From The Hamiltonis principle

$$\int_{t_1}^{t_2} \delta(T - \Pi)\, dt = 0$$

$$\Rightarrow \int_{t_1}^{t_2} \delta\left[\int_{\Omega} \frac{m}{2}\left(\dot{u}_0^2 + \dot{v}_0^2 + \dot{w}^2\right) d\Omega - \int_{V} \frac{1}{2}\{\epsilon\}^T\{\sigma\}\, dV + \int_{\Omega} \{u_0\ v_0\ w\} \begin{Bmatrix} q_x \\ q_y \\ q_z \end{Bmatrix} d\Omega\right] dt$$

$$\Rightarrow \int_{\Omega}\int_{t_1}^{t_2} m\left(\dot{u}_0 \delta\dot{u}_0 + \dot{v}_0 \delta\dot{v}_0 + \dot{w}\delta\dot{w}\right) dt\, d\Omega -$$

$$\int_{t}^{t_2} \left\{\int_{V} \delta\{\epsilon\}^T\{\sigma\}\, dV - \int_{\Omega} \delta\{u_0\ v_0\ w\} \begin{Bmatrix} q_x \\ q_y \\ q_z \end{Bmatrix} d\Omega\right\} dt = 0$$

$$\Rightarrow \underbrace{\int_{\Omega} m\left(\dot{u}_0 \delta u_0 + \dot{v}_0 \delta v_0 + \dot{w}\delta w\right)\Big|_{t_1}^{t_2} d\Omega}_{\mathrm{I}} - \int_{\Omega}\int_{t_1}^{t_2} m\left(\ddot{u}_0 \delta u_0 + \ddot{v}_0 \delta v_0 + \ddot{w}\delta w\right) dt\, d\Omega$$

$$\underbrace{- \int_{t_1}^{t_2} \left\{\int_{V} \delta\{\epsilon\}^T\{\sigma\}\, dV - \int_{\Omega} \delta\{u_0\ v_0\ w\} \begin{Bmatrix} q_x \\ q_y \\ q_z \end{Bmatrix} d\Omega\right\} dt}_{\mathrm{II}} = 0$$

Term I is not an integral over time

Term II is an itegral over time (t_1 to t_2)

Their sum can always be zero for any t_1 and t_2 only

when I and II are individually zero.

Hence,

$$\int_{t_1}^{t_2}\left[\int_{\Omega} m(\ddot{u}_0\,\delta u_0+\ddot{v}_0\,\delta v_0+\ddot{w}\,\delta w)\,d\Omega+\int_{\Omega}\delta\{\epsilon\}^T\{\sigma\}\,d\Omega - \int_{\Omega}\delta\{u_0\ v_0\ w\}\begin{Bmatrix}q_x\\ q_y\\ q_z\end{Bmatrix}d\Omega\right]dt=0$$

As the time integral is zero for any t_1 and t_2, the integrand must be zero

$$\Rightarrow \int_{\Omega} m(\ddot{u}_0\,\delta u_0+\ddot{v}_0\,\delta v_0+\ddot{w}\,\delta w)\,d\Omega+\int_{V}\delta\{\epsilon\}^T\{\sigma\}\,d\Omega - \int_{\Omega}\delta\{u_0\ v_0\ w\}\begin{Bmatrix}q_x\\ q_y\\ q_z\end{Bmatrix}d\Omega=0$$

$$\Rightarrow \int_{\Omega} m\,\delta\{u_0\ v_0\}^T\begin{Bmatrix}\ddot{u}_0\\ \ddot{v}_0\end{Bmatrix}d\Omega+\int_{\Omega} m\,\delta w\,\ddot{w}\,d\Omega$$

$$\cancel{+\int_{\Omega}\delta\{\epsilon_0\}^T} + \int_{\Omega}\sum_{k=1}^{N_L}\int_{z_{k-1}}^{z_k}\Big(\delta\{\epsilon_0\}^T[Q]_k\,\delta\{\epsilon_0\}+z\,\delta\{\epsilon_0\}^T[Q]_k\,\delta\{k\}$$

$$+z\,\delta\{k\}[Q]_k\{\epsilon_0\}+z^2\delta\{k\}[Q]_k\{k\}\Big)\,dz\,d\Omega$$

$$-\int_{\Omega}\delta\{u_0\ v_0\}\begin{Bmatrix}q_x\\ q_y\end{Bmatrix}d\Omega-\int_{\Omega}\delta w\,q_z\,d\Omega=0$$

$$\left[\ \{\epsilon_0\}=\begin{Bmatrix}\epsilon_{x0}\\ \epsilon_{y0}\\ \epsilon_{s0}\end{Bmatrix}\qquad k=\begin{Bmatrix}k_x\\ k_y\\ k_s\end{Bmatrix}\ \right]$$

$$\Rightarrow \int_{\Omega} m\,\delta\{u_0\ v_0\}^T \begin{Bmatrix} \ddot{u}_0 \\ \ddot{v}_0 \end{Bmatrix} d\Omega + \int_{\Omega} m\,\delta w\,\ddot{w}\,d\Omega + \int_{\Omega} \Big(\delta\{\epsilon_0\}^T [A] \{\epsilon_0\}$$

$$+ \delta\{\epsilon_0\}[B]\delta\{k\} + \delta\{k\}[B]\{\epsilon_0\} + \delta\{k\}[D]\{k\} \Big) d\Omega$$

$$- \int_{\Omega} \delta\{u_0\ v_0\} \begin{Bmatrix} q_x \\ q_y \end{Bmatrix} d\Omega - \int_{\Omega} \delta w\, q_z\, d\Omega = 0 \quad \rightarrow (*)$$

As per the approximations for the displacement components

$$\cancel{\begin{Bmatrix} u_0 \\ v_0 \end{Bmatrix} = \begin{bmatrix} \phi_{u1} \cdots \phi_{up}\ \phi_{v1} \cdots \phi_{vp} \end{bmatrix}}$$

$$\begin{Bmatrix} u_0 \\ v_0 \end{Bmatrix} = \begin{bmatrix} \phi_{u1} & \cdots & \phi_{up} & 0 & \cdots & 0 \\ 0 & \cdots & 0 & \phi_{v1} & \cdots & \phi_{vp} \end{bmatrix} \begin{Bmatrix} b_{u1} \\ \vdots \\ b_{up} \\ b_{v1} \\ \vdots \\ b_{vp} \end{Bmatrix}$$

$$= [\phi_1]_{2\times 2p} \{b_1\}_{2p\times 1}$$

$$w = \begin{bmatrix} \phi_{w1} & \cdots & \phi_{wp} \end{bmatrix} \begin{Bmatrix} b_{w1} \\ \vdots \\ b_{wp} \end{Bmatrix} = [\phi_2]\{b_2\}$$

$$\{\epsilon_0\} = \begin{bmatrix} \phi_{u1,x} & \cdots & \phi_{up,x} & 0 & \cdots & 0 \\ 0 & \cdots & 0 & \phi_{v1,y} & \cdots & \phi_{vp,y} \\ \phi_{u1,y} & \cdots & \phi_{up,y} & \phi_{v1,x} & \cdots & \phi_{vp,x} \end{bmatrix} \begin{Bmatrix} b_{u1} \\ \vdots \\ b_{up} \\ b_{v1} \\ \vdots \\ b_{vp} \end{Bmatrix}$$

$$= [G_1]\{b_1\}$$

$$\{k\} = \begin{bmatrix} -\phi_{w1,xx} & \cdots & -\phi_{wp,xx} \\ -\phi_{w1,yy} & \cdots & -\phi_{wp,yy} \\ -2\phi_{w1,xy} & \cdots & -2\phi_{wp,xy} \end{bmatrix} \begin{Bmatrix} b_{w1} \\ \vdots \\ b_{wp} \end{Bmatrix}$$

$$= [G_2]\{b_2\}$$

Eq. (α) can be written as:

$$\int_\Omega m\,\delta\{b_1\}^T[\phi_1]^T[\phi_1]\delta\{\ddot{b}_1\}\,d\Omega + \int_\Omega m\,\delta\{b_2\}^T[\phi_2]^T[\phi_2]\delta\{\ddot{b}_2\}\,d\Omega$$

$$+ \int_\Omega \delta\{b_1\}^T[G_1]^T[A][G_1]\delta\{b_1\}\,d\Omega + \int_\Omega \delta\{b_1\}^T[G_1]^T[B][G_2]\delta\{b_2\}\,d\Omega$$

$$+ \int_\Omega \delta\{b_2\}^T[G_2]^T[B][G_1]\delta\{b_1\}\,d\Omega + \int_\Omega \delta\{b_2\}^T[G_2]^T[D][G_2]\delta\{b_2\}\,d\Omega$$

$$- \int_\Omega \delta\{b_1\}^T[\phi_1]^T\begin{Bmatrix} q_x \\ q_y \end{Bmatrix}d\Omega - \int_\Omega \delta\{b_2\}^T[\phi_2]^T\{q_z\}\,d\Omega = 0$$

$\cancel{\delta\{b_1\}^T}$

$\cancel{\int_\Omega m[\phi_1]^T[\phi_1]\delta\{\ddot{b}_1\}d\Omega}$

$\delta\{b_1\}$ and $\delta\{b_2\}$ are arbitrary. Their coefficients should be zero individually.

Hence, $[M_1]\{\ddot{b}_1\} + [K_1]\{b_1\} = \{F_1\}$

$[M_2]\{\ddot{b}_2\} + [K_2]\{b_2\} = \{F_2\}$

$\cancel{M_1}$ $[M_1] = \int_\Omega m[\phi_1]^T[\phi_1]\,d\Omega$

$[K_1] = \int_\Omega [G_1][A][G_1]\,d\Omega + \int_\Omega [G_1][B][G_2]\,d\Omega$