Problem Solving Guide

1 Introduction

Are you having trouble with solving dynamics problems? Do you often haven't got a clue where to start? Then this problem solving guide might come in handy for you.

In this problem guide, the dynamics course has been split up in a lot of small subjects. For every subject, it is mentioned...

- How to recognize it. Now you will know what problem corresponds to what subject!
- The important equations of that subject. This should help you solve the problem.
- The frequency of occurrence. How often does this subject occur on exams? Quite handy to know, isn't it?

However, dynamics is a difficult subject. A question hardly ever corresponds to just one subject. So in this problem solving guide, it is also often mentioned which subjects are linked in some way.

This problem solving guide contains various bold lines. If you only read these bold lines and the equations, you will, in fact, read a summary of this document. If you want further clarification of the bold lines, then you should also read the normal text next to it.

I would advise you to read this problem solving guide carefully. Then try to solve dynamics exam problems, using this problem solving guide. And then you just might be able to solve those difficult dynamics problems you were never able to solve.

2 General plan of approach

2.1 When to use?

At every question. The plan of approach below contains tips that can help you at every problem. So you should definitely apply this at every problem! Study it carefully.

2.2 Plan of Approach

1. **Read the question twice.** You may wonder, why twice? This is because you often miss things the first time you read something.
2. **While reading, look for keywords, and underline them.** There are several keywords that are important for dynamics problems. Just to mention a few:
   - **Initially at rest.** This means that the initial velocity V_0 is zero.
   - **Massless.** This means that the mass m is zero. It also means that the moment of inertia I is zero.
   - **In the horizontal plane.** When something moves in the horizontal plane, you do not have to consider gravity.
   - **In the vertical plane.** In this case you do have to consider gravity. The gravitational acceleration points downward and has magnitude $g = 9.81 m/s^2$.
   - $g = 9.81m/s^2$**.** This is a subtle hint. If they give the value for g, you probably will have to use it. So in this case, do consider gravity.
   - **Do/do not neglect friction.** Just do what it says. If you can neglect friction, it's great! If you can not neglect friction, you probably have to use static and dynamic friction coefficients.

   Besides these keywords, there are also a few keywords belonging to certain subjects. We will discuss them in the paragraphs belonging to those subjects.
3. **Underline all given values.** You can even write them down on a piece of scrap paper. This is rather important. It gives you a good overview of everything that is given.
4. **Look at what is asked.** This is just as important as the previous point. You can also write down what is asked. (For example, if the velocity in point B is asked, write down $V_B =$?) This once more helps you get an overview.
5. **Make a drawing.** Dynamics is a tangible subject. It is about real-life situations. So you can perfectly draw pictures about it! So, ALWAYS draw a picture (unless, perhaps, if one is already drawn in the exam). In this picture you can then draw velocities/accelerations (if you are dealing with a motion-problem) or forces/moments (if you are dealing with a forces-problem).
6. **Try to figure out what subject the question is about.** If you know the subject a problem corresponds to, you're well on your way to solve the problem. You now know what equations you can (and often should) use. See if you can combine these equations with the variables that are given and asked. Then you only have to work out a few equations to find your solution.
7. **Write down the solution.** This is a very important step. Without it you won't get any points. You may think, "Duh... of course!" However, things often go wrong in this step. Students often just write down numbers and equations, without clarifying what they're doing. Instead, continuously apply the following two steps, while writing down your solution:
   - First say in words what you will do.
   - Then do exactly that.

   For example, your solution can start with "First we calculate the moment of inertia", after which you actually calculate the moment of inertia (with equations). Then you can continue with, "Now we insert it into Newton's second law for rotations", after which you apply $\sum \mathbf{M} = I\alpha$. You should build up your entire solution like this. Try to hand in something you're proud of.

There is also one other habit I would advise you to get used to. Try to work with equations in letters. Only fill in numbers when you have already solved the whole system of equations. This prevents you getting lost in numbers and units.

8. **Check your work.** This is a step which only takes a couple of seconds, but can get you a LOT of points. Many errors are made due to negligence. After you have finished writing down your solution, read it over one more time. While reading, check for the following things.
   - **Is my solution correct and logical?** Quickly glance over your equations. Are there any obvious errors in them? If so, then fix them. If a person on a bike travels at $120m/s$, you probably did something wrong. (Okay, unless the bike has a rocket engine installed on it.)
   - **Do I need to fill in numbers in the final equation?** Often you need to derive an expression for something. (This is the case if no values are given, or just very few.) Then your final answer will just be that expression. But sometimes values are given. Do not forget to insert them into your equation and calculate a final answer!
   - **Do I need to write a unit behind my answer?** If you do need to give a number as a final answer, don't forget the unit! They will subtract points if you forget it.
   - **Have I answered the question?** Sometimes they don't ask for a value, but for a conclusion. For example, they can ask if a bungee jumper will survive his jump from a bridge with height $h = 90m$. If you calculate that he will travel a distance of $108m$ before he stops, then you can conclude that they'd better call the memorial service.

2.3 General exam tips

What do you do when you first see your exam? Panic? Hopefully not. Instead, this is what you should do.

1. **The first couple of minutes, browse through the entire exam. Then start with the easy problems.** (The easy problems are usually problems 1 and 5.) You can also start with problems belonging to subjects you're good at. Solve those easy problems first. It would be a waste if you spend all your time on difficult problems, while leaving the easy points behind.

   By the way, you're allowed to make the problems in any order you like. Just make sure you clearly indicate what solution corresponds to what problem.
2. **When you seem to get stuck on a question, continue with another one.** If you are looking at a question for 15 minutes and still haven't got a clue how to solve it, continue with another question. It would be a shame if you eventually do not have enough time left to solve problems you were able to solve.
3. **Always write something down.** Sometimes you haven't got a clue how to solve a problem. However, just write down some equations. Then fill in some values. You can score points with that! Even calculating a moment of inertia can get you quite some points. You will be surprised by the difference this can make.
4. **The last five minutes of your exam, look through your work for minor errors.** Always do this, even if you haven't finished a couple of questions yet. It is almost always worth while to check your work. In fact, you can often pull up your grade by 0.5 point in those five minutes. Now THAT is efficient.

3 Forces & Motion

Let's take a look at the basic equations of motion. How can we find the velocity and acceleration of an object?

3.1 Newton's second law

3.1.1 When to use?

When a force or acceleration is asked. If a force is asked, then usually an acceleration is given, and vice verse. Also often the mass of an object is given.

3.1.2 Equations

The following equation is the most important equation in physics. Remember it well!

$$\sum \mathbf{F} = m\mathbf{a}. \tag{3.1}$$

3.1.3 Frequency of occurrence

Always. You will always need to apply Newton's second law during an exam. However, it is often part of a bigger subject.

3.2 One-dimensional motion

3.2.1 When to use?

When a formula for s, v or a is given. Without such a formula, you can't use the equations corresponding to this subject.

Another point of recognition is that an object moves in a straight line. (A rocket goes straight up, or a car moves on a straight track.)

3.2.2 Equations

There are three equations for one-dimensional motion. Especially the third one often appears on exams.

$$V = \frac{ds}{dt}, \tag{3.2}$$

$$a = \frac{dV}{dt}, \tag{3.3}$$

$$a\,ds = v\,dv. \tag{3.4}$$

3.2.3 Frequency of occurrence

Low. This subject doesn't appear on the exam very often, and when it does, it usually isn't worth a lot of points (since it is rather easy).

3.3 Relative motion

3.3.1 When to use?

When they ask for a relative velocity/acceleration. You also often need to use the equations of relative motion in combination with other subjects. However, often you already use these equations unconsciously, without knowing you actually apply them.

3.3.2 Equations

The velocity $\mathbf{V_{B/A}}$ of object B with respect to A can be expressed in $\mathbf{V_{A/O}}$ and $\mathbf{V_{B/O}}$, where O is some fixed point. The expression to use is

$$\mathbf{V_{B/A}} = \mathbf{V_{B/O}} - \mathbf{V_{A/O}}. \tag{3.5}$$

It works the same for accelerations. So,

$$\mathbf{a_{B/A}} = \mathbf{a_{B/O}} - \mathbf{a_{A/O}}. \tag{3.6}$$

3.3.3 Frequency of occurrence

High. They don't often ask for relative velocities directly. However, you often do have to use these equations as part of other subjects. But, as was already said, you often do this without knowing that you apply them.

3.4 Rotating bars

3.4.1 When to use?

When a rigid object rotates. Also use this when multiple objects rotate, which are attached to each other in some way. In this case you very often need to use the equations for **relative motion** as well.

3.4.2 Equations

The velocity at some point on the object is given by

$$\mathbf{V} = \omega r \mathbf{e}_\theta. \tag{3.7}$$

Similarly the acceleration is given by

$$\mathbf{a} = \alpha r \mathbf{e}_\theta - \omega^2 r \mathbf{e_r}. \tag{3.8}$$

The minus sign indicates that the centrifugal acceleration points inwards (towards the center of rotation).

3.4.3 Tips

When using the above equations, it is often wise to draw the velocities and accelerations in both directions (radial and tangential) as arrows. Use the above equations to calculate the magnitudes of these arrows. (For example, the magnitude of the acceleration in tangential direction has magnitude αr.)

After having done this, write down the errors in vector form. For example, let's consider an arrow with magnitude $5\sqrt{2}$, pointing to South-East. You should write this arrow as $\begin{bmatrix} 5 \\ -5 \end{bmatrix}$. In the end, add up the vectors to find your final answer. (You often need the equations of relative motion for this.)

3.4.4 Frequency of occurrence

High. Rotating bars appear quite often on an exam. The method of finding the answer isn't very hard. However, a lot of calculations are usually necessary, which often does result in errors.

3.5 (Near-) circular motion

3.5.1 When to use?

When something moves with a non-constant radius r.

3.5.2 Equations

$$\mathbf{r} = r\mathbf{e_r}, \tag{3.9}$$

$$\mathbf{V} = \dot{r}\mathbf{e_r} + r\dot{\theta}\mathbf{e}_\theta, \tag{3.10}$$

$$\mathbf{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\mathbf{e_r} + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\mathbf{e}_\theta. \tag{3.11}$$

3.5.3 Frequency of occurrence

Very low. This subject doesn't appear on an exam often. However, it very rarely occurs that they ask you to derive the above equations yourself. In this case, start with equation (3.9). Then take derivatives and use $d\mathbf{e_r}/dt = \mathbf{e}_\theta\, d\theta/dt$ and $d\mathbf{e}_\theta/dt = -\mathbf{e_r}\, d\theta/dt$.

3.6 Motion for a known path

3.6.1 When to use?

When an object moves along a path with given radius of curvature ρ. Sometimes they give the curvature κ itself. In this case you can use that $\rho = 1/\kappa$.

3.6.2 Equations

$$a_t = \frac{dV}{dt}, \tag{3.12}$$

$$a_r = \frac{V^2}{\rho}. \tag{3.13}$$

3.6.3 Frequency of occurrence

Very low. This subject hardly ever occurs on an exam.

4 Moment of Inertia and Rotations

Rotating things are fun. Playing around with bikes, cars or frisbees proves that. But how can we make things rotate? And what equations apply to them? Let's find that out.

4.1 Moment of Inertia

4.1.1 When to use?

When you need to calculate a moment of inertia. Sometimes they ask for it directly, but usually you need it to calculate something else. However, if you CAN calculate the moment of inertia (without too much trouble), then you SHOULD. You often get points for it.

By the way, if a radius of gyration κ_A about some point A is given, you should immediately calculate the moment of inertia. It is very easy, and you score points right away.

4.1.2 Equations

The (polar) moment of inertia of a bar with length L, with respect to its center of gravity, can be found using

$$I_{bar_{cog}} = \frac{1}{12} m L^2. \tag{4.1}$$

Similarly, for a solid disc and a hollow ring (with negligible thickness) we have

$$I_{disc_{cog}} = \frac{1}{2} m r^2, \tag{4.2}$$

$$I_{ring_{cog}} = m r^2. \tag{4.3}$$

If we know the moment of inertia about the center of gravity, but want to know it about some point A, we can use

$$I_A = I_{cog} + m r_{A,cog}^2. \tag{4.4}$$

And finally, if the radius of gyration κ_A about some point A is given, we should immediately use

$$I_A = m \kappa_A^2. \tag{4.5}$$

4.1.3 Frequency of occurrence

Always. There isn't a single dynamics exam in which you don't have to calculate a moment of inertia.

4.2 Rotations

4.2.1 When to use?

When an angular acceleration is given/asked, but nothing is known about forces/moments and moments. This subject doesn't appear very often, because it's not the most convenient tool. If you don't care about an angular acceleration, you'd better use **energy**. If you want to know, or already know forces and moments, you can better use **Newton's second law for rotations**. Only when an expression is given for ω or α, but no forces or moments are known, should you use the equations of this subject.

4.2.2 Equations

$$\omega = \frac{d\theta}{dt}, \tag{4.6}$$

$$\alpha = \frac{d\omega}{dt}, \tag{4.7}$$

$$\alpha \, d\theta = \omega \, d\omega. \tag{4.8}$$

4.2.3 Frequency of occurrence

Low. This is because (as was already said) other tools are much more convenient to use.

4.3 Newton's second law for rotations

4.3.1 When to use?

When something will rotate, and you want to know a force F, a moment M or the angular acceleration α. In fact, when a force/moment and an angular acceleration are both present in a question, you are bound to be using Newton's second law for rotations.

There is a second way with which you can recognize this type of question. If a moment of inertia I is present and an angular acceleration α. (An exception might occur if it is an angular momentum problem. However, this is rare. Momentum problems are also quite easily recognizable.)

4.3.2 Equations

There is just one very important equation for this subject, namely

$$\sum M_{cog} = I_{cog}\alpha. \tag{4.9}$$

It is advised to apply this equation ONLY about the center of gravity. Otherwise things get complicated.

When solving problems of this type, you also often need to use the equations of rotating bars.

4.3.3 Frequency of occurrence

Always. There is always a question in the dynamics exam where this equation should be applied.

5 Friction, Work & Energy

There are multiple types of energy. These types can be transformed into other types. But the total energy always remains constant. That is a very important law. Let's take a look at what kinds of energies there are.

5.1 Friction

5.1.1 When to use?

When there is a friction force. Another way in which you can easily recognize this kind of problem, is when friction coefficients are given (or asked).

5.1.2 Equations

$$F_w \leq \mu_s N, \tag{5.1}$$

$$F_w = \mu_k N. \tag{5.2}$$

5.1.3 Rules

In problems, an object is often **about to move**. (That's a keyword!) While it is still standing still, use equation (5.1). The frictional force F_w has such a magnitude (and direction) that $\sum \mathbf{F} = m\mathbf{a} = 0$. If F_w can't be that big, the object will move. In that case use equation (5.2). The direction of the friction force F_w is always opposite to the direction of movement.

5.1.4 Frequency of occurrence

Very high. Friction coefficients are present at almost every exam, and often play an important role in problems.

5.2 Energy

5.2.1 When to use?

When certain objects change in height, other objects start moving and/or springs are present. Energy problems are rather characteristic problems. In the equations that correspond to energy are only (angular) velocities V and ω. There are no (angular) accelerations a and α. There usually aren't any forces either (except maybe the frictional force). So when you see that heights and (angular) velocities are given/asked, but nothing is mentioned about (angular) accelerations, you most likely have to use energy.

5.2.2 Equations

Potential energy

The gravitational energy V^g of an object is given by

$$V^g = mgh. \tag{5.3}$$

The elastic energy V^e of an object attached to a spring is given by

$$V^e = \frac{1}{2}k\left(\Delta l\right)^2. \tag{5.4}$$

The spring displacement Δl is given by $l - l_0$, where l is the current length of the spring and l_0 is the unstretched length of the spring.

Kinetic energy

When an object is purely translating ($\omega = 0$), then

$$T = \frac{1}{2}mV^2. \tag{5.5}$$

When an object is purely rotating about a point O, then

$$T = \frac{1}{2}I_O\omega^2. \tag{5.6}$$

In any other case, you have to use

$$T = \frac{1}{2}mV_{cog}^2 + \frac{1}{2}I_{cog}\omega^2. \tag{5.7}$$

Work

The work U caused by some force F is given by

$$U = \int F_s\, ds, \tag{5.8}$$

where F_s is the component of F acting in the direction of movement. If the force acts opposite to the direction of movement, then U is negative.

Often the work caused by friction needs to be calculated. In this case U is always negative.

Energy equation

When applying energy, you virtually always have to use the energy equation, being

$$U_{ext} + E_{initial} = E_{final}. \tag{5.9}$$

Here U_{ext} is the work done by external forces. U_{ext} also contains the frictional work. Do not forget that this frictional work is always negative! By the way, most of the times there are no external forces. In this case $U_{ext} = 0$.

5.2.3 Frequent errors

There are a few things that often go wrong at energy problems. Pay special attention to those things.

- **People mix up the initial and the final situation.** It helps to make two drawings. One for the initial situation and one for the final situation. For every situation, write down the types of energy that are present.
- **People forget types of energy, when using the energy equation.** Just walk through all types of energy. Ask yourself, is there gravitational energy? Is there elastic energy? Is there kinetic energy? Do this for both situations. Also ask yourself if there is any work done by external forces. (Although this is rarely the case.)
- **People take the actual length of the spring while calculating the elastic energy.** Instead, use the difference from the unstretched length.

- **When calculating the kinetic energy, people use the wrong equation.** Ask yourself, is there pure translation? Then use equation (5.5). Is there pure rotation about some point O? Then use equation (5.6). Don't forget to use the moment of inertia with respect to point O! In any other case, use equation (5.7). This equation is always save to use.
- **People forget that work done by friction is negative.** So, remember, work done by friction is always negative!

5.2.4 Frequency of occurrence

Always. There is always a question about energy on the exam.

6 Collisions & Momentum

When objects collide, energy is lost as heat. And we don't know how much heat is created. So we can't apply the conservation of energy law. Instead, we can apply conservation of (angular) momentum. Let's see how we do that.

6.1 Linear momentum

6.1.1 When to use?

When two objects collide in some way. During a collision energy is lost. So you can't apply conservation of energy anymore. Instead, you should apply conservation of momentum. When objects move in straight lines, apply conservation of linear momentum. When objects rotate, apply conservation of angular momentum. (We'll discuss angular momentum later.) In case objects both move and rotate, you might even have to apply both.

Some people often get confused whether to use energy or momentum. There is a nice rule of thumb for that. Just image the situation, as if it is a video clip. If you hear some (knock-like) sound "Tok" in the video clip, then there is some energy lost. You can't use energy anymore, but have to use momentum. (Instead, when everything moves smoothly, and there's no collision sound, then you probably have to use energy.)

6.1.2 Equations

The linear momentum is defined as

$$\mathbf{G} = m\mathbf{V}. \tag{6.1}$$

The rate of change of this linear momentum now is

$$\dot{\mathbf{G}} = m\mathbf{a} = \sum \mathbf{F}. \tag{6.2}$$

The (important) law of conservation of momentum states that

$$\sum \mathbf{G_{initial}} + \sum \mathbf{F_{ext}} t = \mathbf{G_{final}}. \tag{6.3}$$

6.1.3 Frequency of occurrence

Average. Linear momentum doesn't occur very often. But when it occurs, they can ask pretty big questions about it. So be prepared for it.

6.2 Special Collisions

6.2.1 When to use?

When a coefficient of restitution e is given or asked. Sometimes conservation of momentum isn't enough to solve a collision problem. This is where the coefficient of restitution comes in. If it is given or asked, just apply its definition and you'll solve the problem.

Sometimes the coefficient of restitution isn't directly given. Instead, it is given whether the collision is inelastic ($e = 0$) or elastic ($e = 1$). Be aware of that. (By the way, for elastic collisions conservation of energy does apply, but you don't have to use that rule. Just apply the definition of e and you'll be fine.)

6.2.2 Equations

The coefficient of restitution is defined as

$$e = -\frac{V_{A_{final}} - V_{B_{final}}}{V_{A_{initial}} - V_{B_{initial}}}. \tag{6.4}$$

Special care should be payed to the direction. If we define a velocity to the right as positive, then a velocity to the left is negative.

In case an object A (like a ball) collides with an immovable object B (like the ground), then it can be assumed that $V_{B_{initial}} = V_{B_{final}} = 0$.

6.2.3 Frequency of occurrence

Rare. These types of collisions aren't discussed very often.

6.3 Angular momentum

6.3.1 When to use?

When a rotating object collides with something. Once more you can apply the rule of thumb discussed during linear momentum. If you hear a loud "Tok", then you shouldn't use energy. Instead, use (angular) momentum.

6.3.2 Equations

Point-masses

The magnitude of the angular momentum of a point-mass A, with respect to some other point B, is given by

$$H_B = mrV_\theta. \tag{6.5}$$

Here, r is the distance between A and B. Also, V_θ is the component of the velocity perpendicular to the line AB.

Let's suppose the radius r stays constant. In this case the rate of change of the angular momentum is given by

$$\dot{H}_B = mra = \sum M_B. \tag{6.6}$$

Rigid bodies

The angular momentum of a rigid body, about its center of gravity, is given by

$$H_{cog} = I_{cog}\omega. \tag{6.7}$$

If you want to know the angular momentum with respect to some other point B, then use

$$H_B = H_{cog} \pm mrV_{cog/B}. \tag{6.8}$$

Here r is the distance between the center of gravity and point B. Also, $V_{cog/B}$ is the velocity of the center of gravity, with respect to point B. By the way, the $\pm$ sign should be a plus if H_{cog} and $mrV_{cog/B}$ point in the same direction (so both clockwise or both counter-clockwise). In any other case it should be a minus sign.

Law of conservation of momentum

The law of conservation of momentum states that, for some (fixed) point A, we have

$$\sum H_{A_{initial}} + \sum M_A t = \sum H_{A_{final}}. \tag{6.9}$$

Once more, special care should be payed to the direction. If you define clockwise rotations as positive, then counter-clockwise rotations should be considered as negative.

6.3.3 Frequency of occurrence

Average. Just like angular momentum, it doesn't occur often. But when it occurs, a big question can be asked about it.