

(8)

$$\Rightarrow \int_{\Omega} m \delta \{u_0 \ v_0\}^T \begin{Bmatrix} \ddot{u}_0 \\ \ddot{v}_0 \end{Bmatrix} d\Omega + \int_{\Omega} m \delta w \ddot{w} d\Omega + \left( \delta \{t_0\}^T [A] \{t_0\} + \delta \{t_0\} [B] \delta \{k\} + \delta \{k\} [B] \{t_0\} + \delta \{k\} [D] \{k\} \right) d\Omega - \int_{\Omega} \delta \{u_0 \ v_0\} \begin{Bmatrix} q_x \\ q_y \end{Bmatrix} d\Omega - \int_{\Omega} \delta w q_z d\Omega = 0$$

As per the approximations for the displacement components

~~$$\begin{Bmatrix} u_0 \\ v_0 \end{Bmatrix} = \begin{bmatrix} \phi_{u1} & \dots & \phi_{up} & \phi_{ur} & \dots & \phi_{vp} \end{bmatrix}$$~~

$$\begin{Bmatrix} u_0 \\ v_0 \end{Bmatrix} = \begin{bmatrix} \phi_{u1} & \dots & \phi_{up} & 0 & \dots & 0 \\ 0 & \dots & 0 & \phi_{v1} & \dots & \phi_{vp} \end{bmatrix} \begin{Bmatrix} b_{u1} \\ \vdots \\ b_{up} \\ b_{v1} \\ \vdots \\ b_{vp} \end{Bmatrix}$$

$$= [\Phi_1]_{2 \times 2p} \begin{Bmatrix} b_1 \end{Bmatrix}_{2p \times 1}$$

$$w = [\phi_{w1} \dots \phi_{wp}] \begin{Bmatrix} b_{w1} \\ \vdots \\ b_{wp} \end{Bmatrix} = [\Phi_2] \{b_2\}$$

$$\{t_0\} = \begin{bmatrix} \phi_{u1,x} & \dots & \phi_{up,x} & 0 & \dots & 0 \\ 0 & \dots & 0 & \phi_{v1,y} & \dots & \phi_{vp,y} \\ \phi_{u1,y} & \dots & \phi_{up,y} & \phi_{v1,x} & \dots & \phi_{vp,x} \end{bmatrix} \begin{Bmatrix} b_{u1} \\ \vdots \\ b_{up} \\ b_{v1} \\ \vdots \\ b_{vp} \end{Bmatrix}$$

$$= [G_1] \{b_1\}$$

$$\{k\} = \begin{bmatrix} -\phi_{w1,xx} & \dots & -\phi_{wp,xx} \\ -\phi_{w1,yy} & \dots & -\phi_{wp,yy} \\ -2\phi_{w1,xy} & \dots & -2\phi_{wp,xy} \end{bmatrix} \begin{Bmatrix} b_{w1} \\ \vdots \\ b_{wp} \end{Bmatrix}$$

$$= [G_2] \{b_2\}$$

Eq. (a) can be written as

$$\int_{\Omega} m \delta\{b_1\}^T [\phi_1]^T [\phi_1] \delta\{b_1\} d\Omega + \int_{\Omega} m \delta\{b_2\}^T [\phi_2]^T [\phi_2] \delta\{b_2\} d\Omega$$

$$+ \int_{\Omega} \delta\{b_1\}^T [G_1]^T [A] [G_1] \delta\{b_1\} d\Omega + \int_{\Omega} \delta\{b_1\}^T [G_1]^T [B] [G_2] \delta\{b_2\} d\Omega$$

$$+ \int_{\Omega} \delta\{b_2\}^T [G_2]^T [B] [G_1] \delta\{b_1\} d\Omega + \int_{\Omega} \delta\{b_2\}^T [G_2]^T [D] [G_2] \delta\{b_2\} d\Omega$$

$$- \int_{\Omega} \delta\{b_1\}^T [\phi_1]^T \begin{Bmatrix} q_x \\ q_y \end{Bmatrix} d\Omega - \int_{\Omega} \delta\{b_2\}^T [\phi_2]^T \begin{Bmatrix} q_x \\ q_y \end{Bmatrix} d\Omega = 0$$

~~$\delta\{b_1\}$~~

$\delta\{b_1\}$  and  $\delta\{b_2\}$  are arbitrary

~~$\int_{\Omega} m [\phi_1]^T [\phi_1] \delta\{b_1\} d\Omega$~~

Their coefficients should be zero individually

Hence,  $[M_1] \{b_1\} + [K_1] \{b_1\} = \{F_1\}$

$$[M_2] \{b_2\} + [K_2] \{b_2\} = \{F_2\}$$

$$\underline{M_{1,1}} = [M_1] = \int_{\Omega} m [\phi_1]^T [\phi_1] d\Omega$$

$$[K_1] = \int_{\Omega} [G_1] [A] [G_1] d\Omega + \int_{\Omega} [G_1] [B] [G_2] d\Omega$$

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$$\{F_1\} = \int_{\Omega} [\phi_1]^T \begin{Bmatrix} q_x \\ q_y \end{Bmatrix} d\Omega$$

$$[M_2] = \int_{\Omega} m [\phi_2]^T [\phi_2] d\Omega$$

$$[K_2] = \int_{\Omega} [G_2]^T [B] [G_1] d\Omega + \int_{\Omega} [G_2]^T [D] [G_2] d\Omega$$

$$\{F_2\} = \int_{\Omega} [\phi_2]^T \begin{Bmatrix} q_x \\ q_y \end{Bmatrix} d\Omega$$