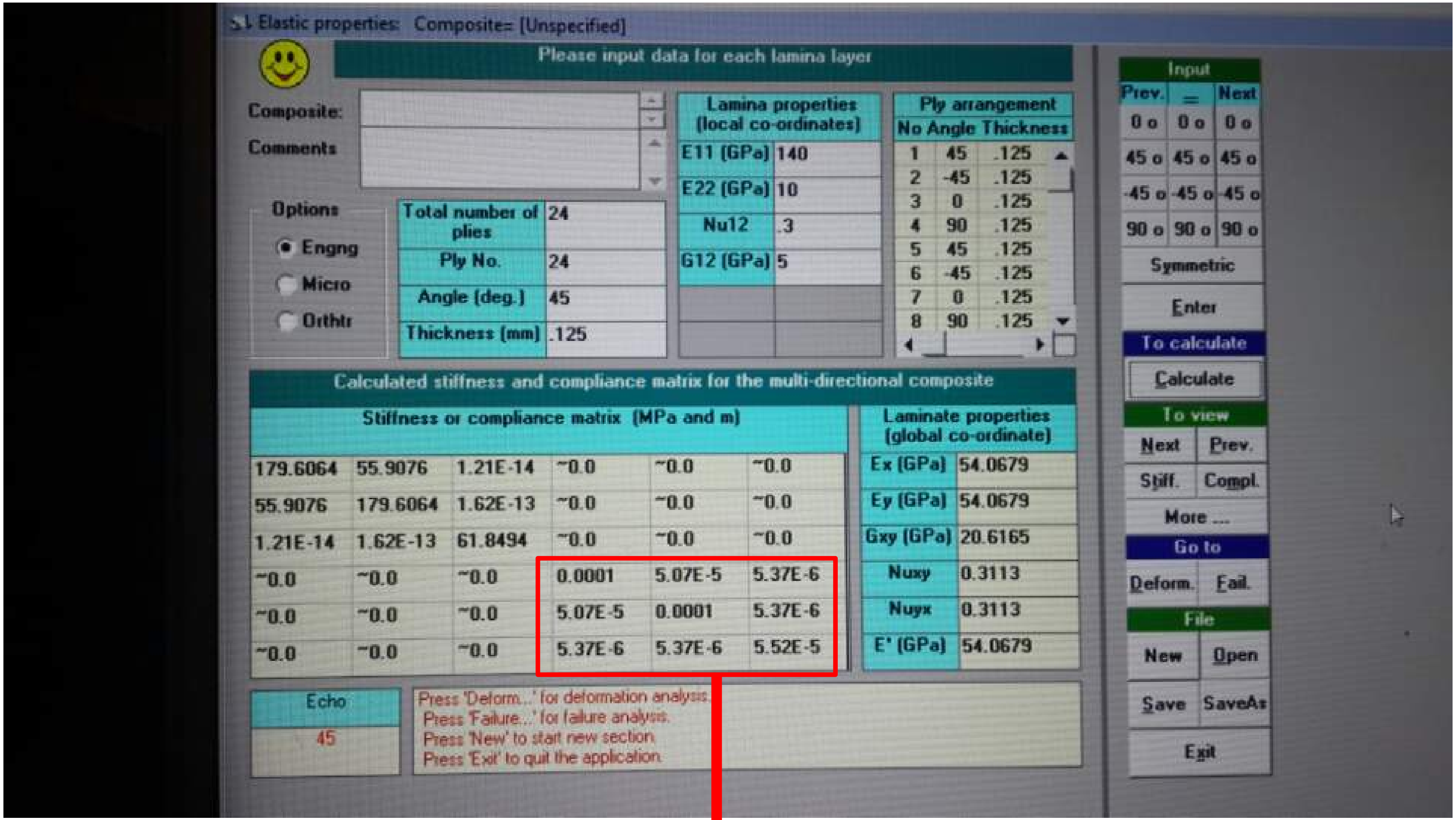
Buckling calculation – using ESDU

$$\frac{a}{b} = \frac{1000}{250} = 4$$

$$\frac{a}{b}\left(\frac{D_{22}}{D_{11}}\right)^{\frac{1}{4}} = 4 \times 0.97 = 3.88$$ → Asymptote value

Ends/sides simply supported → c=2

ESDU Fig.1 → k_0 =20



D_{ij} : in order to match the unit as N-mm, 10^9 times must be applied to these values

$$N_{Xcrit} = \frac{K_0(D_{11}D_{22})}{b^2} + \frac{C\Pi^2(D_{12}+2D_{66})}{b^2}$$

$$= \frac{20\times 125'678}{250^2} + \frac{2\Pi^2\times 161'058}{250^2}$$

$$= 40.22 + 50.87$$

$$= 91.09 N/mm$$

$\rightarrow \sigma_{Xcrit} = \frac{91.09}{3}$ = 30.36 N/mm^2 Note <<FPF Buckling occurs!

- **Compression with Al alloy panel of equal thickness**

$$\sigma_{cr} = KE\left(\frac{t}{b}\right)^2$$

$$= 3.62 \times 70'000\left(\frac{3}{250}\right)^2$$

= 36.5 N/mm^2

*Note : Composite panel direct Compression buckling strength
= 83% of ally panel buckling strength!

Aspect ratio=a/b=1000/250=4
K'=4 from graph "SS-SS" case

$$\sigma_{cr} = K' \cdot \frac{\pi^2 \cdot E}{12(1-\nu^2)} \cdot \left(\frac{t}{b}\right)^2$$

K=4x0.904=3.62

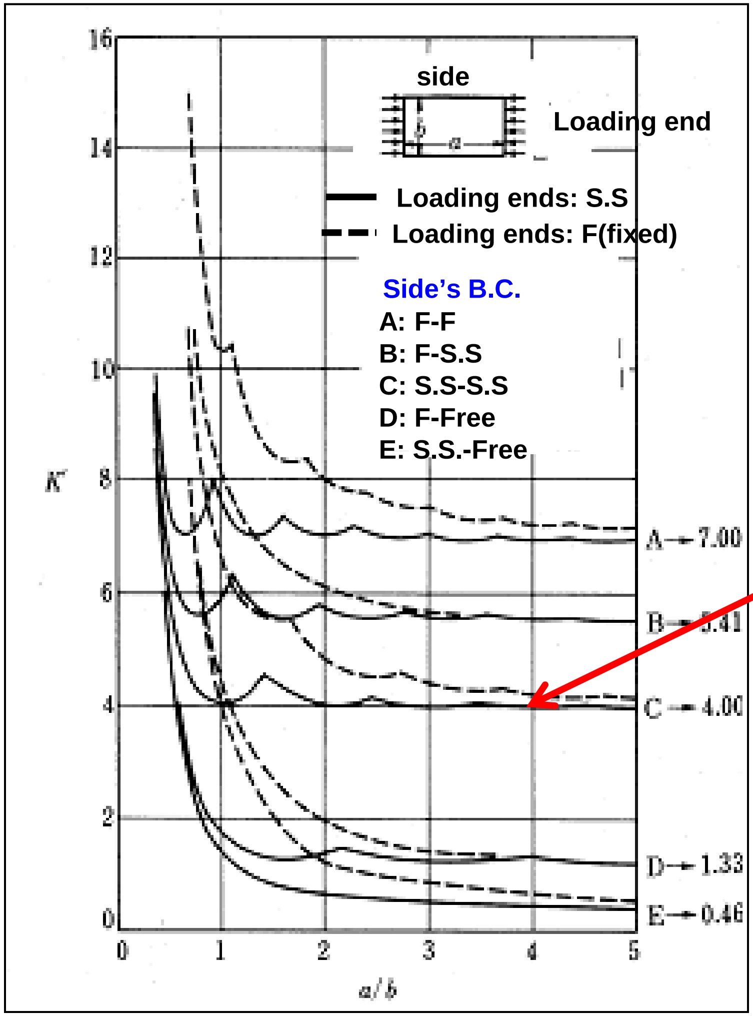


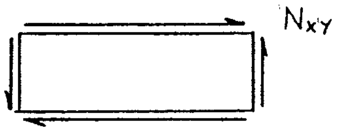
Fig. 12.7

- Depending on support B.C., different K=K'

$$\sigma_{cr} = K' \cdot \frac{\pi^2 \cdot E}{12(1-\nu^2)} \cdot \left(\frac{t}{b}\right)^2$$

(ⅱ) pure shear loading

B1(ⅱ) Pure in-plane shear loading



Laminate Analysis

- Using unit N_{xy} load intensity

- Bending stiffness
- Laminate strength values for comparison

First ply failure occurs @ N_{xy} = 309 N/mm in mode 2-2/T, layer 2(-45°)

i.e. @ τ_{xy} =309/3=103 $\mathrm{N/mm^2}$

Buckling Calculation – using ESDU

$$* \; D_0 = (D_{11} + 2D_{66}) = 161058$$

$$* \frac{a}{b}\left(\frac{D_{22}}{D_{11}}\right)^{\frac{1}{2}} = 3.88 \quad , \qquad \frac{D_0}{(D_{11} \cdot D_{22})^{\frac{1}{2}}} = \frac{161058}{125687} = 1.28$$

ESDU Fig 5 All edges simply supported

$$\rightarrow \frac{N_{bxy} a \cdot b}{(D_{11} \cdot D_{22})^{\frac{1}{2}}} = 105 \qquad N_{bxy} = N_{xycrit}$$

$$\rightarrow N_{bxy} = \frac{105(D_{11}D_{22})^{\frac{1}{2}}}{a \cdot b}$$

$$= \frac{105 \times 125'687}{1000 \times 250}$$

i.e. $N_{xy_{crit}} = 52.8 N/mm$

and $\tau_{xy_{crit}} = \frac{52.8}{3}$ = $\underline{\underline{17.6}}\, N/mm^2$ <<FPF Buckling occurs!

Comparison with AL alloy panel of equal thickness

$$\tau_{xy_{crit}} = KE\left(\frac{t}{b}\right)^2$$

$$= 5 \times 70'000\left(\frac{3}{250}\right)^2$$

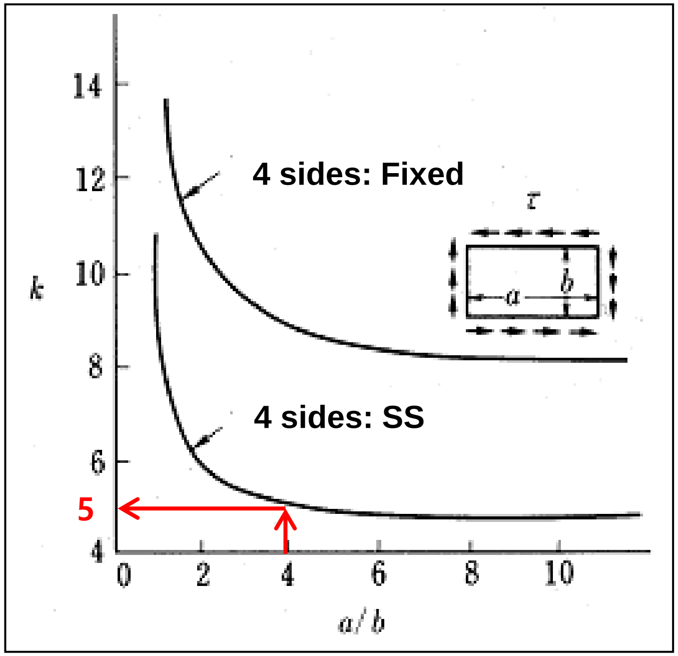
= $\underline{\underline{50.4}}\, N/mm^2$

*Note: composite panel shear buckling strength

= $\underline{\underline{35\%}}$ of AL ally panel buckling strength!

Shear buckling coefficient of plate with different B.C.

$$\tau_{cr} = KE\left(\frac{t}{b}\right)^2$$



C. Thin wall section calculations

HW # 5

Using : Laminate analysis / thin-wall analysis

. Calculate section stiffnesses and stress for sections and loading cases given in the following example

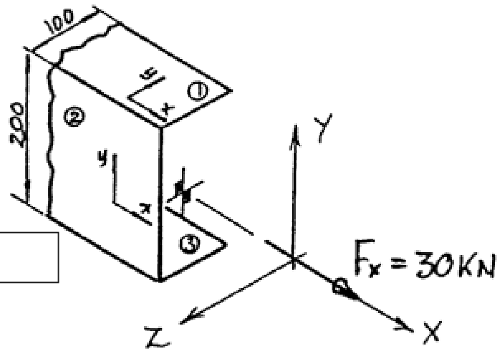
- All laminates are based on material UD-HSCFEP
- Ply thickness 0.125mm
- Neglect thermal effects

C 1 Axial loading

C1.1

Top/btm Flange Laminates : QI=(0,+45,-45,90)s

Web Laminate AP=(+45,-45)s

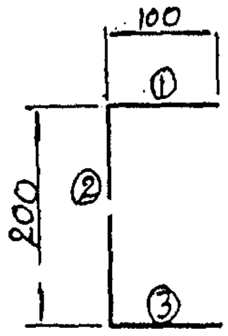


C1.1

Appendix C. Thin wall section solutions

(For laminate runs see examples A2.1)

THIN WALL SECTION
EXAMPLE C1.1



Section details:

$$Layup_{1,3} = QI = (0,+45,-45,90)_S$$

$$\rightarrow \ t_{1,3} = 1mm$$

$$E_{x_{1,3}} = 54.1 \times 10^3 N/mm^2$$

Axial stiffness can be obtained by CLT, ROM or Carpet methods. Here used CLT.

$$Layup_2 = AP = (+45,-45)_S$$

$$\rightarrow \ t_2 = 0.5mm$$

$$E_{x_2} = 17.7 \times 10^3 N/mm^2$$

Axial load $F_x = 30KN$

Thru centroid

Total section area

$$\sum A_i = \begin{Bmatrix} 1 \times 100 \\ +0.5 \times 200 \\ +1 \times 100 \end{Bmatrix} = 300mm^2$$

Effective section axial stiffness

$$\overline{AE} = \sum A_i E_{xi} = \left\{ \begin{array}{l} (1\times 100)\times 54.1\times 10^3 \\ +(0.5\times 200)\times 17.7\times 10^3 \\ +(1\times 100)\times 54.1\times 10^3 \end{array} \right\} = 12.59\times 10^6 \, N/\ strain$$

Equivalent isotropic section modulus

$$\overline{E} = \frac{\sum A_i E_{xi}}{\sum A_i} = \frac{12.59}{300}\times 10^6 = 41.97\times 10^3 \, N/mm^2$$

Element axial stresses

①top flange $\sigma_{x1} = \frac{F_x}{\sum A_i}\cdot\frac{E_{x1}}{\overline{E}} = \frac{30\times 10^3}{300}\times\frac{54.1\times 10^3}{41.97\times 10^3} = 128.9 N/mm^2$

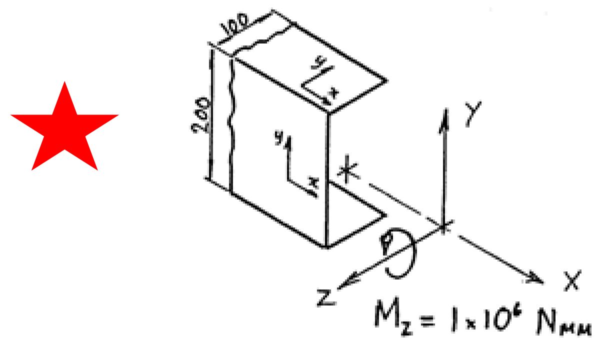
②web $\sigma_{x2} = \frac{F_x}{\sum A_i}\cdot\frac{E_{x2}}{\overline{E}} = \frac{30\times 10^3}{300}\times\frac{17.7\times 10^3}{41.97\times 10^3} = 42.2 N/mm^2$

③btm flange $\sigma_{x3} = 128.9 N/mm^2$ - by symmetry

C 2 Bending loading

C2.1

Top/btm Flange laminates : QI=(0,+45,-45,90)s
Web Laminates : AP=(+45,-45)s

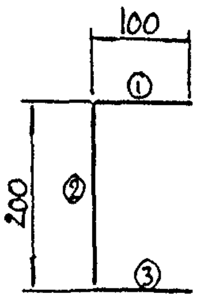


C2.1

EXAMPLE

$$Layup_{1,3} = QI = (0,+45,-45,90)_S$$

$$\rightarrow \ t_{1,3} = 1mm$$



$$E_{x_{1,3}} = 54.1 \times 10^3 \, N/mm$$

$$Layup_2 = AP = (+45,-45)_S$$

$$\rightarrow \ t_2 = 0.5mm$$

$$E_{x_2} = 17.7 \times 10^3 \, N/mm^2$$

Bending Mont $M_Z = 1 \times 10^6 \, N/mm$

About symmetric axis

Total section area

$$\sum A_i = \begin{Bmatrix} 1 \times 100 \\ +0.5 \times 200 \\ +1 \times 100 \end{Bmatrix} = 300mm^2$$

Effective section axial stiffness

$$\overline{AE} = \sum A_i E_{xi} = \begin{Bmatrix} (1 \times 100) \times 54.1 \times 10^3 \\ +(0.5 \times 200) \times 17.7 \times 10^3 \\ +(1 \times 100) \times 54.1 \times 10^3 \end{Bmatrix} = 12.59 \times 10^6 \, N/strain$$

Equivalent isotropic section modulus

$$\overline{E} = \frac{\sum A_i E_{xi}}{\sum A_i} = \frac{12.59\times10^6}{300} = 41.97\times10^3\, N/mm^2$$

Effective section bending stiffness (rigidity)

$$\overline{EI_z} = \sum E_{xi} I_{zi}$$

$$= \begin{Bmatrix} 54.1\times10^3 \times \{(100\times1)\times100^2\} \\ +17.7\times10^3 \times \{0.5\times200^3/12\} \\ +54.1\times10^3 \times \{(100\times1)\times100^2\} \end{Bmatrix} = \begin{Bmatrix} 54.1\times10^{10} \\ +0.59\times10^{10} \\ +5.41\times10^{10} \end{Bmatrix} = 11.41\times10^{10}\, N/mm^2$$

Equivalent section isotropic second moment of area

$$\overline{I_z} = \frac{\sum E_{xi} I_{zi}}{\overline{E}} = \frac{11.41\times10^{10}}{41.97\times10^3} = 2.72\times10^6\, mm^4$$

Element bending(axial) stress

①top flange $\sigma_{x1} = \pm\frac{M_z Y_1}{\overline{I}_z}\cdot\frac{E_{x1}}{\overline{E}} = \frac{-(1\times10^6)\times100}{2.72\times10^6}\times\frac{54.1\times10^3}{41.97\times10^3} = -47.5N/\ mm^2$

②web $\sigma_{x2} = \pm\frac{M_z Y_2}{\overline{I}}\cdot\frac{E_{x2}}{\overline{E}} = \pm\frac{(1\times10^6)\times100}{2.72\times10^6}\times\frac{17.7\times10^3}{41.97\times10^3} = \pm15.5N/\ mm^2$

③btm flange $\sigma_{xi} = +47.5N/mm^2$

Element loading intensities

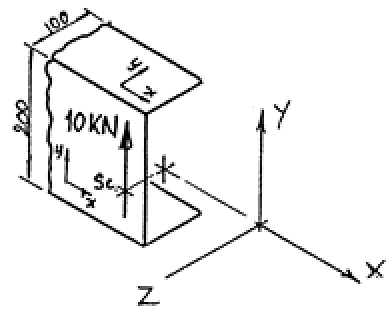
- calculate from $N_{xi} = \sigma_{xi}\times t_i \;\rightarrow\; N/mm$

C 3 Shear loading

C3.1

Top/btm Flange Laminates : QI=(0,+45,-45,90)s

Web Laminates : AP=(+45,-45)s



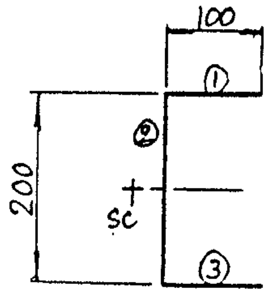
C3.1

Thin WALL SECTION
EXAMPLE C3.1

$$Layup_{1,3} = QI = (0,+45,-45,90)_S$$

$$\rightarrow \quad t_{1,3} = 1mm$$

$$E_{x_{1,3}} = 54.1 \times 10^3 \, N/mm$$



$$Layup_2 = AP = (+45,-45)_S$$

$$\rightarrow \quad t_2 = 0.5mm$$

$$E_{x_2} = 17.7 \times 10^3 \, N/mm^2$$

Shear load F_y =10KN

Thru shear centre

// l to principal axis

Section total area

$$\sum A_i = \begin{Bmatrix} 1 \times 100 \\ +1.5 \times 200 \\ +1 \times 100 \end{Bmatrix} = 300mm^2$$

Effective section axial stiffness

$$\overline{AE} = \sum A_i E_i = \begin{Bmatrix} (1 \times 100) \times 54.1 \times 10^3 \\ +(0.5 \times 200) \times 17.7 \times 10^3 \\ +(1 \times 100) \times 54.1 \times 10^3 \end{Bmatrix} = 12.59 \times 10^6 \, N/strain$$

Equivalent isotropic section modulus

$$\overline{E} = \frac{\sum A_i E_{xi}}{\sum A_i} = \frac{12.59 \times 10^6}{300} = 41.97 \times 10^3 \, N/mm^2$$

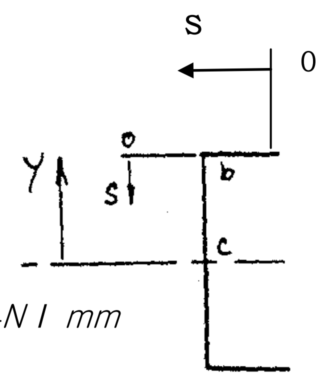
Effective section bending stiffness (rigidity)

$$\overline{EI_z} = \sum E_{xi} I_{zi}$$

$$= \begin{Bmatrix} 54.1 \times 10^3 \times \{(100 \times 1) \times 100^2\} \\ +17.7 \times 10^3 \times \{0.5 \times 200^3 / 12\} \\ +54.1 \times 10^3 \times \{(100 \times 1) \times 100^2\} \end{Bmatrix} = \begin{Bmatrix} 5.41 \times 10^{10} \\ +0.59 \times 10^{10} \\ +5.41 \times 10^{10} \end{Bmatrix} = 11.41 \times 10^{10} \, N/mm^2$$

Equivalent section isotropic second momentof area

$$\overline{I_z} = \frac{\sum E_{xi} I_{zi}}{\overline{E}} = \frac{11.41 \times 10^{10}}{41.97 \times 10^3} = 2.72 \times 10^6 \, mm^4$$



Element shear flows

$$q_{xy_i} = N_{xy_i} = \frac{F_y}{\overline{I_z}} \int t y_i \, ds \cdot \frac{E_{xi}}{\overline{E}}$$ Evaluate integral around section

①top flange : y=constant=100 , t =constant =1

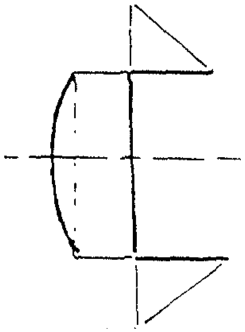
$$N_{xyb} = \frac{10000}{2.72 \times 10^6} \times \int_0^{100} 1 \times 100 ds \cdot \frac{54.1 \times 10^3}{41.97 \times 10^3} = 47.4 N/mm$$

At b point $[100s]_0^{100} = [100 \cdot 100 - 100 \times 0] = 10000$

②web y=100-s , t=constant =0.5

$$N_{xy_c} = 47.4 + \frac{10000}{2.72\times10^6}\times\int_0^{100} 0.5\times(100-s)ds\times\frac{17.7\times10^3}{41.97\times10^3} = 47.4+3.88 = 51.28$$

=51.28：c point $\left[100s - \frac{s^2}{2}\right]_0^{100} = 10000 - 100^2 / 2 = 5000$



③btm flange –by symmetry

Element shear stress $\tau_{xyi} = N_{xyi} / t_i$

C 4 Torsion loading

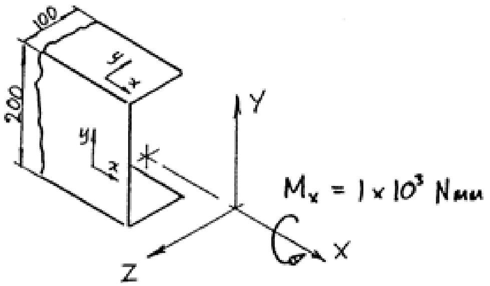
C4.1

Flange and web Laminates : AP=(+45,-45)

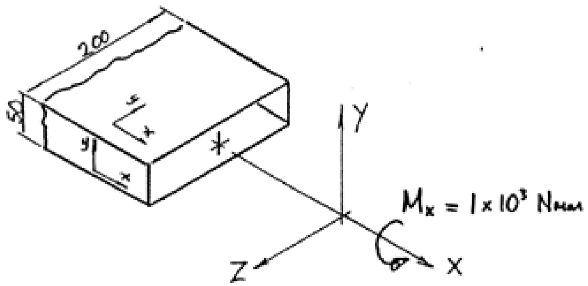
C4.2

Top/btm Flange laminates : QI=(0,+45,-45,90)s

Web Laminates : AP=(+45,-45)s



C4.1

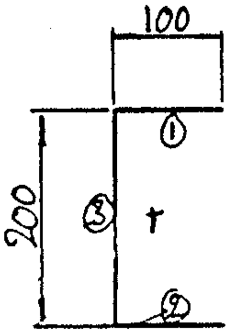


C4.2

THIN WALL SECTION
EXAMPLE C4.1

Torsional loading

$$M_x = 1 \times 10^3 \, N/mm$$



All elements : $Layup_{1,2,3} = (+45, -45)_s$

$\rightarrow$ t= 0.5mm

$$E^m_x = 17.7 \times 10^3$$

$$G^b_{xy} = 20.9 \times 10^3$$

Effective section torsional stiffness

$$\overline{GJ} = \sum G^b_{xy} \frac{bt^3}{3}$$

$$= \left\{ \begin{array}{l} 1 \times 20.9 \times 10^3 \times \{100 \times 0.5^3 / 3\} \\ +1 \times 20.9 \times 10^3 \times \{200 \times 0.5^3 / 3\} \\ +1 \times 20.9 \times 10^3 \times \{100 \times 0.5^3 / 3\} \end{array} \right\} = 0.34 \times 10^6 \, N/mm^2$$

Rate of twist

$$\frac{d\theta}{dx} = \frac{T}{\overline{GJ}} = \frac{1 \times 10^3}{0.348 \times 10^6} = 2.87 \times 10^{-3} \, rad/mm$$

Max shear stress

$$\tau_{xy_{\max}} = \pm G^b_{xy} t \cdot \frac{d\theta}{dx} = \pm 20.9 \times 10^3 \times 0.5 \times 2.87 \times 10^{-3} = \pm 30 \, N/mm^2$$

THIN WALL SECTION
EXAMPLE C4.2

– Skin $Layup_{1,3} = (0.+45,-45,90)$

$\rightarrow$ t=1mm

$$E_x^m = 54.1 \times 10^3 N$$

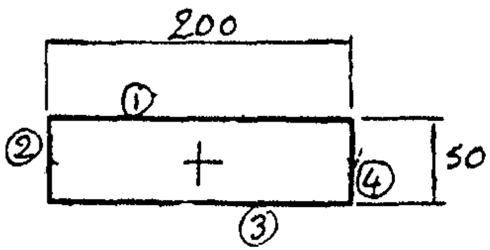
$$G_{xy}^m = 20.6 \times 10^3 N$$

– Web $Layup_{2,4} = AP = (+45,-45)_S$

$\rightarrow$ t= 0.5mm

$$E_x^m = 17.7 \times 10^3 N/mm^2$$

$$G_{xy}^m = 36.4 \times 10^3 N/mm^2$$



Torsional loading

$$M_x = 1 \times 10^3 Nmm$$

Section enclosed area

A=200×50=10,000 mm^2 1

Effective section torsional stiffness

$$\overline{GJ} = 4A^2 / \int \frac{dS}{G_{xy}^m t} = 4A^2 / \sum \frac{b_i}{G_{xyi}^m t_i}$$

$$= 4 \times 10000^2 / \left\{ \begin{array}{l} 200/(20.6 \times 10^3 \times 1) \\ +50/(36.4 \times 10^3 \times 0.5) \\ +200/(20.6 \times 10^3 \times 1) \\ +50/(36.4 \times 10^3 \times 0.5) \end{array} \right\} = 1.612 \times 10^{10} Nmm^2$$

Rate of twist

$$\frac{d\theta}{dx} = \frac{T}{\overline{\overline{GJ}}} = \frac{1\times10^3}{1.612\times10^3} = 6.205\times10^{-8} rad / mm$$

section shear flow (constant)

$$q_{xy} = N_{xy} = \frac{T}{2A} = \frac{1 \times 10^3}{2 \times 10000} = 0.05N / mm$$

Appendix D. Data ★

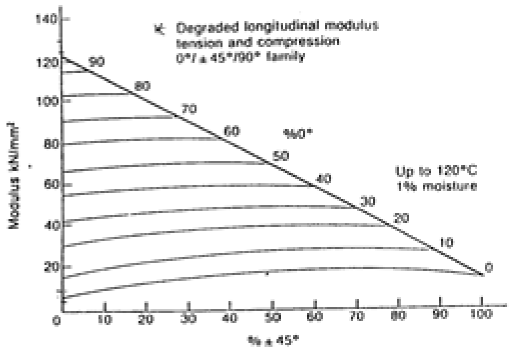
Typical Generic Material Data

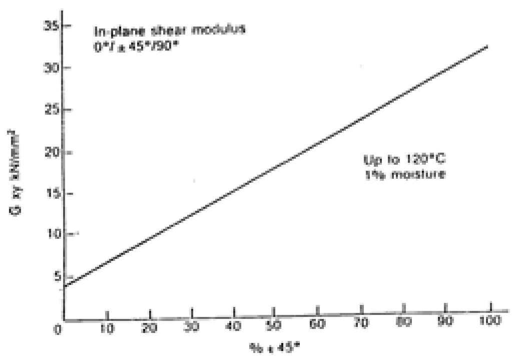
For uni-directional tape and woven fabric composites

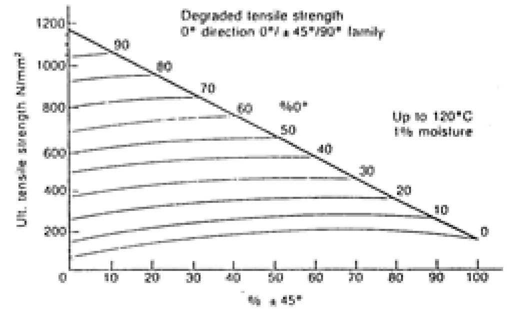
UD Tapes	Stiffness				Strengths				
	KN / mm^2				Nmm^2				
(60%vf)	E_1	E_2	G_{12}	ν_{12}	σ_{1t}	σ_{1c}	σ_{2t}	σ_{2c}	τ_{12}
HSCREP	140	10	5	0.3	1500	-1200	50	-250	70
HMCREP	180	8	5	0.3	1000	-850	40	-200	60
EGFEP	40	8	4	0.25	1000	-600	30	-110	40
KFEP	75	6	2	0.34	1300	-280	30	-40	60
Typical ply thickness 0.125-0.2mm									

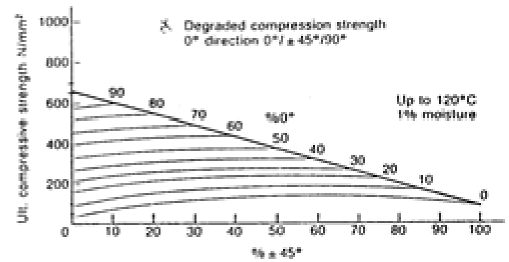
Fabric	Stiffness				Strengths				
	KN / mm^2				Nmm^2				
(50%Vf)	E_1	E_2	G_{12}	ν_{12}	σ_{1t}	σ_{1c}	σ_{2t}	σ_{2c}	τ_{12}
HSCREP	70	70	5	0.10	600	-570	600	-570	90
HMCREP	85	85	5	0.10	350	-150	350	-150	35
EGFEP	25	25	4	0.20	440	-425	440	-425	40
KFEP	30	30	5	0.20	480	-190	480	-190	50
Typical ply thickness 0.25-0.4mm									

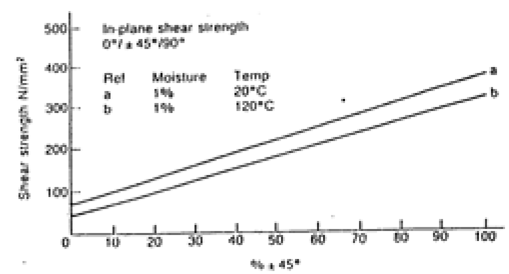
Note :
An initial estimate of strains can be made assuming linear elasticity to failure
E.g. $\varepsilon_{1t} = \sigma_{1t} / E_1 \; etc$


Degraded longitudinal modulus
tension and compression
0°/ ± 45°/90° family
Modulus kN/mm²
%0°
Up to 120°C
1% moisture
% ± 45°


In-plane shear modulus
0°/ ± 45°/90°
G xy kN/mm²
Up to 120°C
1% moisture
% ± 45°


Degraded tensile strength
0° direction 0°/ ± 45°/90° family
Ult. tensile strength N/mm²
%0°
Up to 120°C
1% moisture
% ± 45°


Degraded compression strength
0° direction 0°/ ± 45°/90°
Ult. compressive strength N/mm²
%0°
Up to 120°C
1% moisture
% ± 45°


In-plane shear strength
0°/ ± 45°/90°
Shear strength N/mm²
Ref Moisture Temp
a 1% 20°C
b 1% 120°C
% ± 45°

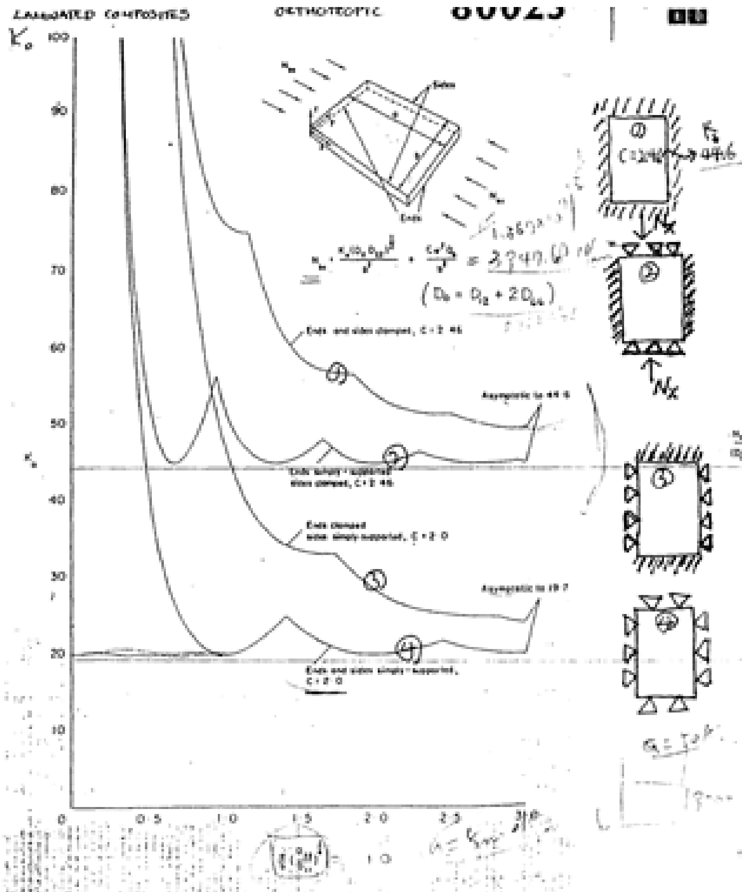


FIGURE 1 UNIAXIAL LOAD COMBINATIONS OF CLAMPED AND SIMPLY-SUPPORTED EDGE CONDITIONS

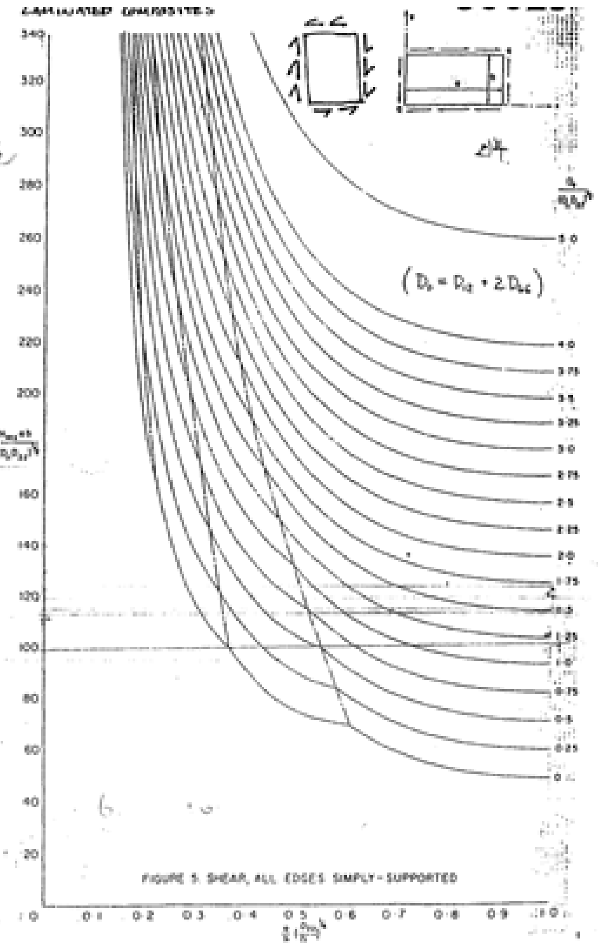


FIGURE 5 SHEAR, ALL EDGES SIMPLY-SUPPORTED

ESDU 80023

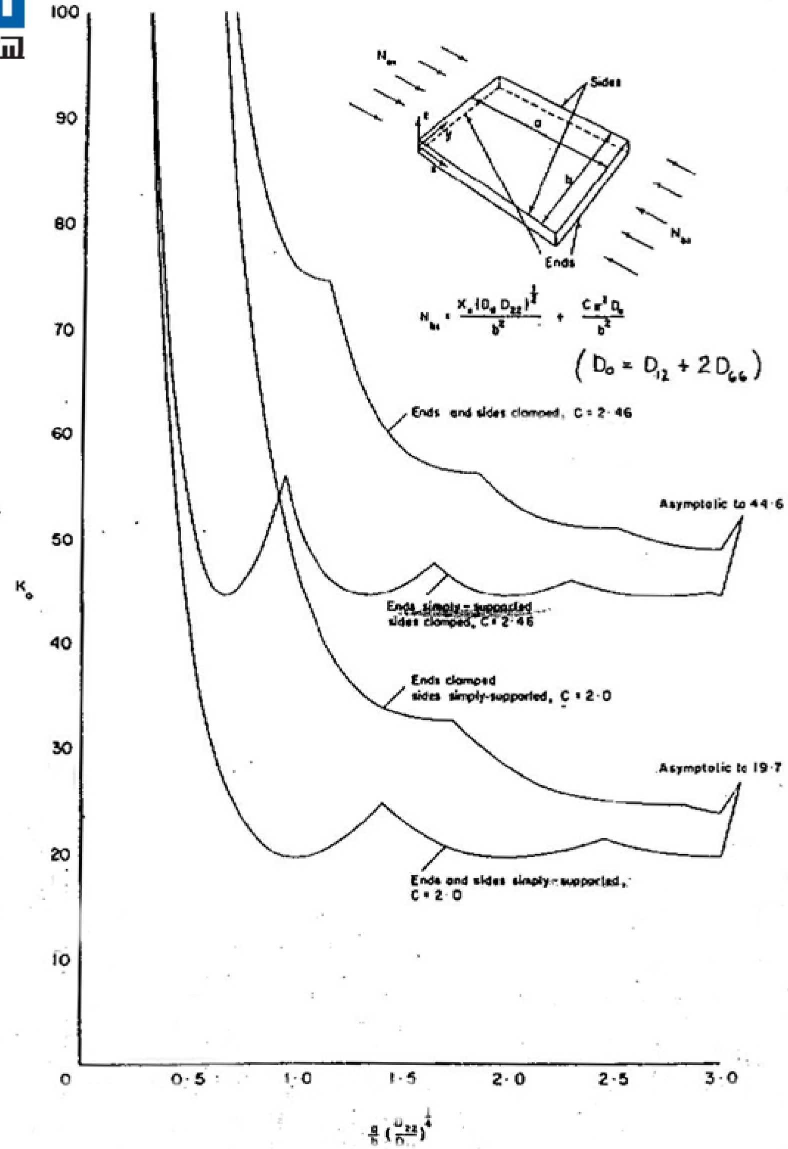


FIGURE 1. UNIAXIAL LOAD COMBINATIONS OF CLAMPED AND

ESDU 80023

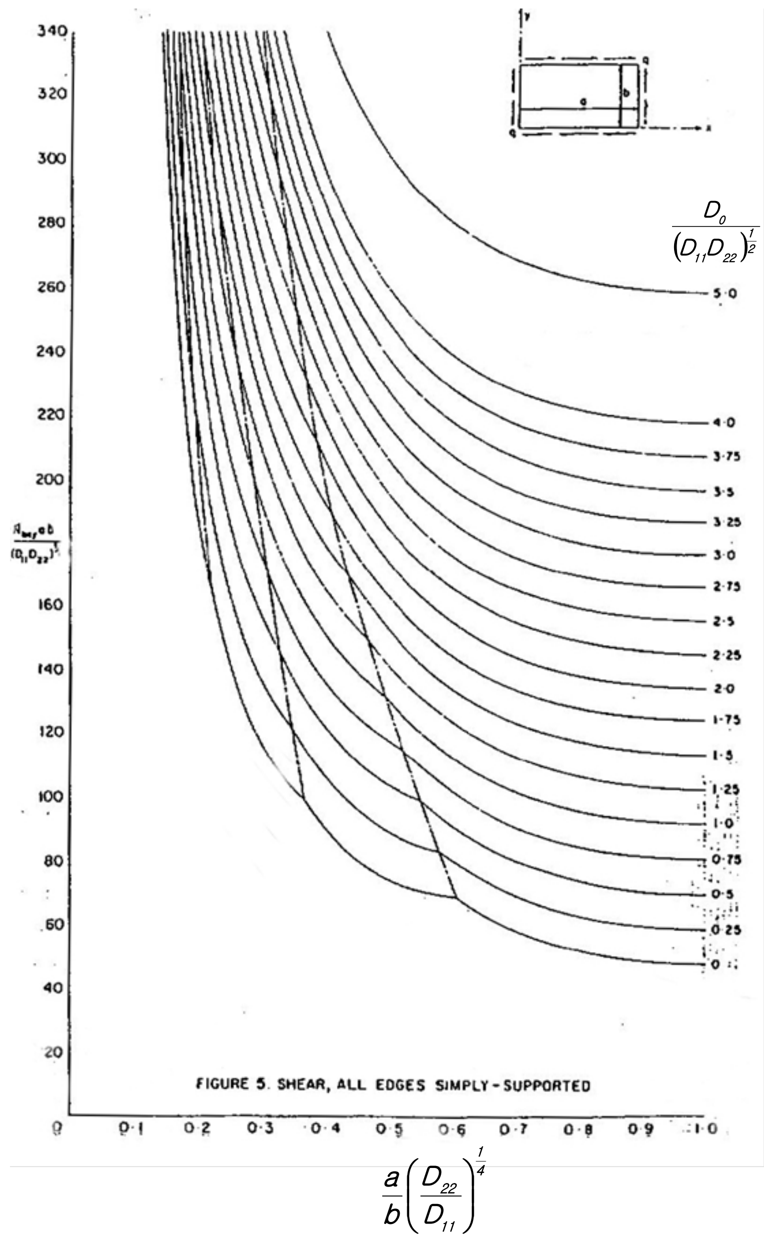


FIGURE 5. SHEAR, ALL EDGES SIMPLY-SUPPORTED

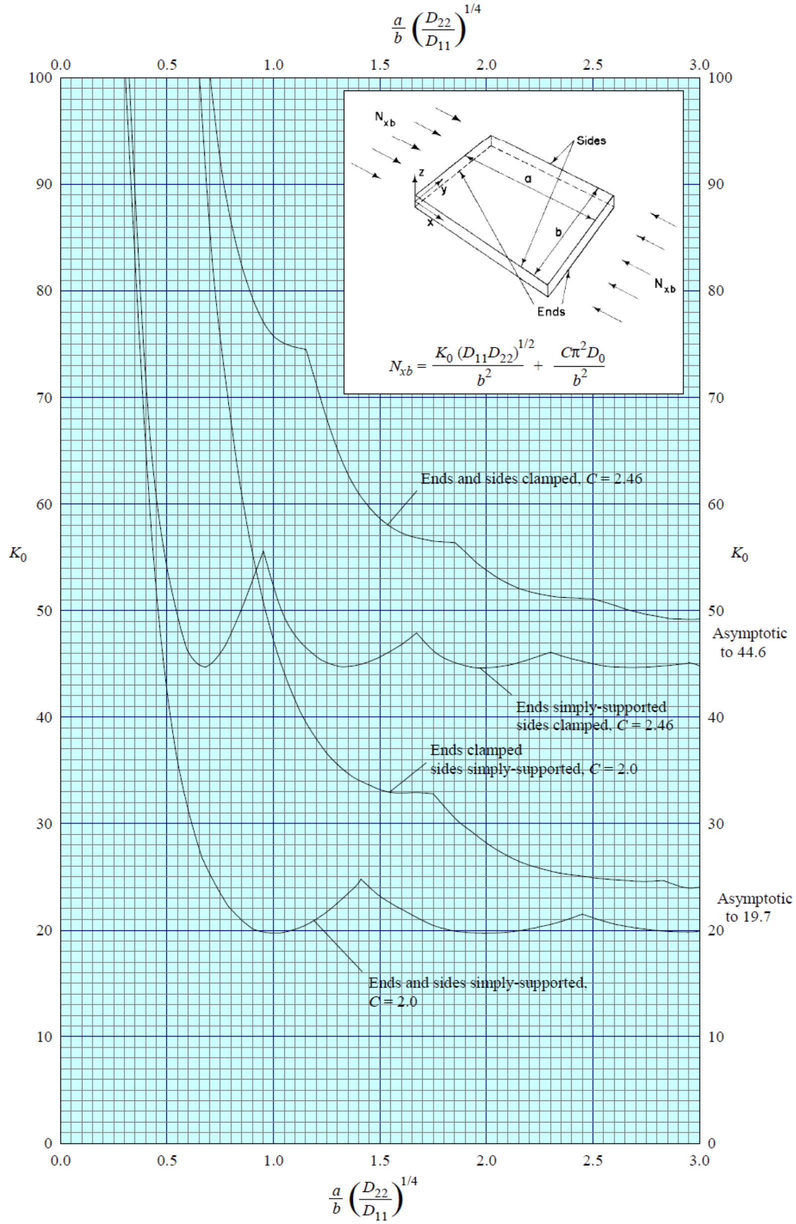


FIGURE 1 UNIAXIAL LOAD. COMBINATIONS OF CLAMPED AND SIMPLY-SUPPORTED EDGE CONDITIONS

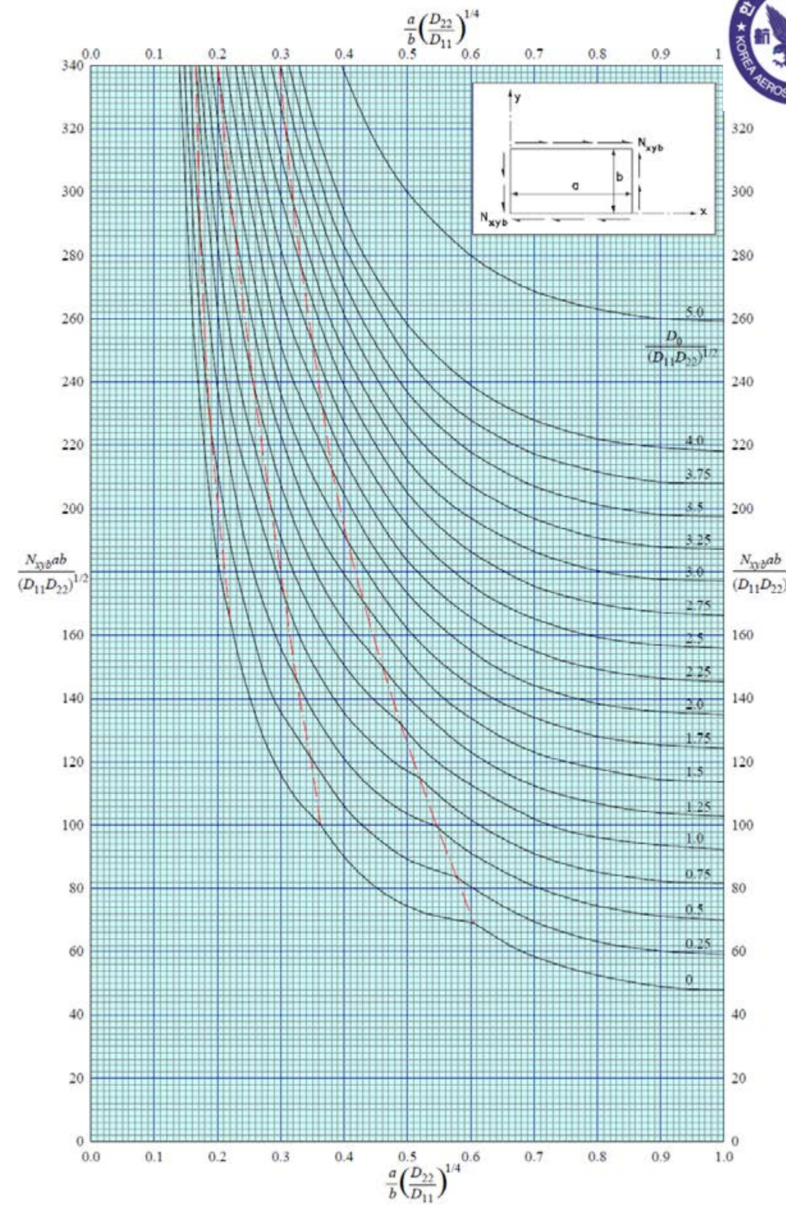
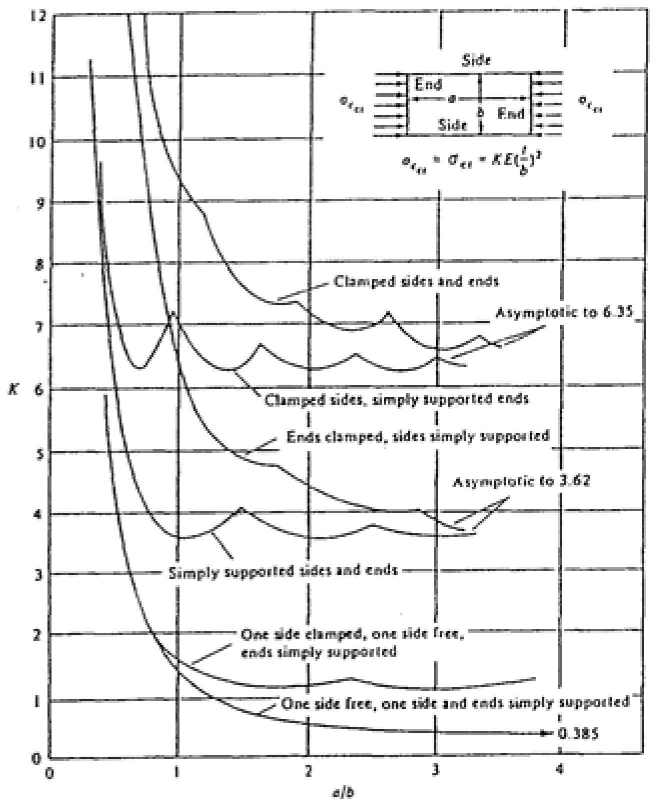
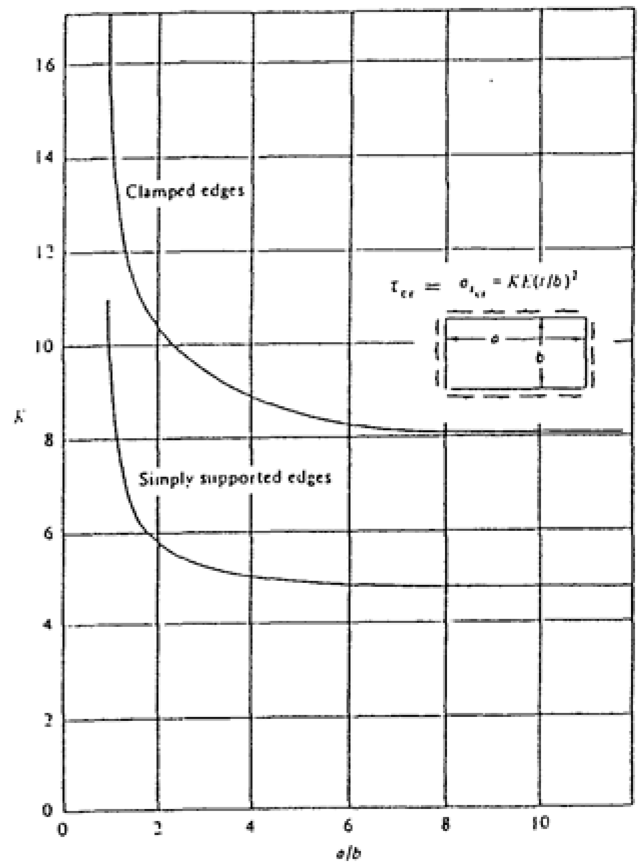


FIGURE 5 SHEAR, ALL EDGES SIMPLY-SUPPORTED



for ν = 0.3



for ν = 0.3