Dynamics Summary

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AE1130-II Hibbeler 14th Edition

Contents

Chapter 12 Kinematics of a particle

12.2 Rectilinear kinematics

$$v = \frac{ds}{dt} \tag{1}$$

$$a = \frac{dv}{dt} \tag{2}$$

Velocity as function of time:
$a_c : constant \quad acceleration$

$$\int dv = \int a_c dt \tag{3}$$

Position as function of time:

$$\int ds = \int (v_0 + a_c t) dt \tag{4}$$

Velocity as a function of position:

$$\int v dv = \int a_c ds \tag{5}$$

12–18.

The acceleration of a rocket traveling upward is given by $a = (6 + 0.02s)\ \mathrm{m/s^2}$, where s is in meters. Determine the time needed for the rocket to reach an altitude of $s = 100$ m. Initially, $v = 0$ and $s = 0$ when $t = 0$.

SOLUTION

$a\,ds = v\,dv$

$\int_0^s (6 + 0.02\,s)\,ds = \int_0^v v\,dv$

$6\,s + 0.01\,s^2 = \frac{1}{2}v^2$

$v = \sqrt{12\,s + 0.02\,s^2}$

$ds = v\,dt$

$\int_0^{100} \frac{ds}{\sqrt{12\,s + 0.02\,s^2}} = \int_0^t dt$

$\frac{1}{\sqrt{0.02}} \ln \left[\sqrt{12s + 0.02s^2} + s\sqrt{0.02} + \frac{12}{2\sqrt{0.02}} \right]_0^{100} = t$

$t = 5.62$ s **Ans.**

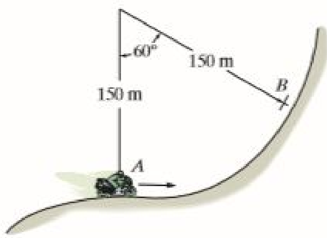
Figure 1: Exercise of section 12.2

12.7 Curvilinear motion: Normal an Tangential components

$$a_n = \frac{v^2}{\rho} \tag{6}$$

12–138.

The motorcycle is traveling at 40 m/s when it is at A. If the speed is then decreased at $\dot{v} = -(0.05\,s)$ m/s^2, where s is in meters measured from A, determine its speed and acceleration when it reaches B.



SOLUTION

Velocity. The velocity of the motorcycle along the circular track can be determined by integrating $v\,dv = a_t ds$ with the initial condition $v = 40$ m/s at $s = 0$. Here, $a_t = -0.05s$.

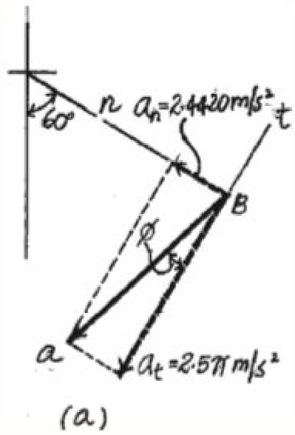
$$\int_{40\text{ m/s}}^{v} v\,dv = \int_0^s -0.05\,s\,ds$$

$$\left.\frac{v^2}{2}\right|_{40\text{ m/s}}^{v} = -0.025\,s^2\Big|_0^s$$

$$v = \left\{\sqrt{1600 - 0.05\,s^2}\right\}\text{ m/s}$$

At B, $s = r\theta = 150\left(\frac{\pi}{3}\right) = 50\pi$ m. Thus

$$v_B = v|_{s=50\pi\text{m}} = \sqrt{1600 - 0.05(50\pi)^2} = 19.14\text{ m/s} = 19.1\text{ m/s} \qquad \textbf{Ans.}$$



Acceleration. At B, the tangential and normal components are

$$a_t = 0.05(50\pi) = 2.5\pi\text{ m/s}^2$$

$$a_n = \frac{v_B^2}{\rho} = \frac{19.14^2}{150} = 2.4420\text{ m/s}^2$$

Thus, the magnitude of the acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(2.5\pi)^2 + 2.4420^2} = 8.2249\text{ m/s}^2 = 8.22\text{ m/s} \qquad \textbf{Ans.}$$

And its direction is defined by angle ϕ measured from the negative t-axis, Fig. a.

$$\phi = \tan^{-1}\left(\frac{a_n}{a_t}\right) = \tan^{-1}\left(\frac{2.4420}{2.5\pi}\right)$$

$$= 17.27° = 17.3° \qquad \textbf{Ans.}$$

Figure 2: Exercise of section 12.7

12.8 Curvilinear motion: Cylindrical components

$$v = v_r u_r + v_\theta u_\theta \tag{7}$$

$$v_r = \dot{r} \tag{8}$$

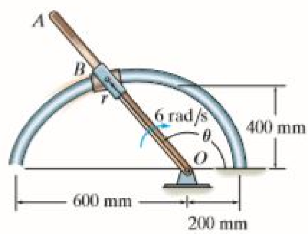
$$v_\theta = r\dot{\theta} \tag{9}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 \tag{10}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} \tag{11}$$

12–173.

Determine the magnitude of the acceleration of the slider blocks in Prob. 12–172 when $\theta = 150°$.



SOLUTION

Acceleration. Using the chain rule, the first and second time derivatives of r can be determined

$$r = 200(2 - \cos\theta)$$

$$\dot{r} = 200(\sin\theta)\dot{\theta} = \{200(\sin\theta)\dot{\theta}\}\ \text{mm/s}$$

$$\ddot{r} = \{200[(\cos\theta)\dot{\theta}^2 + (\sin\theta)\ddot{\theta}]\}\ \text{mm/s}^2$$

Here, since $\dot{\theta}$ is constant, $\ddot{\theta} = 0$. Since $\dot{\theta}$ is in the opposite sense to that of positive θ, $\dot{\theta} = -6$ rad/s. Thus, at $\theta = 150°$

$$r = 200(2 - \cos 150°) = 573.21\ \text{mm}$$

$$\dot{r} = 200(\sin 150°)(-6) = -600\ \text{mm/s}$$

$$\ddot{r} = 200\left[(\cos 150°)(-6)^2 + \sin 150°(0)\right] = -6235.38\ \text{mm/s}^2$$

The radial and transverse components of the acceleration are

$$a_r = \ddot{r} - r\dot{\theta}^2 = -6235.38 - 573.21\,(-6)^2 = -26870.77\ \text{mm/s}^2 = -26.87\ \text{m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 573.21(0) + 2(-600)(-6) = 7200\ \text{mm/s}^2 = 7.20\ \text{m/s}^2$$

Thus, the magnitude of the acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-26.87)^2 + 7.20^2} = 27.82\ \text{m/s}^2 = 27.8\ \text{m/s}^2 \qquad \textbf{Ans.}$$

These components are shown in Fig. *a*.

Figure 3: Exercise of section 12.8

12.10 Relative motion of two particles using translating axes

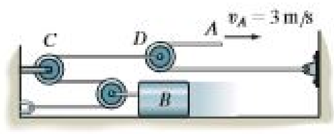
$$r_B = r_A + r_{B/A} \tag{12}$$

$$v_B = v_A + v_{B/A} \tag{13}$$

$$a_B = a_A + a_{B/A} \tag{14}$$

12–202.

If the end A of the cable is moving at $v_A = 3\ \text{m/s}$, determine the speed of block B.



SOLUTION

Position Coordinates. The positions of pulley B, D and point A are specified by position coordinates s_B, s_D and s_A respectively as shown in Fig. *a*. The pulley system consists of two cords which give

$$2\,s_B + s_D = l_1 \qquad \textbf{(1)}$$

and

$$(s_A - s_D) + (b - s_D) = l_2$$

$$s_A - 2\,s_D = l_2 - b \qquad \textbf{(2)}$$

Time Derivative. Taking the time derivatives of Eqs. (1) and (2), we get

$$2v_B + v_D = 0 \qquad \textbf{(3)}$$

$$v_A - 2v_D = 0 \qquad \textbf{(4)}$$

Eliminate v_0 from Eqs. (3) and (4),

$$v_A + 4v_B = 0 \qquad \textbf{(5)}$$

Here $v_A = +3\ \text{m/s}$ since it is directed toward the positive sense of s_A.

Thus

$$3 + 4v_B = 0$$

$$v_B = -0.75\ \text{m/s} = 0.75\ \text{m/s} \leftarrow \qquad \textbf{Ans.}$$

The negative sign indicates that $\mathbf{v}_D$ is directed toward the negative sense of $\mathbf{s}_B$.

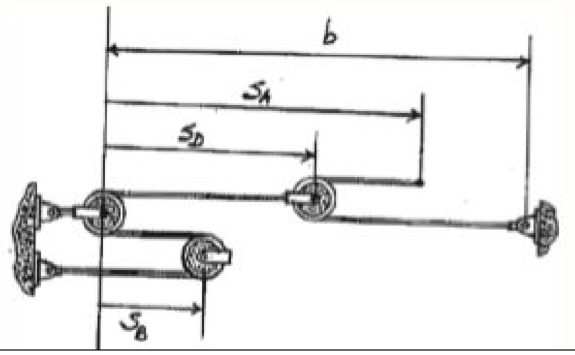


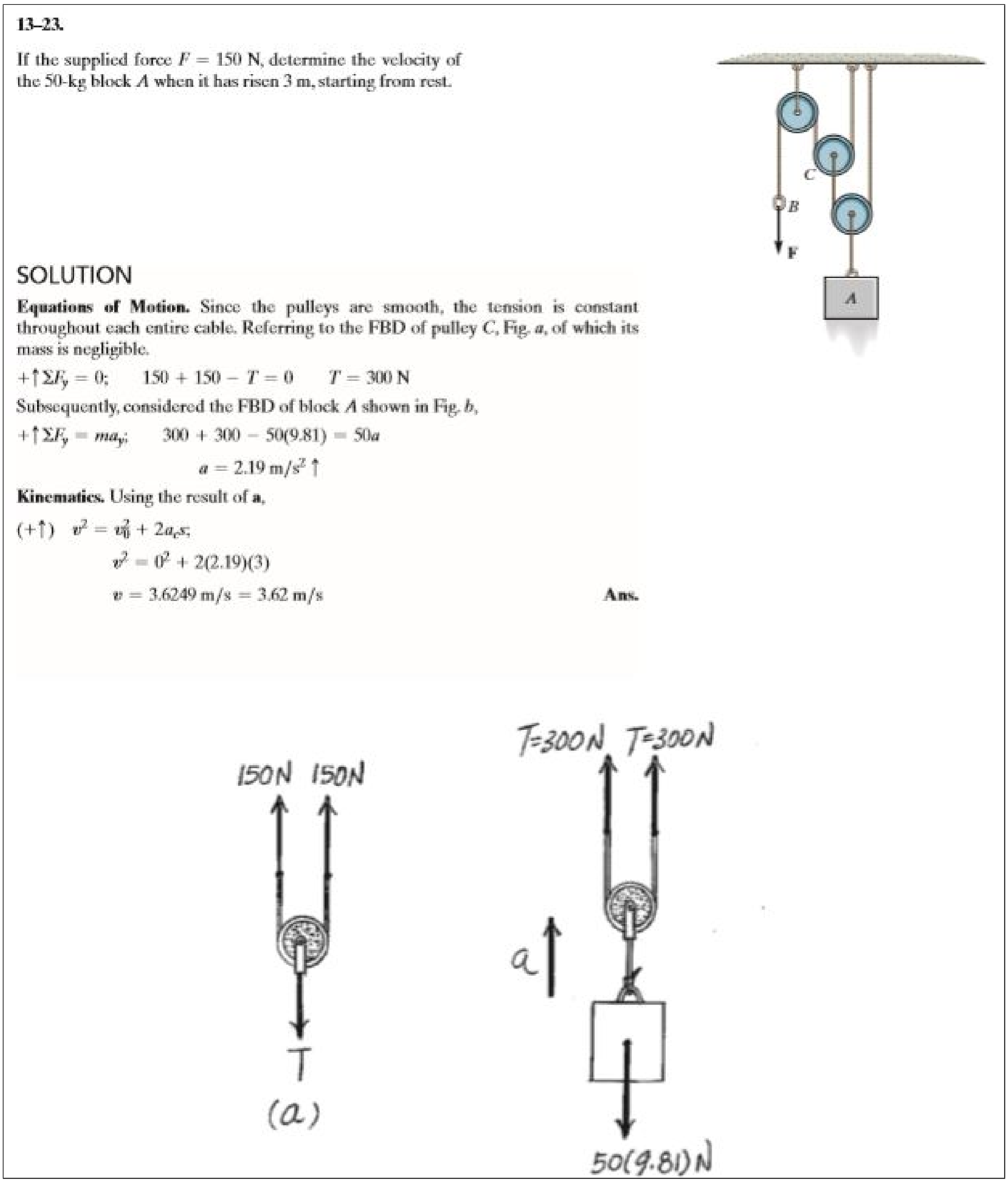
Figure 4: Exercise of section 12.10

Chapter 13 Kinematics of a particle: Force and Acceleration

13.4 Equations of Motion: Rectangular Coordinates

$$\sum F_x = ma_x \tag{15}$$

$$\sum F_y = ma_y \tag{16}$$



13–23.

If the supplied force $F = 150$ N, determine the velocity of the 50-kg block A when it has risen 3 m, starting from rest.

SOLUTION

Equations of Motion. Since the pulleys are smooth, the tension is constant throughout each entire cable. Referring to the FBD of pulley C, Fig. *a*, of which its mass is negligible.

$+\uparrow \Sigma F_y = 0;\qquad 150 + 150 - T = 0 \qquad T = 300\text{ N}$

Subsequently, considered the FBD of block A shown in Fig. *b*,

$+\uparrow \Sigma F_y = ma_y;\qquad 300 + 300 - 50(9.81) = 50a$

$a = 2.19\text{ m/s}^2 \uparrow$

Kinematics. Using the result of **a**,

$(+\uparrow)\quad v^2 = v_0^2 + 2a_c s;$

$v^2 = 0^2 + 2(2.19)(3)$

$v = 3.6249\text{ m/s} = 3.62\text{ m/s}$ **Ans.**

Figure 5: Exercise of section 13.4

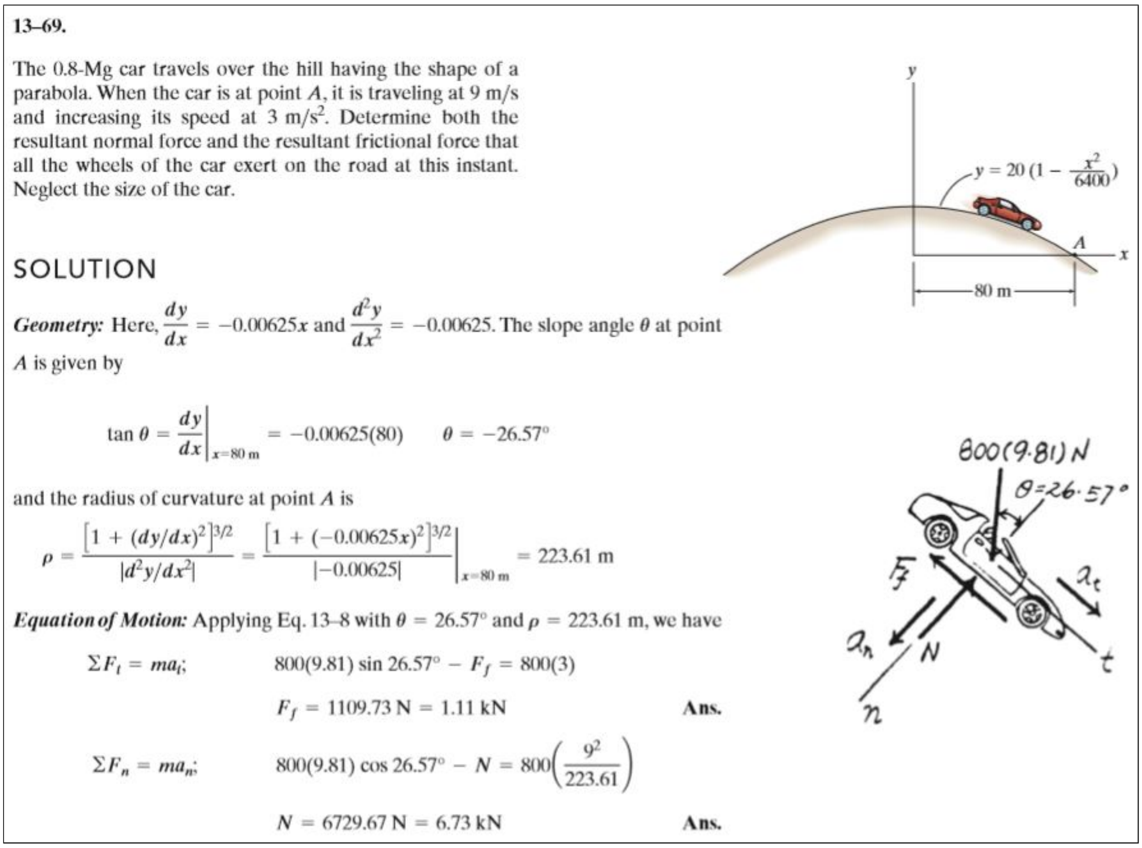
13.5 Equations of Motion: Normal and Tangential axes

$$\sum F_n = ma_n \tag{17}$$

$$\sum F_t = ma_t \tag{18}$$

$$\sum F_r = m(\ddot{r} - r\dot{\theta}^2) \tag{19}$$

$$\sum F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta}) \tag{20}$$



13–69.

The 0.8-Mg car travels over the hill having the shape of a parabola. When the car is at point A, it is traveling at 9 m/s and increasing its speed at 3 m/s^2. Determine both the resultant normal force and the resultant frictional force that all the wheels of the car exert on the road at this instant. Neglect the size of the car.

SOLUTION

Geometry: Here, $\dfrac{dy}{dx} = -0.00625x$ and $\dfrac{d^2y}{dx^2} = -0.00625$. The slope angle θ at point A is given by

$$\tan\theta = \left.\frac{dy}{dx}\right|_{x=80\text{ m}} = -0.00625(80) \qquad \theta = -26.57^\circ$$

and the radius of curvature at point A is

$$\rho = \frac{\left[1 + (dy/dx)^2\right]^{3/2}}{|d^2y/dx^2|} = \left.\frac{\left[1 + (-0.00625x)^2\right]^{3/2}}{|-0.00625|}\right|_{x=80\text{ m}} = 223.61\text{ m}$$

Equation of Motion: Applying Eq. 13–8 with $\theta = 26.57^\circ$ and $\rho = 223.61$ m, we have

$\Sigma F_t = ma_t;$ $\quad 800(9.81)\sin 26.57^\circ - F_f = 800(3)$

$F_f = 1109.73\text{ N} = 1.11\text{ kN}$ **Ans.**

$\Sigma F_n = ma_n;$ $\quad 800(9.81)\cos 26.57^\circ - N = 800\left(\dfrac{9^2}{223.61}\right)$

$N = 6729.67\text{ N} = 6.73\text{ kN}$ **Ans.**

Figure 6: Exercise of section 13.5

Chapter 14 Kinematics of a particle: Work and Energy

14.1 The work of a Force

$$W = \int F dr = \int F\cos\theta ds \tag{21}$$

Work of a spring:

$$W_s = \int F_s ds = \int -ksds = -\frac{1}{2}ks^2 + C \tag{22}$$

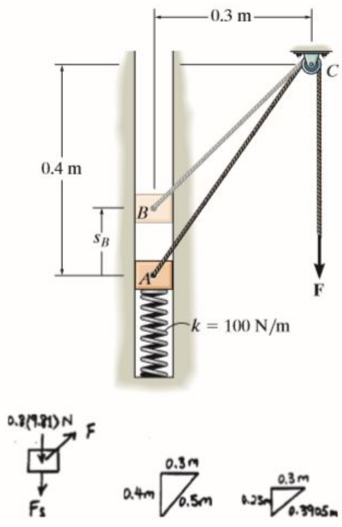
14.2 Principle of Work and Energy

$$\sum \int F_t ds = \int mvdv \tag{23}$$

$$T_1 + \sum U_{1-2} = T_2 \tag{24}$$

***14–32.**

The block has a mass of 0.8 kg and moves within the smooth vertical slot. If it starts from rest when the *attached* spring is in the unstretched position at A, determine the *constant* vertical force F which must be applied to the cord so that the block attains a speed $v_B = 2.5$ m/s when it reaches B; $s_B = 0.15$ m. Neglect the size and mass of the pulley. *Hint:* The work of **F** can be determined by finding the difference Δl in cord lengths AC and BC and using $U_F = F\,\Delta l$.



SOLUTION

$$l_{AC} = \sqrt{(0.3)^2 + (0.4)^2} = 0.5 \text{ m}$$

$$l_{BC} = \sqrt{(0.4 - 0.15)^2 + (0.3)^2} = 0.3905 \text{ m}$$

$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + F(0.5 - 0.3905) - \frac{1}{2}(100)(0.15)^2 - (0.8)(9.81)(0.15) = \frac{1}{2}(0.8)(2.5)^2$$

$$F = 43.9 \text{ N} \qquad \textbf{Ans.}$$

Figure 7: Exercise of section 14.3

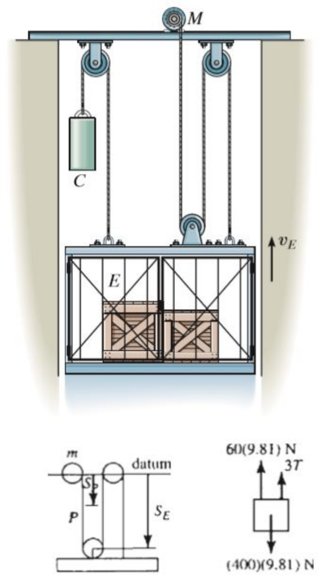
14.4 Power and Efficiency

$$P = \frac{dW}{dt} \tag{25}$$

$$\varepsilon = \frac{power \quad output}{power \quad input} \tag{26}$$

14–55.

The elevator E and its freight have a total mass of 400 kg. Hoisting is provided by the motor M and the 60-kg block C. If the motor has an efficiency of $\epsilon = 0.6$, determine the power that must be supplied to the motor when the elevator is hoisted upward at a constant speed of $v_E = 4$ m/s.



SOLUTION

Elevator:

Since $a = 0$,

$$+\uparrow \Sigma F_y = 0; \qquad 60(9.81) + 3T - 400(9.81) = 0$$

$$T = 1111.8 \text{ N}$$

$$2s_E + (s_E - s_P) = l$$

$$3v_E = v_P$$

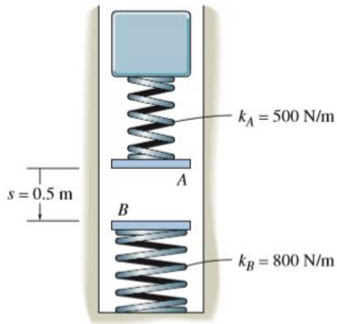
Since $v_E = -4$ m/s, $\qquad v_P = -12$ m/s

$$P_i = \frac{\mathbf{F}\cdot\mathbf{v}_P}{e} = \frac{(1111.8)(12)}{0.6} = 22.2 \text{ kW} \qquad \textbf{Ans.}$$

Figure 8: Exercise of section 14.4

14–87.

The block has a mass of 20 kg and is released from rest when $s = 0.5$ m. If the mass of the bumpers A and B can be neglected, determine the maximum deformation of each spring due to the collision.



SOLUTION

Datum at initial position:

$T_1 + V_1 = T_2 + V_2$

$$0 + 0 = 0 + \frac{1}{2}(500)s_A^2 + \frac{1}{2}(800)s_B^2 + 20(9.81)\left[-(s_A + s_B) - 0.5\right] \qquad \textbf{(1)}$$

Also, $F_s = 500s_A = 800s_B \qquad s_A = 1.6s_B$ **(2)**

Solving Eqs. (1) and (2) yields:

$s_B = 0.638$ m **Ans.**

$s_A = 1.02$ m **Ans.**

Figure 9: Exercise of section 14.6

Chapter 15 Kinematics of a particle: Impulse and Momentum

15.1 Principle of linear Impulse and Momentum

$$\sum \int F dt = m \int dv \tag{27}$$

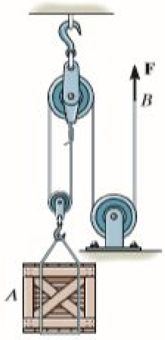
$$m\vec{v_1} + \sum \int \vec{F} dt = m\vec{v_2} \tag{28}$$

15.2 Principle of linear Impulse and Momentum for a system of particles

$$\sum m(\vec{v_i})_1 + \sum \int \vec{F_i} dt = \sum m(\vec{v_i})_2 \tag{29}$$

15–27.

The 20-kg crate is lifted by a force of $F = (100 + 5t^2)$ N, where t is in seconds. Determine the speed of the crate when $t = 3$ s, starting from rest.



SOLUTION

Principle of Impulse and Momentum. At $t = 0$, $F = 100$ N. Since at this instant, $2F = 200\text{ N} > W = 20(9.81) = 196.2$ N, the crate will move the instant force **F** is applied. Referring to the FBD of the crate, Fig. *a*,

$$(+\uparrow) \quad m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y dt = m(v_y)_2$$

$$0 + 2\int_0^{3\text{ s}} (100 + 5t^2)dt - 20(9.81)(3) = 20v$$

$$2\left(100t + \frac{5}{3}t^3\right)\Big|_0^{3\text{ s}} - 588.6 = 20v$$

$$v = 5.07 \text{ m/s}$$ **Ans.**

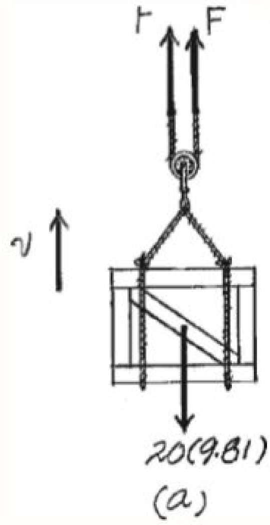
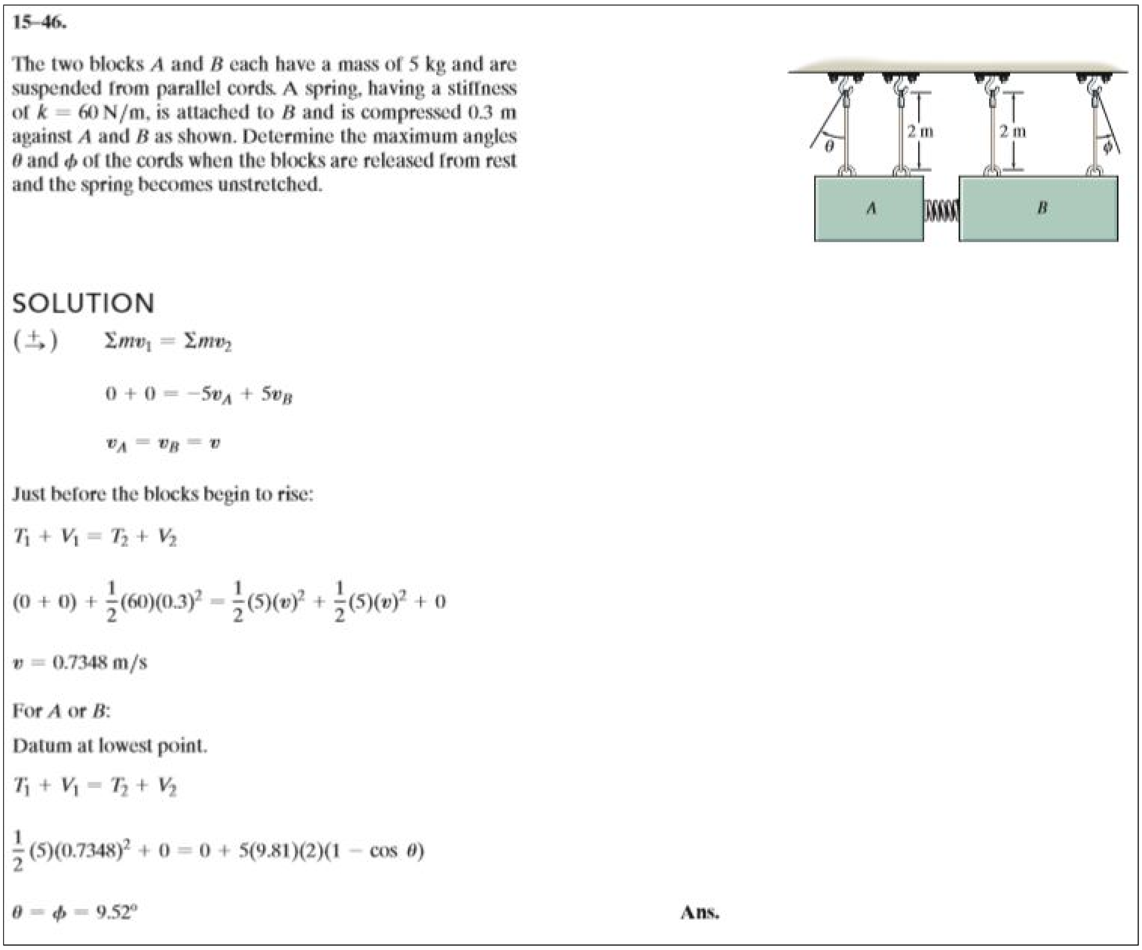


Figure 10: Exercise of section 15.2



15–46.

The two blocks A and B each have a mass of 5 kg and are suspended from parallel cords. A spring, having a stiffness of $k = 60$ N/m, is attached to B and is compressed 0.3 m against A and B as shown. Determine the maximum angles θ and ϕ of the cords when the blocks are released from rest and the spring becomes unstretched.

SOLUTION

$(\xrightarrow{+})\qquad \Sigma m v_1 = \Sigma m v_2$

$$0 + 0 = -5v_A + 5v_B$$

$$v_A = v_B = v$$

Just before the blocks begin to rise:

$$T_1 + V_1 = T_2 + V_2$$

$$(0 + 0) + \frac{1}{2}(60)(0.3)^2 = \frac{1}{2}(5)(v)^2 + \frac{1}{2}(5)(v)^2 + 0$$

$$v = 0.7348 \text{ m/s}$$

For A or B:

Datum at lowest point.

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(5)(0.7348)^2 + 0 = 0 + 5(9.81)(2)(1 - \cos\theta)$$

$\theta = \phi = 9.52^\circ$ **Ans.**

Figure 11: Exercise of section 15.3

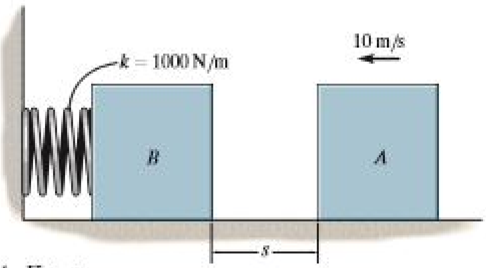
15.4 Impact

$$m_a(v_a)_1 + m_b(v_b)_1 = m_a(v_a)_2 + m_b(v_b)_2 \tag{30}$$

$$e = \frac{(v_b)_2 - (v_a)_2}{(v_a)_1 - (v_b)_1} \tag{31}$$

15–61.

The 15-kg block A slides on the surface for which $\mu_k = 0.3$. The block has a velocity $v = 10\,\text{m/s}$ when it is $s = 4$ m from the 10-kg block B. If the unstretched spring has a stiffness $k = 1000$ N/m, determine the maximum compression of the spring due to the collision. Take $e = 0.6$.



SOLUTION

Principle of Work and Energy. Referring to the FBD of block A, Fig. a, motion along the y axis gives $N_A = 15(9.81) = 147.15$ N. Thus the friction is $F_f = \mu_k N_A = 0.3(147.15) = 44.145$ N.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(15)(10^2) + (-44.145)(4) = \frac{1}{2}(15)(v_A)_1^2$$

$$(v_A)_1 = 8.7439\,\text{m/s} \leftarrow$$

Conservation of Momentum.

$$(\overset{+}{\leftarrow}) \quad m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2$$

$$15(8.7439) + 0 = 15(v_A)_2 + 10(v_B)_2$$

$$3(v_A)_2 + 2(v_B)_2 = 26.2317 \qquad \textbf{(1)}$$

Coefficient of Restitution.

$$(\overset{+}{\leftarrow}) \quad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}; \qquad 0.6 = \frac{(v_B)_2 - (v_A)_2}{8.7439 - 0}$$

$$(v_B)_2 - (v_A)_2 = 5.2463 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2)

$$(v_B)_2 = 8.3942\,\text{m/s} \leftarrow \quad (v_A)_2 = 3.1478\,\text{m/s} \leftarrow$$

Conservation of Energy. When block B stops momentarily, the compression of the spring is maximum. Thus, $T_2 = 0$.

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(10)(8.3942^2) + 0 = 0 + \frac{1}{2}(1000)x_{max}^2$$

$$x_{max} = 0.8394\ \text{m} = 0.839\ \text{m} \qquad \textbf{Ans.}$$

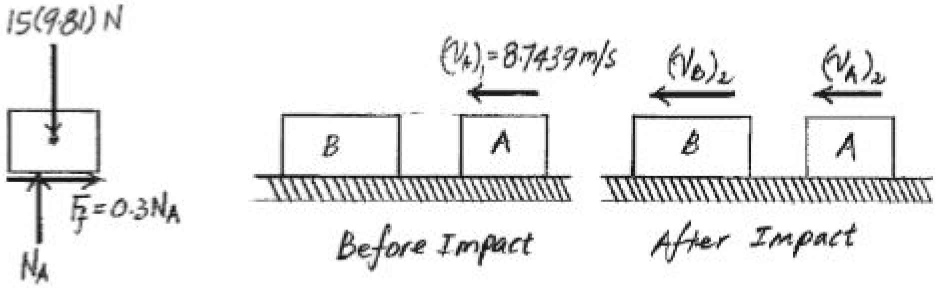


Figure 12: Exercise of section 15.4

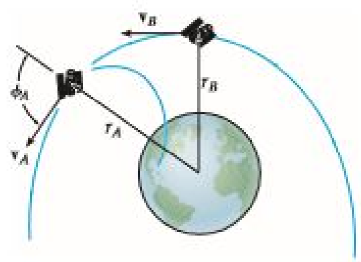
15.7 Principle of Angular Impulse and Momentum

$$\int M dt = \int (r \times F) dt \tag{32}$$

$$(r \times mv)_1 + \sum \int M_O dt = (r \times mv)_2 \tag{33}$$

15–113.

An earth satellite of mass 700 kg is launched into a free-flight trajectory about the earth with an initial speed of $v_A = 10$ km/s when the distance from the center of the earth is $r_A = 15$ Mm. If the launch angle at this position is $\phi_A = 70^\circ$, determine the speed v_B of the satellite and its closest distance r_B from the center of the earth. The earth has a mass $M_e = 5.976(10^{24})$ kg. *Hint:* Under these conditions, the satellite is subjected only to the earth's gravitational force, $F = GM_e m_s/r^2$, Eq. 13–1. For part of the solution, use the conservation of energy.



SOLUTION

$$(H_O)_1 = (H_O)_2$$

$$m_s(v_A \sin\phi_A)r_A = m_s(v_B)r_B$$

$$700[10(10^3)\sin 70^\circ](15)(10^6) = 700(v_B)(r_B) \qquad \textbf{(1)}$$

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}m_s(v_A)^2 - \frac{GM_e m_s}{r_A} = \frac{1}{2}m_s(v_B)^2 - \frac{GM_e m_s}{r_B}$$

$$\frac{1}{2}(700)[10(10^3)]^2 - \frac{66.73(10^{-12})(5.976)(10^{24})(700)}{[15(10^6)]} = \frac{1}{2}(700)(v_B)^2 - \frac{66.73(10^{-12})(5.976)(10^{24})(700)}{r_B} \qquad \textbf{(2)}$$

Solving,

$v_B = 10.2$ km/s **Ans.**

$r_B = 13.8$ Mm **Ans.**

Figure 13: Exercise of section 15.7

Chapter 16 Planar kinematics of a Rigid Body

16.3 Rotation about a fixed axis

$$\omega = \frac{d\theta}{dt} \tag{34}$$

$$\alpha = \frac{d\omega}{dt} \tag{35}$$

Constant Angular Acceleration:

$$\omega = \omega_0 + \alpha_c t \tag{36}$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2 \tag{37}$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2 \tag{38}$$

Acceleration:

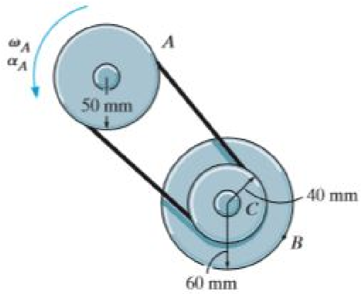
$$a_t = \alpha r \tag{39}$$

$$a_n = \omega^2 r \tag{40}$$

$$a = a_t + a_n = \alpha \times r - \omega^2 r \tag{41}$$

16–10.

At the instant $v_A = 5\ \text{rad/s}$, pulley A is given a constant angular acceleration $\alpha_A = 6\ \text{rad/s}^2$. Determine the magnitude of acceleration of point B on pulley C when A rotates 2 revolutions. Pulley C has an inner hub which is fixed to its outer one and turns with it.



SOLUTION

Angular Motion. Since the angular acceleration of pulley A is constant, we can apply

$$\omega_A^2 = (\omega_A)_0^2 + 2\alpha_A[\theta_A - (\theta_A)_0];$$

$$\omega_A^2 = 5^2 + 2(6)[2(2\pi) - 0]$$

$$\omega_A = 13.2588\ \text{rad/s}$$

Since pulleys A and C are connected by a non-slip belt,

$$\omega_C r_C = \omega_A r_A; \qquad \omega_C(40) = 13.2588(50)$$

$$\omega_C = 16.5735\ \text{rad/s}$$

$$\alpha_C r_C = \alpha_A r_A; \qquad \alpha_C(40) = 6(50)$$

$$\alpha_C = 7.50\ \text{rad/s}^2$$

Motion of Point B. The tangential and normal component of acceleration of point B can be determined from

$$(a_B)_t = \alpha_C r_B = 7.50(0.06) = 0.450\ \text{m/s}^2$$

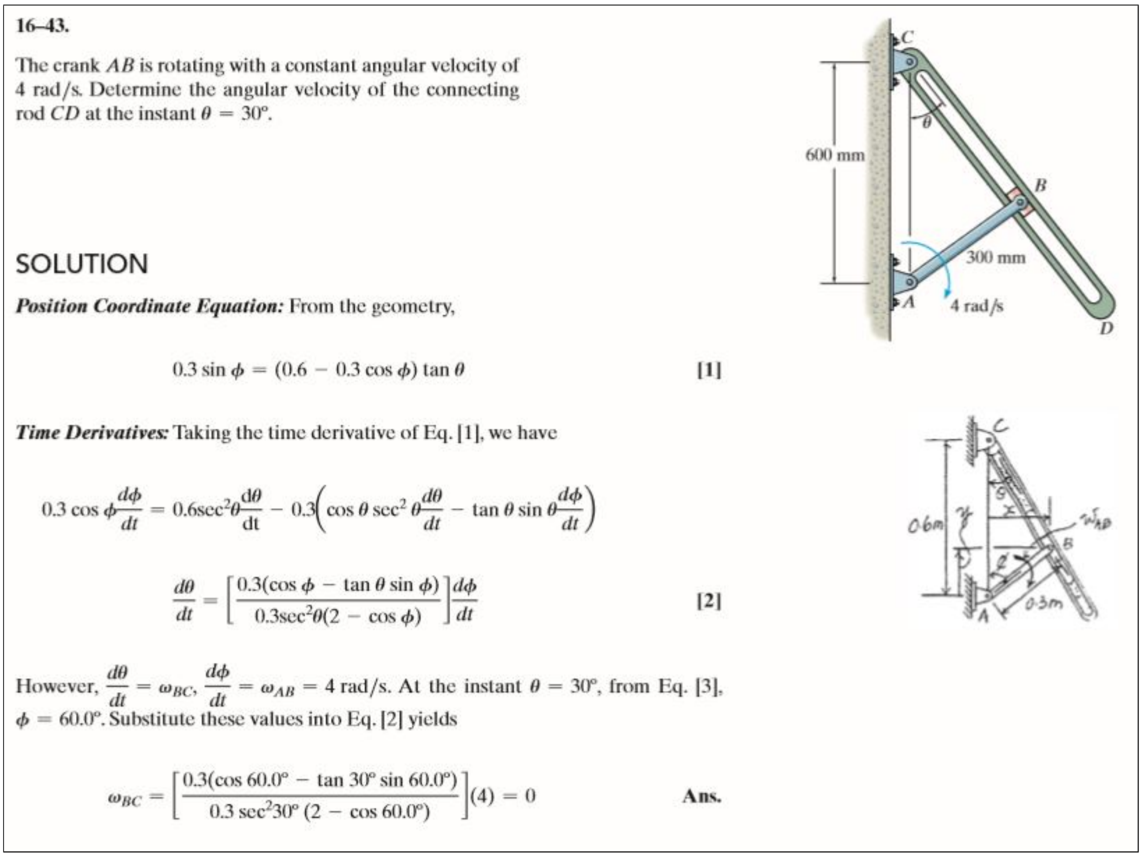
$$(a_B)_n = \omega_C^2 r_B = (16.5735^2)(0.06) = 16.4809\ \text{m/s}^2$$

Thus, the magnitude of a_B is

$$a_B = \sqrt{(a_B)_t^2 + (a_B)_n^2} = \sqrt{0.450^2 + 16.4809^2}$$

$$= 16.4871\ \text{m/s}^2 = 16.5\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Figure 14: Exercise of section 16.3



16–43.

The crank AB is rotating with a constant angular velocity of 4 rad/s. Determine the angular velocity of the connecting rod CD at the instant $\theta = 30°$.

SOLUTION

Position Coordinate Equation: From the geometry,

$$0.3 \sin \phi = (0.6 - 0.3 \cos \phi) \tan \theta \qquad \textbf{[1]}$$

Time Derivatives: Taking the time derivative of Eq. [1], we have

$$0.3 \cos \phi \frac{d\phi}{dt} = 0.6\sec^2\theta\frac{d\theta}{dt} - 0.3\left(\cos\theta \sec^2\theta \frac{d\theta}{dt} - \tan\theta \sin\theta \frac{d\phi}{dt}\right)$$

$$\frac{d\theta}{dt} = \left[\frac{0.3(\cos\phi - \tan\theta\sin\phi)}{0.3\sec^2\theta(2 - \cos\phi)}\right]\frac{d\phi}{dt} \qquad \textbf{[2]}$$

However, $\frac{d\theta}{dt} = \omega_{BC}$, $\frac{d\phi}{dt} = \omega_{AB} = 4$ rad/s. At the instant $\theta = 30°$, from Eq. [3], $\phi = 60.0°$. Substitute these values into Eq. [2] yields

$$\omega_{BC} = \left[\frac{0.3(\cos 60.0° - \tan 30° \sin 60.0°)}{0.3 \sec^2 30° \, (2 - \cos 60.0°)}\right](4) = 0 \qquad \textbf{Ans.}$$

Figure 15: Exercise of section 16.4

16.5 Relative motion analysis: Velocity

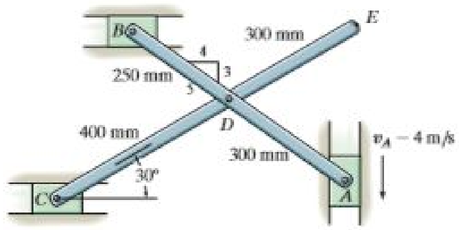
$$v_b = v_a + v_{b/a} \tag{42}$$

$$\vec{v}_b = \vec{v}_a + \vec{\omega} \times \vec{r}_{b/a} \tag{43}$$

$$\vec{r}_{b/a} = | r | \cdot \vec{AB} \tag{44}$$

16–73.

If the slider block A is moving downward at $v_A = 4$ m/s, determine the velocity of point E at the instant shown.



SOLUTION

See solution to Prob. 16–87.

$$v_E = v_D + v_{E/D}$$

$$\vec{v}_E = 4\downarrow + \underset{{}^{4}\!\nearrow^{3}_{5}}{2.727} + \underset{\nwarrow 30^\circ}{(5.249)(0.3)}$$

$(\overset{+}{\rightarrow})\qquad (v_E)_x = 0 + 2.727\left(\dfrac{3}{5}\right) + 5.249(0.3)(\sin 30^\circ)$

$(+\downarrow)\qquad (v_E)_y = 4 - 2.727\left(\dfrac{4}{5}\right) + 5.249(0.3)(\cos 30^\circ)$

$(v_E)_x = 2.424 \text{ m/s} \rightarrow$

$(v_E)_y = 3.182 \text{ m/s} \downarrow$

$\mathbf{v}_E = \sqrt{(2.424)^2 + (3.182)^2} = 4.00 \text{ m/s}$ **Ans.**

$\theta = \tan^{-1}\left(\dfrac{3.182}{2.424}\right) = 52.7^\circ$ **Ans.**

Also:

See solution to Prob. 16–87.

$\mathbf{v}_E = \mathbf{v}_D + \boldsymbol{\omega}_{CE} \times \mathbf{r}_{E/D}$

$\mathbf{v}_E = (1.636\mathbf{i} - 1.818\mathbf{j}) + (-5.25\mathbf{k}) \times \{\cos 30^\circ(0.3)\mathbf{i} - 0.4 \sin 30^\circ(0.3)\mathbf{j}\}$

$\mathbf{v}_E = \{2.424\mathbf{i} - 3.182\mathbf{j}\}$ m/s

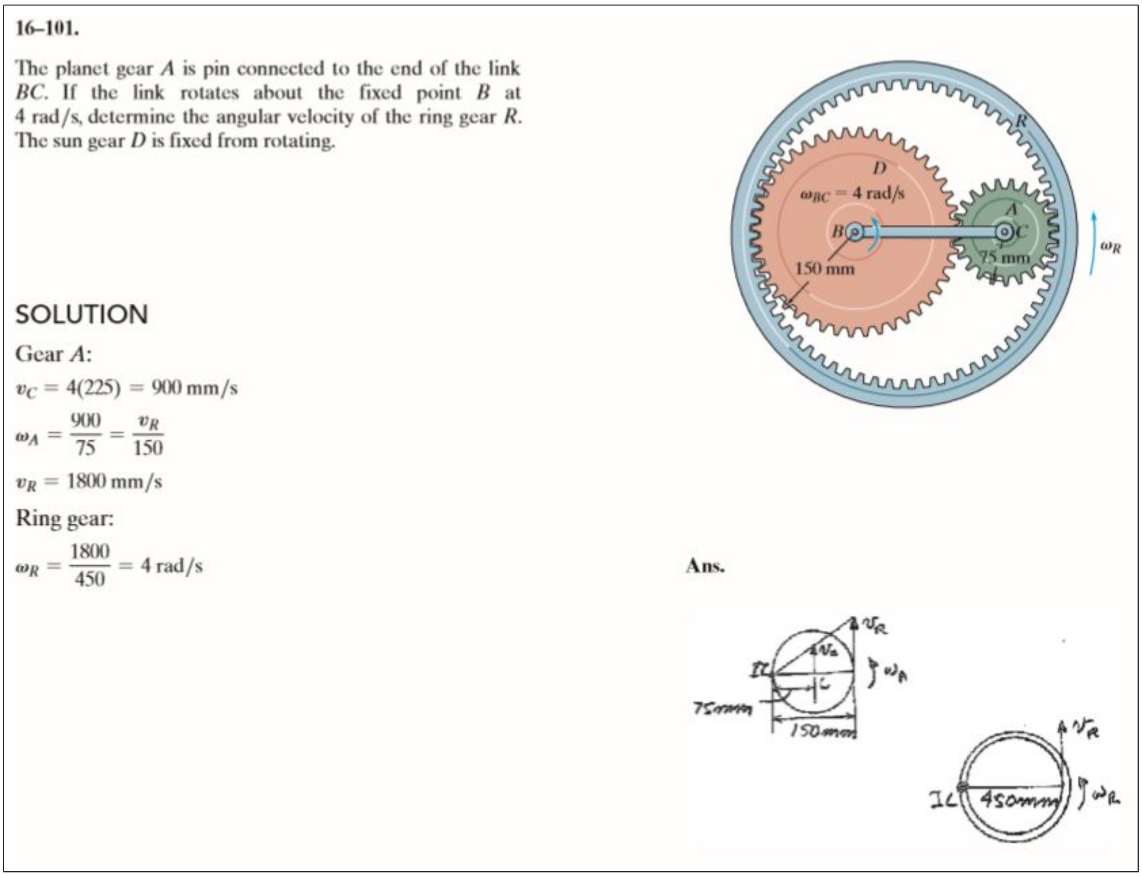
$v_E = \sqrt{(2.424)^2 + (3.182)^2} = 4.00 \text{ m/s}$ **Ans.**

$\theta = \tan^{-1}\left(\dfrac{3.182}{2.424}\right) = 52.7^\circ$ **Ans.**

Figure 16: Exercise of section 16.5

16.6 Instantaneous center of zero velocity

To locate the IC we can use the fact that the velocity of a point on the body is always perpendicular to the relative position vector directed from the IC to the point.



16–101.

The planet gear A is pin connected to the end of the link BC. If the link rotates about the fixed point B at 4 rad/s, determine the angular velocity of the ring gear R. The sun gear D is fixed from rotating.

SOLUTION

Gear A:

$v_C = 4(225) = 900 \text{ mm/s}$

$\omega_A = \frac{900}{75} = \frac{v_R}{150}$

$v_R = 1800 \text{ mm/s}$

Ring gear:

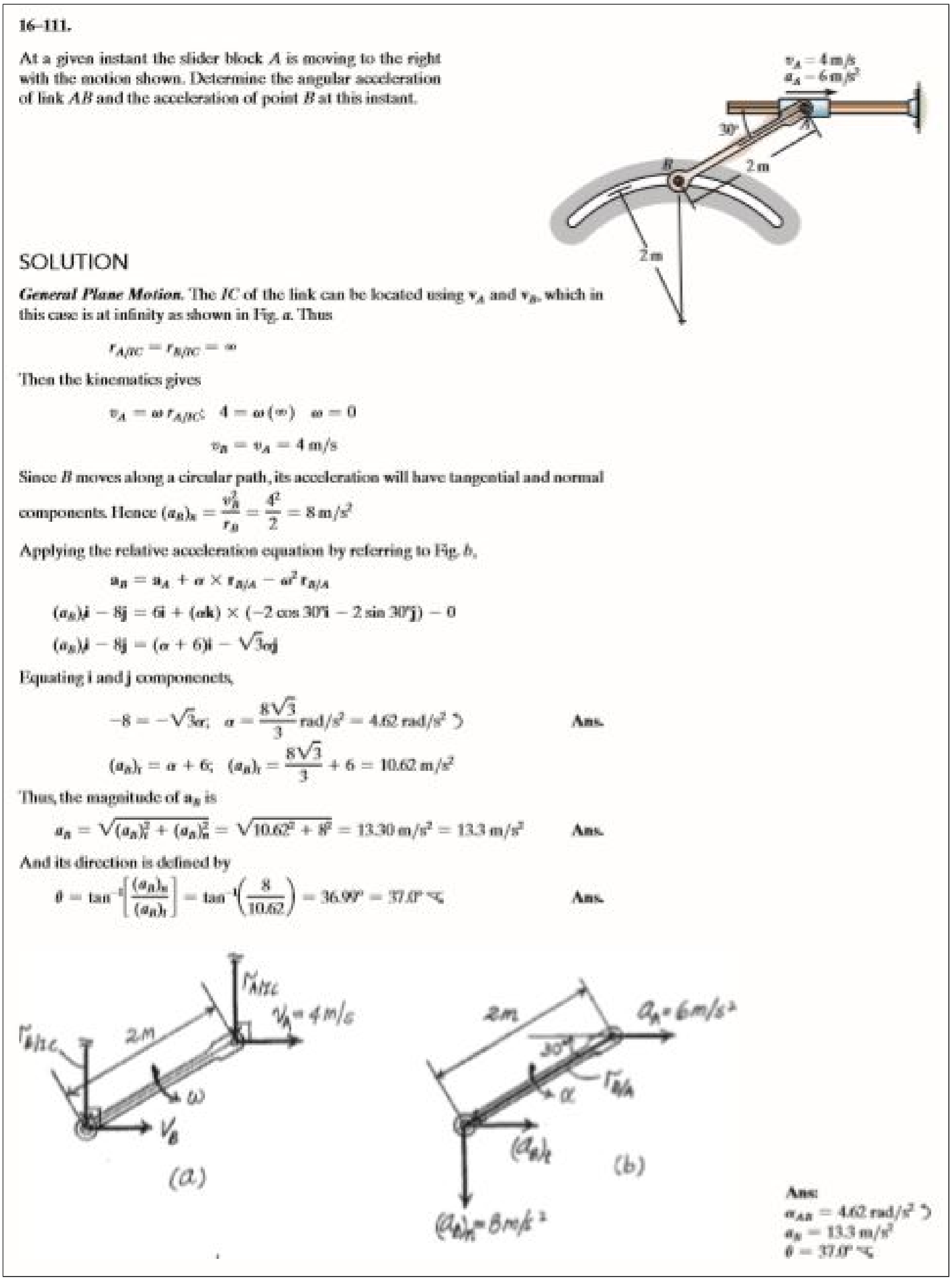
$\omega_R = \frac{1800}{450} = 4 \text{ rad/s}$ **Ans.**

Figure 17: Exercise of section 16.6

16.7 Relative motion analysis: Acceleration

$$a_b = a_a + (a_{b/a})_t + (a_{b/a})_n \tag{45}$$

$$\vec{a_b} = \vec{a_a} + \alpha \times \vec{r_{b/a}} - \omega^2 \cdot \vec{r_{b/a}} \tag{46}$$



16–111.

At a given instant the slider block A is moving to the right with the motion shown. Determine the angular acceleration of link AB and the acceleration of point B at this instant.

SOLUTION

General Plane Motion. The IC of the link can be located using $\mathbf{v}_A$ and $\mathbf{v}_B$, which in this case is at infinity as shown in Fig. a. Thus

$$r_{A/IC} = r_{B/IC} = \infty$$

Then the kinematics gives

$$v_A = \omega\, r_{A/IC};\quad 4 = \omega(\infty) \quad \omega = 0$$

$$v_B = v_A = 4 \text{ m/s}$$

Since B moves along a circular path, its acceleration will have tangential and normal components. Hence $(a_B)_n = \dfrac{v_B^2}{r_B} = \dfrac{4^2}{2} = 8 \text{ m/s}^2$

Applying the relative acceleration equation by referring to Fig. b,

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha} \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{B/A}$$

$$(a_B)_t\mathbf{i} - 8\mathbf{j} = 6\mathbf{i} + (\alpha\mathbf{k}) \times (-2\cos 30°\mathbf{i} - 2\sin 30°\mathbf{j}) - \mathbf{0}$$

$$(a_B)_t\mathbf{i} - 8\mathbf{j} = (\alpha + 6)\mathbf{i} - \sqrt{3}\alpha\mathbf{j}$$

Equating $\mathbf{i}$ and $\mathbf{j}$ componenets,

$$-8 = -\sqrt{3}\alpha;\quad \alpha = \frac{8\sqrt{3}}{3}\text{ rad/s}^2 = 4.62 \text{ rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$

$$(a_B)_t = \alpha + 6;\quad (a_B)_t = \frac{8\sqrt{3}}{3} + 6 = 10.62 \text{ m/s}^2$$

Thus, the magnitude of $\mathbf{a}_B$ is

$$a_B = \sqrt{(a_B)_t^2 + (a_B)_n^2} = \sqrt{10.62^2 + 8^2} = 13.30 \text{ m/s}^2 = 13.3 \text{ m/s}^2 \qquad \textbf{Ans.}$$

And its direction is defined by

$$\theta = \tan^{-1}\left[\frac{(a_B)_n}{(a_B)_t}\right] = \tan^{-1}\left(\frac{8}{10.62}\right) = 36.99° = 37.0° \qquad \textbf{Ans.}$$

Ans:
$\alpha_{AB} = 4.62$ rad/s² $\circlearrowright$
$a_B = 13.3$ m/s²
$\theta = 37.0°$

Figure 18: Exercise of section 16.7

17 Planar kinematics of a rigid body: Force and Acceleration

17.1 Mass moment of inertia

$$I = I_G + md^2 \tag{47}$$

17.3 Equation of motion: Translation

Rectilinear Translation:

$$\sum F_x = m(a_G)_x \tag{48}$$

$$\sum F_y = m(a_G)_y \tag{49}$$

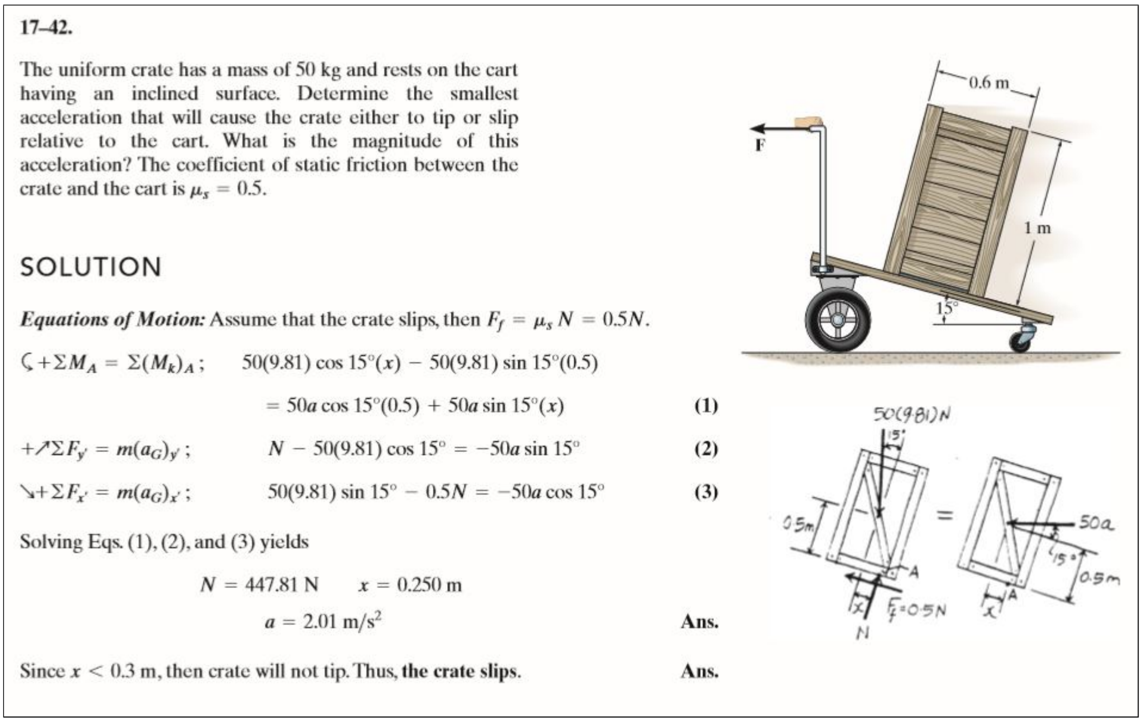
$$\sum M_G = 0 \tag{50}$$

Curvilinear Translation:

$$\sum F_n = m(a_G)_n \tag{51}$$

$$\sum F_t = m(a_G)_t \tag{52}$$

$$\sum M_G = 0 \tag{53}$$



17–42.

The uniform crate has a mass of 50 kg and rests on the cart having an inclined surface. Determine the smallest acceleration that will cause the crate either to tip or slip relative to the cart. What is the magnitude of this acceleration? The coefficient of static friction between the crate and the cart is $\mu_s = 0.5$.

SOLUTION

Equations of Motion: Assume that the crate slips, then $F_f = \mu_s N = 0.5N$.

$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A$; $\quad 50(9.81)\cos 15^\circ(x) - 50(9.81)\sin 15^\circ(0.5)$

$= 50a\cos 15^\circ(0.5) + 50a\sin 15^\circ(x)$ **(1)**

$+\nearrow \Sigma F_{y'} = m(a_G)_{y'}$; $\quad N - 50(9.81)\cos 15^\circ = -50a\sin 15^\circ$ **(2)**

$\searrow + \Sigma F_{x'} = m(a_G)_{x'}$; $\quad 50(9.81)\sin 15^\circ - 0.5N = -50a\cos 15^\circ$ **(3)**

Solving Eqs. (1), (2), and (3) yields

$N = 447.81$ N $\quad x = 0.250$ m

$a = 2.01\ \text{m/s}^2$ **Ans.**

Since $x < 0.3$ m, then crate will not tip. Thus, **the crate slips.** **Ans.**

Figure 19: Exercise of section 17.3

17.4 Equation of motion: Rotation about fixed axis

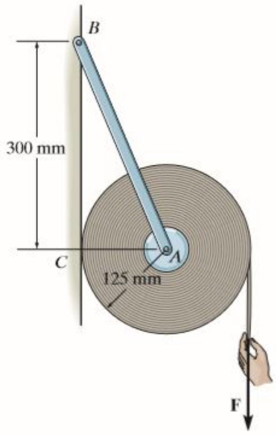
$$\sum F_n = m(a_G)_n = m\omega^2 r_G \tag{54}$$

$$\sum F_t = m(a_G)_t = m\alpha r_G \tag{55}$$

$$\sum M_G = I_G\alpha \tag{56}$$

17–70.

The 20-kg roll of paper has a radius of gyration $k_A = 90$ mm about an axis passing through point A. It is pin supported at both ends by two brackets AB. If the roll rests against a wall for which the coefficient of kinetic friction is $\mu_k = 0.2$, determine the constant vertical force F that must be applied to the roll to pull off 1 m of paper in $t = 3$ s starting from rest. Neglect the mass of paper that is removed.



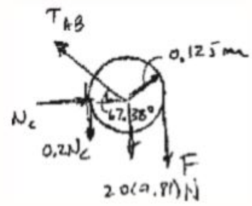
SOLUTION

$(+\downarrow)\ s = s_0 + v_0 t + \frac{1}{2}a_C t^2$

$1 = 0 + 0 + \frac{1}{2}a_C(3)^2$

$a_C = 0.222\ \text{m/s}^2$

$\alpha = \frac{a_C}{0.125} = 1.778\ \text{rad/s}^2$



$\xrightarrow{+}\ \Sigma F_x = m(a_{G})_x;\qquad N_C - T_{AB}\cos 67.38^\circ = 0$

$+\uparrow \Sigma F_y = m(a_G)_y;\qquad T_{AB}\sin 67.38^\circ - 0.2N_C - 20(9.81) - F = 0$

$\circlearrowleft + \Sigma M_A = I_A\alpha;\qquad -0.2N_C(0.125) + F(0.125) = 20(0.09)^2(1.778)$

Solving:

$N_C = 99.3$ N

$T_{AB} = 258$ N

$F = 22.1$ N **Ans.**

Figure 20: Exercise of section 17.4

17.5 Equation of motion: General plane of motion

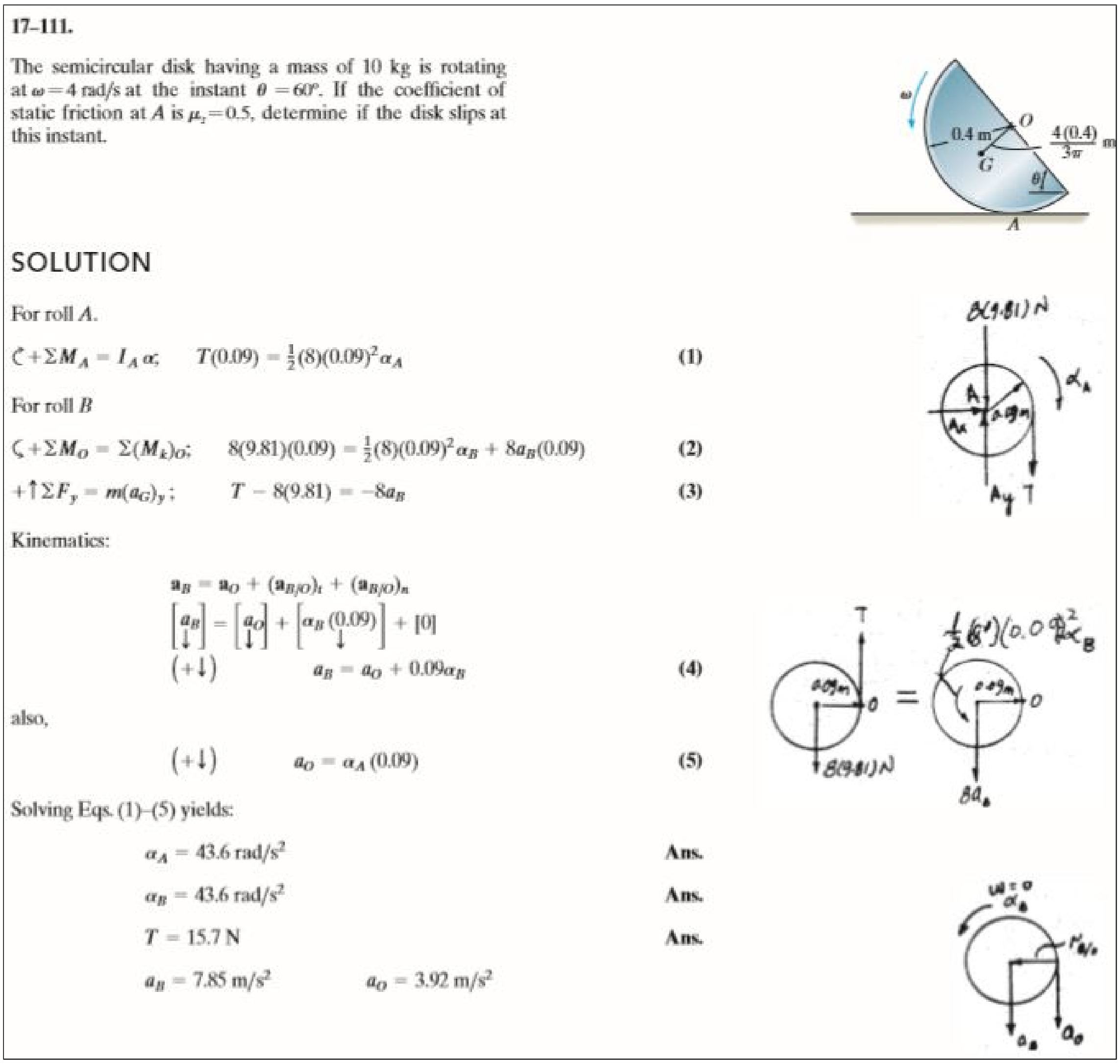
$$\sum F_x = m(a_G)_x \tag{57}$$

$$\sum F_y = m(a_G)_y \tag{58}$$

$$\sum M_G = I_G\alpha \tag{59}$$

$$\sum M_P = \sum(M_k)_P = I_G\alpha + (ma_G)d_{G/P} \tag{60}$$

$$\sum M_{IC} = I_{IC}\alpha \tag{61}$$



17–111.

The semicircular disk having a mass of 10 kg is rotating at $\omega = 4$ rad/s at the instant $\theta = 60°$. If the coefficient of static friction at A is $\mu_s = 0.5$, determine if the disk slips at this instant.

SOLUTION

For roll A.

$$\circlearrowleft + \Sigma M_A = I_A \alpha; \qquad T(0.09) = \tfrac{1}{2}(8)(0.09)^2 \alpha_A \qquad (1)$$

For roll B

$$\circlearrowleft + \Sigma M_O = \Sigma (M_k)_O; \qquad 8(9.81)(0.09) = \tfrac{1}{2}(8)(0.09)^2 \alpha_B + 8a_B(0.09) \qquad (2)$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad T - 8(9.81) = -8a_B \qquad (3)$$

Kinematics:

$$\mathbf{a}_B = \mathbf{a}_O + (\mathbf{a}_{B/O})_t + (\mathbf{a}_{B/O})_n$$

$$\begin{bmatrix} a_B \\ \downarrow \end{bmatrix} = \begin{bmatrix} a_O \\ \downarrow \end{bmatrix} + \begin{bmatrix} \alpha_B(0.09) \\ \downarrow \end{bmatrix} + [0]$$

$$(+\downarrow) \qquad a_B = a_O + 0.09\alpha_B \qquad (4)$$

also,

$$(+\downarrow) \qquad a_O = \alpha_A(0.09) \qquad (5)$$

Solving Eqs. (1)–(5) yields:

$\alpha_A = 43.6$ rad/s^2 **Ans.**

$\alpha_B = 43.6$ rad/s^2 **Ans.**

$T = 15.7$ N **Ans.**

$a_B = 7.85$ m/s^2 $\qquad a_O = 3.92$ m/s^2

Figure 21: Exercise of section 17.5

Chapter 18 Planar Kinetics of a Rigid Body: Work and Energy

18.1 Kinetic Energy

Translation:

$$T = \frac{1}{2} m v_G^2 \tag{62}$$

Rotation about a fixed axis:

$$T = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2 \tag{63}$$

$$T = \frac{1}{2} I_O \omega^2 \tag{64}$$

General plane motion:

$$T = \frac{1}{2} I_{IC} \omega^2 \tag{65}$$

18.2 The work of a force

$$W = \int F dr = \int F \cos\theta ds \tag{66}$$

Work of a spring:

$$W_s = \int F_s ds = \int -ksds = -\frac{1}{2}ks^2 + C \tag{67}$$

18.2 The work of a couple moment

$$U_M = \int M d\theta \tag{68}$$

18.4 Principle of work and energy

$$T_1 + \sum U_{1-2} = T_2 \tag{69}$$

18–10.

The spool has a mass of 40 kg and a radius of gyration of $k_O = 0.3$ m. If the 10-kg block is released from rest, determine the distance the block must fall in order for the spool to have an angular velocity $\omega = 15$ rad/s. Also, what is the tension in the cord while the block is in motion? Neglect the mass of the cord.

SOLUTION

Kinetic Energy. Since the system is released from rest, $T_1 = 0$. The final velocity of the block is $v_b = \omega r = 15(0.3) = 4.50$ m/s. The mass moment of inertia of the spool about O is $I_O = mk_O^2 = 40(0.3^2) = 3.60$ Kg · m². Thus

$$T_2 = \frac{1}{2} I_O \omega^2 + \frac{1}{2} m_b v_b^2$$

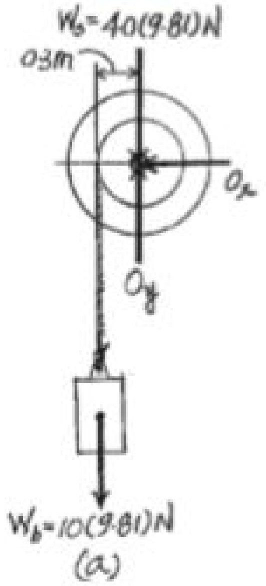
$$= \frac{1}{2}(3.60)(15^2) + \frac{1}{2}(10)(4.50^2)$$

$$= 506.25 \text{ J}$$

For the block, $T_1 = 0$ and $T_2 = \frac{1}{2} m_b v_b^2 = \frac{1}{2}(10)(4.50^2) = 101.25$ J

Work. Referring to the FBD of the system Fig. *a*, only $\mathbf{W}_b$ does work when the block displaces s vertically downward, which it is positive.

$$U_{W_b} = W_b s = 10(9.81)s = 98.1\, s$$



Referring to the FBD of the block, Fig. *b*. $\mathbf{W}_b$ does positive work while $\mathbf{T}$ does negative work.

$$U_T = -Ts$$

$$U_{W_b} = W_b s = 10(9.81)(s) = 98.1\, s$$

Principle of Work and Energy. For the system,

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 98.1s = 506.25$$

$$s = 5.1606 \text{ m} = 5.16 \text{ m} \qquad \textbf{Ans.}$$

For the block using the result of s,

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 98.1(5.1606) - T(5.1606) = 101.25$$

$$T = 78.48 \text{ N} = 78.5 \text{ N} \qquad \textbf{Ans.}$$

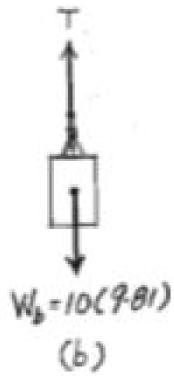
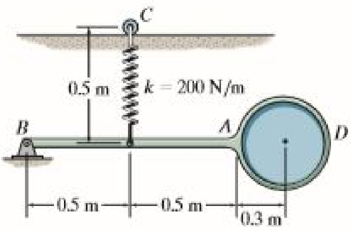


Figure 22: Exercise of section 18.4

***18–60.**

The pendulum consists of a 6-kg slender rod fixed to a 15-kg disk. If the spring has an unstretched length of 0.2 m, determine the angular velocity of the pendulum when it is released from rest and rotates clockwise 90° from the position shown. The roller at C allows the spring to always remain vertical.



SOLUTION

Kinetic Energy. The mass moment of inertia of the pendulum about B is $I_B = \left[\frac{1}{12}(6)(1^2) + 6(0.5^2)\right] + \left[\frac{1}{2}(15)(0.3^2) + 15(1.3^2)\right] = 28.025\ \text{kg}\cdot\text{m}^2$. Thus

$$T = \frac{1}{2}I_B\omega^2 = \frac{1}{2}(28.025)\,\omega^2 = 14.0125\,\omega^2$$

Since the pendulum is released from rest, $T_1 = 0$.

Potential Energy. with reference to the datum set in Fig. *a*, the gravitational potential energies of the pendulum when it is at positions ① and ② are

$$(V_g)_1 = m_r g(y_r)_1 + m_d g(y_d)_1 = 0$$

$$(V_g)_2 = m_r g(y_r)_2 + m_d g(y_d)_2$$

$$= 6(9.81)(-0.5) + 15(9.81)(-1.3)$$

$$= -220.725\ \text{J}$$

The stretch of the spring when the pendulum is at positions ① and ② are

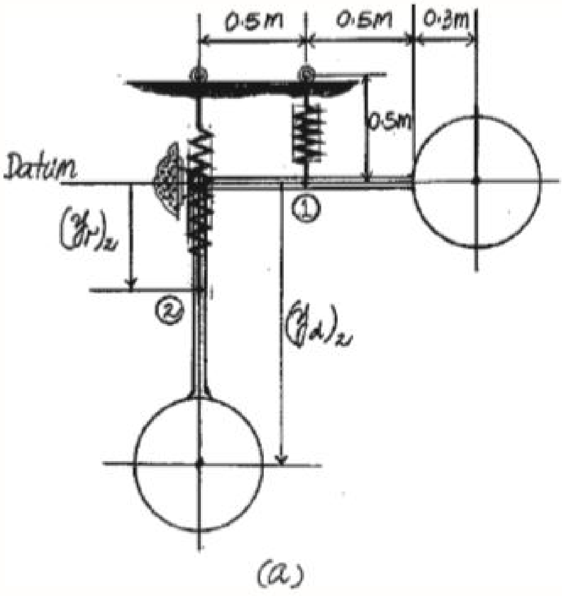
$$x_1 = 0.5 - 0.2 = 0.3\ \text{m}$$

$$x_2 = 1 - 0.2 = 0.8\ \text{m}$$

Thus, the initial and final elastic potential energies of the spring are

$$(V_e)_1 = \frac{1}{2}kx_1^2 = \frac{1}{2}(200)(0.3^2) = 9.00\ \text{J}$$

$$(V_e)_2 = \frac{1}{2}kx_2^2 = \frac{1}{2}(200)(0.8^2) = 64.0\ \text{J}$$



Conservation of Energy.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + (0 + 9.00) = 14.0125\omega^2 + (-220.725) + 64.0$$

$$\omega = 3.4390\ \text{rad/s} = 3.44\ \text{rad/s} \qquad \textbf{Ans.}$$

Figure 23: Exercise of section 18.5

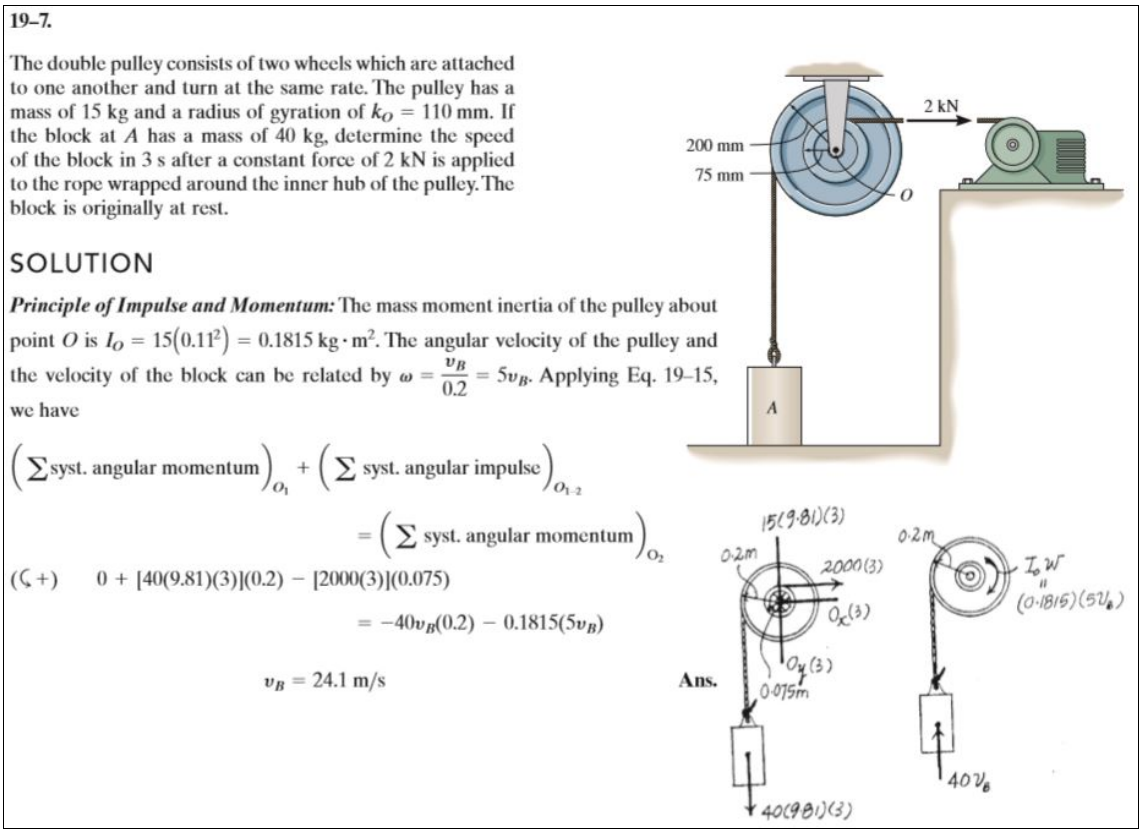
Chapter 19 Planar kinetics of a rigid body: Impulse and Momentum

19.2 Principle of Impulse and Momentum

$$m(v_{Gx})_1 + \sum \int F_x dt = m(v_{Gx})_2 \tag{70}$$

$$m(v_{Gy})_1 + \sum \int F_y dt = m(v_{Gy})_2 \tag{71}$$

$$I_G\omega_1 + \sum \int M_G dt = I_G\omega_2 \tag{72}$$



19–7.

The double pulley consists of two wheels which are attached to one another and turn at the same rate. The pulley has a mass of 15 kg and a radius of gyration of $k_O = 110$ mm. If the block at A has a mass of 40 kg, determine the speed of the block in 3 s after a constant force of 2 kN is applied to the rope wrapped around the inner hub of the pulley. The block is originally at rest.

SOLUTION

Principle of Impulse and Momentum: The mass moment inertia of the pulley about point O is $I_O = 15\left(0.11^2\right) = 0.1815\ \text{kg}\cdot\text{m}^2$. The angular velocity of the pulley and the velocity of the block can be related by $\omega = \frac{v_B}{0.2} = 5v_B$. Applying Eq. 19–15, we have

$$\left(\sum \text{syst. angular momentum}\right)_{O_1} + \left(\sum \text{syst. angular impulse}\right)_{O_{1\text{-}2}} = \left(\sum \text{syst. angular momentum}\right)_{O_2}$$

$$(\circlearrowleft +) \qquad 0 + [40(9.81)(3)](0.2) - [2000(3)](0.075) = -40v_B(0.2) - 0.1815(5v_B)$$

$$v_B = 24.1\ \text{m/s} \qquad \textbf{Ans.}$$

Figure 24: Exercise of section 19.2

19.3 Conservation of Momentum

Linear momentum:

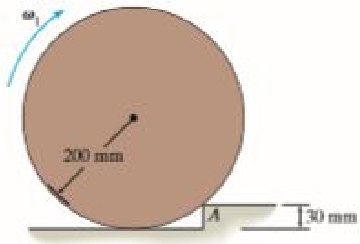
$$(\sum syst.linear.momentum)_1 = (\sum syst.linear.momentum)_2 \tag{73}$$

Angular Momentum:

$$(\sum syst.angular.momentum)_{O1} = (\sum syst.angular.momentum)_{O2} \tag{74}$$

19–50.

The 20-kg disk strikes the step without rebounding. Determine the largest angular velocity ω_1 the disk can have and not lose contact with the step, A.



SOLUTION

Conservation of Angular Momentum. The mass moment of inertia of the disk about its mass center is $I_G = \frac{1}{2}mr^2 = \frac{1}{2}(20)(0.2^2) = 0.4\ \text{kg}\cdot\text{m}^2$. Since no slipping occurs, $v_G = \omega r = \omega(0.2)$. Referring to the impulse and momentum diagram, Fig. *a*, we notice that angular moment is conserved about point A since **W** is nonimpulsive. Thus,

$$(H_A)_1 = (H_A)_2$$

$$20[\omega_1(0.2)](0.17) + 0.4\,\omega_1 = 0.4\,\omega_2 + 20[\omega_2(0.2)](0.2)$$

$$\omega_1 = 1.1111\,\omega_2 \qquad \textbf{(1)}$$

Equations of Motion. Since the requirement is the disk is about to lose contact with the step when it rotates about A, $N_A \simeq 0$. Here $\theta = \cos^{-1}\left(\frac{0.17}{0.2}\right) = 31.79°$. Consider the motion along n direction,

$$+\searrow \Sigma F_n = M(a_G)_n; \qquad 20(9.81)\cos 31.79° = 20[\,\omega_2^2(0.2)\,]$$

$$\omega_2 = 6.4570\ \text{rad/s}$$

Substitute this result into Eq. (1)

$$\omega_1 = 1.1111(6.4570) = 7.1744\ \text{rad/s} = 7.17\ \text{rad/s} \qquad \textbf{Ans.}$$

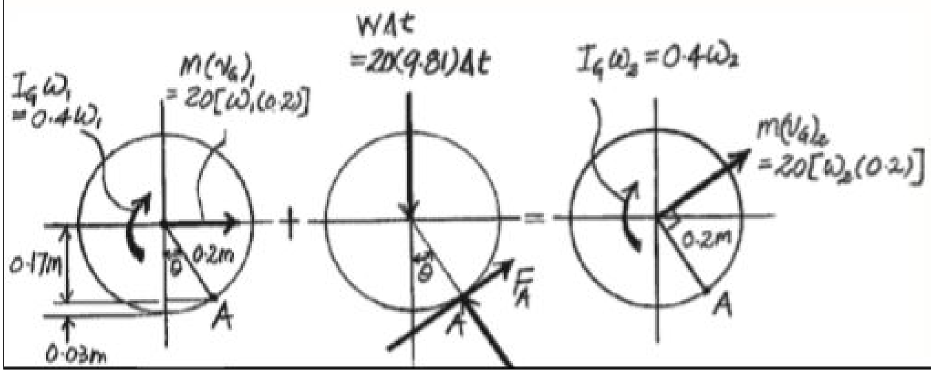


Figure 25: Exercise of section 19.3