

Gas Turbines Summary

1. Basics of gas turbines

In this first chapter, we're going to look at the basics of gas turbines. What are they? When were they developed? How do they work? And how do we perform basic calculations?

1.1 Introduction to gas turbines

This is where our journey into the world of gas turbines takes off. And we start with a very important question: what is a gas turbine?

1.1.1 What is a gas turbine?

A **gas turbine** is a machine delivering mechanical power or thrust. It does this using a gaseous working fluid. The mechanical power generated can be used by, for example, an industrial device. The outgoing gaseous fluid can be used to generate thrust.

In the gas turbine, there is a continuous flow of the working fluid. This working fluid is initially compressed in the **compressor**. It is then heated in the **combustion chamber**. Finally, it goes through the **turbine**. The turbine converts the energy of the gas into mechanical work. Part of this work is used to drive the compressor. The remaining part is known as the **net work** of the gas turbine.

1.1.2 History of gas turbines

We can distinguish two important types of gas turbines. There are **industrial gas turbines** and there are **jet engine gas turbines**. Both types of gas turbines have a short but interesting history.

Industrial gas turbines were developed rather slowly. This was because, to use a gas turbine, a high initial compression is necessary. This rather troubled early engineers. Due to this, the first working gas turbine was only made in 1905 by the Frenchman **Rateau**. The first gas turbine for power generation became operational in 1939 in Switzerland. It was developed by the company **Brown Boveri**.

Back then, gas turbines had a rather low thermal efficiency. But they were still useful. This was because they could start up rather quickly. They were therefore used to provide power at peak loads in the electricity network. In the 1980's, natural gas made its breakthrough as fuel. Since then, gas turbines have increased in popularity.

The first time a gas turbine was considered as a jet engine, was in 1929 by the Englishman **Frank Whittle**. However, he had trouble finding funds. The first actual jet aircraft was built by the German **Von Ohain** in 1939. After world war 2, the gas turbine developed rapidly. New high-temperature materials, new cooling techniques and research in aerodynamics strongly improved the efficiency of the jet engine. It therefore soon became the primary choice for many applications.

Currently, there are several companies producing gas turbines. The biggest producer of both industrial gas turbines and jet engines is **General Electric (GE)** from the USA. **Rolls Royce** and **Pratt & Whitney** are also important manufacturers of jet engines.

1.1.3 Gas turbine topics

When designing a gas turbine, you need to be schooled in various topics. To define the compressor and the turbine, you need to use **aerodynamics**. To get an efficient combustion, knowledge on **thermodynamics** is required. And finally, to make sure the engine survives the big temperature differences and high forces, you must be familiar with **material sciences**.

Gas turbines come in various sizes and types. Which kind of gas turbine to use depends on a lot of criteria. These criteria include the required **power output**, the bounds on the **volume** and **weight** of the turbine, the **operating profile**, the **fuel type**, and many more. Industrial gas turbines can deliver a power from $200kW$ to $240MW$. Similarly, jet engines can deliver thrust from $40N$ to $400kN$.

1.2 The ideal gas turbine cycle

To start examining a gas turbine in detail, we make a few simplifications. By doing this, we wind up with the **ideal gas turbine**. How do we analyze such a turbine?

1.2.1 Examining the cycle

Let's examine the thermodynamic process in an ideal gas turbine. The cycle that is present is known as the **Joule-Brayton cycle**. This cycle consists of five important points.

We start at position 1. After the gas has passed through the **inlet**, we are at position 2. The inlet doesn't do much special, so $T_1 = T_2$ and $p_1 = p_2$. (Since the properties in points 1 and 2 are equal, point 1 is usually ignored.) The gas then passes through the **compressor**. We assume that the compression is performed isentropically. So, $s_2 = s_3$. The gas is then heated in the **combustor**. (Point 4.) This is done isobarically (at constant pressure). So, $p_3 = p_4$. Finally, the gas is expanded in the turbine. (Point 5.) This is again done isentropically. So, $s_4 = s_5$. The whole process is visualized in the enthalpy-entropy diagram shown in figure 1.1.

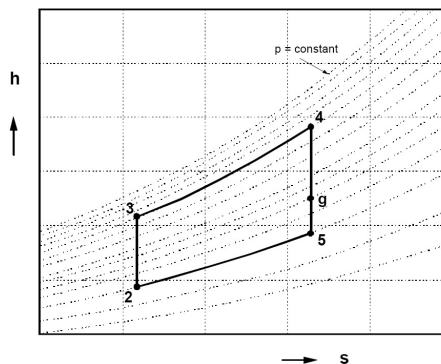


Figure 1.1: The enthalpy-entropy diagram for an ideal cycle.

We can make a distinction between open and closed cycles. In an **open cycle**, atmospheric air is used. The exhaust gas is released back into the atmosphere. This implies that $p_5 = p_1 = p_{atm}$. In a **closed cycle**, the same working fluid is circulated through the engine. After point 5, it passes through a **cooler**, before it again arrives at point 1. Since the cooling is performed isobarically, we again have $p_5 = p_1$.

When examining the gas turbine cycle, we do make a few assumptions. We assume that the working fluid is a **perfect gas** with constant specific heats c_p and c_v . Also, the specific heat ratio k (sometimes also denoted by γ) is constant. We also assume that the kinetic/potential energy of the working fluid

does not vary along the gas turbine. Finally, pressure losses, mechanical losses and other kinds of losses are ignored.

1.2.2 The steps of the cycle

Let's examine the various steps in the gas turbine cycle. During step 2 – 3, the compressor requires a certain **compressor power** \dot{W}_{2-3} . During step 3 – 4, a certain amount of **heat input** \dot{Q}_{3-4} is added. And, during step 5 – 2, a certain amount of **waste heat** \dot{Q}_{5-2} is discarded. These quantities can be found using

$$\dot{W}_{2-3} = \dot{m}c_p(T_3 - T_2), \quad \dot{Q}_{3-4} = \dot{m}c_p(T_4 - T_3) \quad \text{and} \quad \dot{Q}_{5-2} = \dot{m}c_p(T_5 - T_2). \quad (1.2.1)$$

Here, \dot{m} denotes the mass flow through the turbine. Now let's examine step 4 – 5. We can split this step up into two parts. We do this, by creating an imaginary point g between points 4 and 5. This point is such that $\dot{W}_{4-g} = \dot{W}_{2-3}$. In other words, the power produced in step 4 – g generates the power needed by the compressor. (\dot{W}_{4-g} is called the **turbine power**). The remaining power is called the **gas power**, and is denoted by $\dot{W}_{g-5} = \dot{W}_{gg}$. Both \dot{W}_{4-g} and \dot{W}_{g-5} are given by

$$\dot{Q}_{4-g} = \dot{m}c_p(T_4 - T_g) \quad \text{and} \quad \dot{Q}_{g-5} = \dot{m}c_p(T_g - T_5). \quad (1.2.2)$$

1.2.3 Performing calculations

Now let's try to make some calculations. We know that steps 2 – 3 and steps 4 – 5 are performed isentropically. Also, $p_2 = p_5$ and $p_3 = p_4$. This implies that

$$\varepsilon = \frac{p_3}{p_2} = \frac{p_4}{p_5} = \left(\frac{T_3}{T_2}\right)^{\frac{k}{k-1}} = \left(\frac{T_4}{T_5}\right)^{\frac{k}{k-1}}, \quad (1.2.3)$$

where ε is the **pressure ratio**. Now let's find a relation for T_g . Since $\dot{W}_{4-g} = \dot{W}_{2-3}$, we must have

$$T_g = T_4 - T_3 + T_2 = T_4 - T_2 \left(\varepsilon^{\frac{k-1}{k}} - 1\right). \quad (1.2.4)$$

The important **specific gas power** $W_{s,gg}$ is now given by

$$W_{s,gg} = \frac{\dot{W}_{gg}}{\dot{m}} = c_p(T_g - T_5) = c_p T_4 \left(1 - \frac{1}{\varepsilon^{\frac{k-1}{k}}}\right) - c_p T_2 \left(\varepsilon^{\frac{k-1}{k}} - 1\right). \quad (1.2.5)$$

Having a high specific gas power is positive. This is because, to get the same amount of power, we need less mass flow. Our gas turbine can thus be smaller.

1.2.4 The efficiency of the cycle

Based on the equation we just derived, we can find the **thermodynamic efficiency** η_{th} of the gas turbine cycle. This rather important parameter is given by

$$\eta_{th} = \frac{E_{useful}}{E_{in}} = \frac{W_{s,gg}}{Q_{s,3-4}} = \frac{T_g - T_5}{T_4 - T_3} = 1 - \frac{T_2}{T_3} = 1 - \frac{1}{\varepsilon^{\frac{k-1}{k}}}. \quad (1.2.6)$$

So the efficiency of the cycle greatly depends on the pressure ratio ε . To get an efficient cycle, the pressure ratio should be as high as possible.

1.2.5 The optimum pressure ratio

Let's suppose we don't want a maximum efficiency. Instead, we want to maximize the power $W_{s,gg}$. The corresponding pressure ratio ε is called the **optimum pressure ratio** ε_{opt} . To find it, we can set $dW_{s,gg}/d\varepsilon = 0$. From this, we can derive that, at these optimum conditions, we have

$$T_3 = T_5 = \sqrt{T_2 T_4}. \quad (1.2.7)$$

It also follows that the optimum pressure ratio itself is given by

$$\varepsilon_{opt} = \left(\frac{T_3}{T_2}\right)^{\frac{k}{k-1}} = \left(\frac{T_4}{T_2}\right)^{\frac{k}{2(k-1)}}. \quad (1.2.8)$$

The corresponding values of $W_{s,gg}$ and η are given by

$$\frac{W_{s,ggmax}}{c_p T_2} = \left(\sqrt{\frac{T_4}{T_2}} - 1\right)^2 \quad \text{and} \quad \eta = 1 - \sqrt{\frac{T_2}{T_4}}. \quad (1.2.9)$$

Note that we have used a non-dimensional version of the specific gas power $W_{s,gg}$.

1.3 Enhancing the cycle

There are various tricks, which we can use to enhance the gas turbine cycle. We will examine the three most important ones.

1.3.1 Heat exchange

The first enhancement we look at the **heat exchanger** (also known as a **recuperator**). Let's suppose we're applying a heat exchanger. After the gas exits the turbine (point 5), it is brought to this heat exchanger. The heat of the exhaust gas is then used, to warm up the gas entering the combustor (point 3). Let's call the point between the heat exchanger and the combustion chamber point 3.5.

It is important to note that a heat exchanger can only be used if $T_5 > T_3$. (Heat only flows from warmer to colder gasses.) And we only have $T_5 > T_3$ if $\varepsilon < \varepsilon_{opt}$. So a heat exchanger is nice for turbines with low pressure ratios.

Now let's examine the effects of a heat exchanger. The gas entering the combustor (at point 3.5) is already heated up a bit. So the combustor needs to add less heat. The heat input $Q_{s,3-4}$ is thus reduced, increasing the efficiency. In an ideal case, we have $T_{3.5} = T_5$. In this case, the heat input is given by

$$Q_{s,3-4} = c_p (T_4 - T_{3.5}) = c_p (T_4 - T_5) = c_p T_4 \left(1 - \frac{1}{\varepsilon^{\frac{k-1}{k}}}\right). \quad (1.3.1)$$

The thermodynamic efficiency of the cycle is now given by

$$\eta_{th} = \frac{W_{s,gg}}{Q_{s,3-4}} = \frac{c_p T_4 \left(1 - \frac{1}{\varepsilon^{\frac{k-1}{k}}}\right) - c_p T_2 \left(\varepsilon^{\frac{k-1}{k}} - 1\right)}{c_p T_4 \left(1 - \frac{1}{\varepsilon^{\frac{k-1}{k}}}\right)} = 1 - \frac{T_2}{T_4} \varepsilon^{\frac{k-1}{k}}. \quad (1.3.2)$$

In this case, the efficiency increases for decreasing temperature ratios. This is contrary to the case without heat exchange. There is a simple reason for this: The lower the pressure ratio, the more heat can be exchanged, and the more the efficiency can be improved by this heat exchange.

1.3.2 Intercooling

We can also try to increase the specific gas power $W_{s,gg}$. One way to do this, is by making sure the compressor requires less power. This is where the **intercooler** comes in.

Let's suppose we're using an intercooler in the compressor. First, the compressor compresses the gas a bit (point 2.3). The applied pressure ratio is $\varepsilon_1 = p_{2.3}/p_2$. Then, the gas is cooled by the intercooler (point 2.5). (This is usually done such that $T_{2.5} = T_2$. Intercooling is also performed isobarically, so $p_{2.3} = p_{2.5}$.) Finally, the gas is compressed again, until we're at point 3. The applied pressure ratio during this step is $\varepsilon_2 = p_3/p_{2.5}$. The total applied pressure ratio is $\varepsilon_{tot} = \varepsilon_1\varepsilon_2 = p_3/p_2$. The entire process of intercooling has been visualized in figure 1.2 (left).

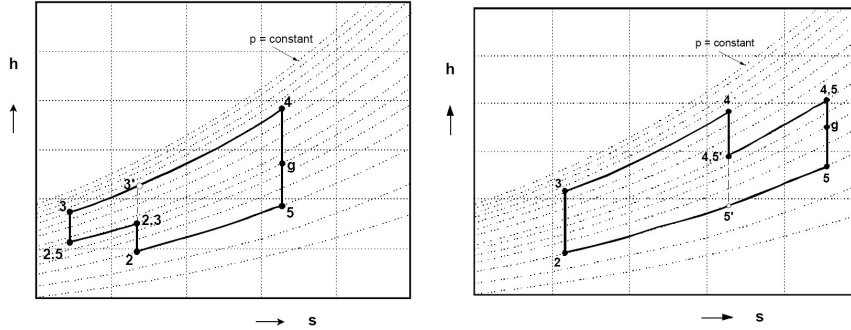


Figure 1.2: The enthalpy-entropy diagram for intercooling (left) and reheating (right).

Thanks to the intercooler, the specific work needed by the compressor has been reduced to

$$W_{s,2-3} = W_{s,2-2.3} + W_{s,2.5-3} = c_p (T_{2.3} - T_2) + c_p (T_3 - T_{2.5}). \quad (1.3.3)$$

The specific gas power $W_{s,gg}$ has increased by the same amount. The increase in specific gas power strongly depends on the pressure ratios ε_1 and ε_2 . It can be shown that, to maximize $W_{s,gg}$, we should have

$$\varepsilon_1 = \varepsilon_2 = \sqrt{\varepsilon_{tot}}. \quad (1.3.4)$$

Intercooling may look promising, but it does have one big disadvantage. The combustor needs to do more work. It also needs to provide the heat that was taken by the intercooler. (This is because T_3 has decreased.) So, while $W_{s,gg}$ increases by a bit, $Q_{s,3-4}$ increases by quite a bit more. Intercooling therefore increases the work output, at the cost of efficiency.

1.3.3 Reheating

An idea similar to intercooling is **reheating**. However, reheating is applied in the turbine. Reheating increases the work done by the turbine. A well-known example of applying reheating is the afterburner in an aircraft.

From point 4, we start by expanding the gas in the turbine. We soon reach point 4.3. (Again, we define $\varepsilon_1 = p_4/p_{4.3}$.) The gas is then reheated, until point 4.5 is reached. (This is done isobarically, so $p_{4.3} = p_{4.5}$.) From this point, the gas is again expanded in a turbine, until points g and 5 are reached. We now have $\varepsilon_2 = p_{4.5}/p_5$ and $\varepsilon_{tot} = \varepsilon_1\varepsilon_2 = p_4/p_5$. The entire process of reheating has been visualized in figure 1.2 (right).

As was said before, reheating increases the work done by the turbine. Let's suppose that $T_4 = T_{4.5}$. In this case, the increase in turbine work is at a maximum if $\varepsilon_1 = \varepsilon_2 = \sqrt{\varepsilon_{tot}}$, just like in the case of the intercooler. Reheating also has the same downside as intercooling. Although the work increases, more heat needs to be added. So the efficiency decreases.

2. Non-ideal gas turbines

Previously, we have considered an ideal gas turbine. But we don't live in an ideal world. So now it's time to get rid of most of the simplifying assumptions.

2.1 Adjustments for the real world

In a non-ideal world, things are often slightly different than in an ideal world. How do we take those differences into account? That's what we'll examine now.

2.1.1 Specific heat

Previously, we have assumed that the specific heat c_p and the specific heat ratio k were constant. However, they are not. They vary because of three reasons. The temperature changes, the pressure changes and the composition of the gas changes. The latter is caused by adding fuel. The change in c_p and k due to pressure changes is usually negligible. However, temperature and composition change do have an important effect.

How do we cope with this? Well, when performing computer calculations, we can simply make several iterations. But, for hand calculations, this is too much work. Instead, we can take mean values for c_p and k , for every step. Often used values are $c_{p,air} = 1000 J/kg K$ and $k_{air} = 1.4$ for the compression stage. Similarly, $c_{p,gas} = 1150 J/kg K$ and $k_{gas} = 1.33$ for the expansion stage in the turbine. By doing this, our calculations are quite accurate. But deviations from the real world of up to 5% may still occur.

2.1.2 Kinetic energy

Previously, we have neglected the kinetic energy of the gas. But the gas often has a non-negligible velocity c . To solve this problem, we use a nice trick. We define the **total enthalpy** h_0 , the **total temperature** T_0 and the **total pressure** p_0 . The total enthalpy is defined as

$$h_0 = h + \frac{1}{2}c^2, \quad (2.1.1)$$

where h is the static enthalpy. With static enthalpy, we mean the enthalpy when not taking into account the velocity. We can derive the total temperature, by using

$$h_0 = c_p T_0 = c_p T + \frac{1}{2}c^2, \quad \text{which gives} \quad T_0 = T + \frac{c^2}{2c_p}. \quad (2.1.2)$$

We can find the total pressure p_0 from the isentropic flow relations. The result will be

$$p_0 = p \left(\frac{T_0}{T} \right)^{\frac{k}{k-1}}. \quad (2.1.3)$$

When using these total values, we don't have to take into account the kinetic energy anymore. That makes life just a bit easier.

2.2 The isentropic and the polytropic efficiency

There are two very important parameters, that strongly influence the gas turbine properties. They are the isentropic and the polytropic efficiency. Let's examine them.

2.2.1 Isentropic efficiency

Let's examine the compressor and the turbine. In reality, they don't perform their work isentropically. To see what does happen, we examine the compressor. In the compressor, the gas is compressed. In an ideal (isentropic) case, the enthalpy would rise from h_{02} to h_{03s} . However, in reality, it rises from h_{02} to h_{03} , which is a bigger increase. Similarly, in an ideal (isentropic) turbine, the enthalpy would decrease from h_{04} to h_{0g} . However, in reality, it decreases from h_{04} to h_{0g} , which is a smaller decrease. Both these changes are visualized in figure 2.1.

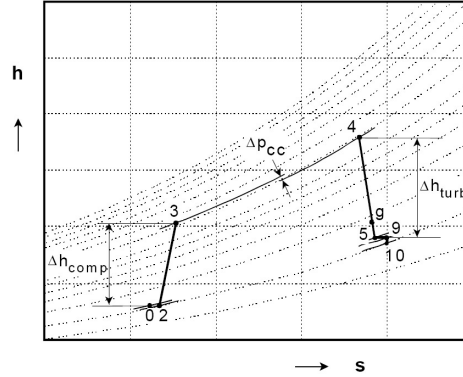


Figure 2.1: The enthalpy-entropy diagram for a non-ideal cycle.

This effect can be expressed in the **isentropic efficiency**. The efficiencies for compression and expansion are, respectively, given by

$$\eta_{is,c} = \frac{h_{03s} - h_{02}}{h_{03} - h_{02}} = \frac{T_{03s} - T_{02}}{T_{03} - T_{02}} \quad \text{and} \quad \eta_{is,t} = \frac{h_{04} - h_{0g}}{h_{04} - h_{0gs}} = \frac{T_{04s} - T_{0g}}{T_{04} - T_{0gs}}. \quad (2.2.1)$$

You may have trouble remembering which difference goes on top of the fraction, and which one goes below. In that case, just remember that we always have $\eta_{is} \leq 1$.

By using the isentropic relations, we can rewrite the above equations. We then find that

$$\eta_{is,c} = \frac{\left(\frac{p_{03}}{p_{02}}\right)^{\frac{k_{air}-1}{k_{air}}} - 1}{\frac{T_{03}}{T_{02}-1}} \quad \text{and} \quad \eta_{is,t} = \frac{\frac{T_{0g}}{T_{04}-1}}{\left(\frac{p_{0g}}{p_{04}}\right)^{\frac{k_{gas}-1}{k_{gas}}} - 1}. \quad (2.2.2)$$

The specific work received by the compressor, and delivered by the turbine, is now given by

$$\dot{W}_{s,c} = \frac{c_{p_{air}} T_{02}}{\eta_{is,c}} \left(\left(\frac{p_{03}}{p_{02}}\right)^{\frac{k_{air}-1}{k_{air}}} - 1 \right) \quad \text{and} \quad \dot{W}_{s,t} = c_{p_{gas}} T_{04} \eta_{is,t} \left(\left(\frac{p_{0g}}{p_{04}}\right)^{\frac{k_{gas}-1}{k_{gas}}} - 1 \right). \quad (2.2.3)$$

Note that, through the definition of p_{0g} , these two quantities must be equal to each other. (That is, as long as the mass flow doesn't change, and there are no additional losses when transmitting the work.)

2.2.2 Polytropic efficiency

Let's examine a compressor with a varying pressure ratio. In this case, it turns out that also the isentropic efficiency varies. That makes it difficult to work with.

To solve this problem, we divide the compression into an infinite number of small steps. All these infinitely small steps have the same isentropic efficiency. This efficiency is known as the **polytropic efficiency**.

The resulting process is also known as a **polytropic** process. This means that there is a **polytropic exponent** n , satisfying

$$\frac{T_0}{T_{0_{initial}}} = \left(\frac{p_0}{p_{0_{initial}}} \right)^{\frac{n_{air}}{n_{air}-1}}. \quad (2.2.4)$$

The polytropic efficiencies for compression $\eta_{\infty c}$ and expansion $\eta_{\infty t}$ are now given by, respectively,

$$\eta_{\infty c} = \frac{k_{air} - 1}{k_{air}} \frac{n_{air}}{n_{air} - 1} = \frac{\ln \left(\frac{p_{03}}{p_{02}} \right)^{\frac{k_{air}-1}{k_{air}}}}{\ln \left(\frac{T_{03}}{T_{02}} \right)} \quad \text{and} \quad \eta_{\infty t} = \frac{k_{gas} - 1}{k_{gas}} \frac{n_{gas}}{n_{gas} - 1} = \frac{\ln \left(\frac{T_{0g}}{T_{04}} \right)}{\ln \left(\frac{p_{0g}}{p_{04}} \right)^{\frac{k_{gas}-1}{k_{gas}}}}. \quad (2.2.5)$$

The full compression/expansion process also has an isentropic efficiency. It is different from the polytropic efficiency. In fact, the relation between the two is given by

$$\eta_{is,c} = \frac{\left(\frac{p_{03}}{p_{02}} \right)^{\frac{k_{air}-1}{k_{air}}} - 1}{\left(\frac{p_{03}}{p_{02}} \right)^{\frac{k_{air}-1}{\eta_{\infty c} k_{air}}} - 1} \quad \text{and} \quad \eta_{is,t} = \frac{\left(\frac{p_{0g}}{p_{04}} \right)^{\eta_{\infty t} \frac{k_{gas}-1}{k_{gas}}} - 1}{\left(\frac{p_{0g}}{p_{04}} \right)^{\frac{k_{gas}-1}{k_{gas}}} - 1}. \quad (2.2.6)$$

There are a few important rules to remember. For compression, the polytropic efficiency is higher than the isentropic efficiency. (So, $\eta_{\infty c} > \eta_{is,c}$.) For expansion, the polytropic efficiency is lower than the isentropic efficiency. (So, $\eta_{\infty t} < \eta_{is,t}$.) Finally, if the pressure ratio increases, then the difference between the two efficiencies increases.

2.3 Losses occurring in the gas turbine

In a non-ideal world, losses occur at several places in the gas turbine. There are also several types of losses. We will examine a few.

2.3.1 Pressure losses

Previously, we have assumed that no pressure losses occurred. This is, of course, not true. Pressure losses occur at several places. First of all, in the combustor. The **combustion chamber pressure loss** is given by $\Delta p_{cc} = p_{03} - p_{04}$. The **combustor pressure loss factor** is now defined as

$$\varepsilon_{cc} = \frac{p_{04}}{p_{03}} = \frac{p_{03} - \Delta p_{cc}}{p_{03}}. \quad (2.3.1)$$

Pressure losses also occur at the inlet and at the exhaust duct. For industrial gas turbines, these pressure losses are defined as

$$\Delta p_{0_{inlet}} = p_{amb} - p_{01} \quad \text{and} \quad \Delta p_{0_{exhaust}} = p_{05} - p_{amb}. \quad (2.3.2)$$

We will examine these pressure differences for jet engines in a later chapter.

2.3.2 Mechanical losses

Losses also occur due to internal friction in the system. These **mechanical losses** are joined together in one term, being the **transmission efficiency** η_m . It is given by

$$\eta_m = \frac{\text{turbine power} - \text{mechanical losses}}{\text{turbine power}}. \quad (2.3.3)$$

2.3.3 Combustor efficiency

Ideally, we will have a **full combustion** of the fuel in the combustion chamber. In this case, we would get the maximum heat out of it. This maximum heat is called the **lower heating value LHV** of the fuel, also known as the **lower calorific value LCV**. However, in reality, we have an **incomplete combustion**. This results in combustion products like carbon monoxide (*CO*) and unburned fuel.

Next to this, heat may also escape. To take this into account, the **combustor efficiency** η_{cc} is used. It is defined as

$$\eta_{cc} = \frac{\dot{m}_{air} c_{p_{gas}} (T_{04} - T_{03})}{\dot{m}_{fuel} LHV}. \quad (2.3.4)$$

2.3.4 Heat exchange

Let's reconsider the heat exchanger. In the previous chapter, we have assumed that, after heat exchange, we had $T_{03.5} = T_{05}$. In other words, the heat of the gas entering the combustor equals the heat of the gas leaving the turbine. In reality, this is of course not the case. We thus have $T_{03.5} < T_{05}$.

To take this effect into account, we use the **heat exchanger effectiveness** E . First, we define the coefficients C_{cold} and C_{hot} as

$$C_{cold} = c_{p_{in,cold}} \dot{m}_{in,cold} \quad \text{and} \quad C_{hot} = c_{p_{in,hot}} \dot{m}_{in,hot}. \quad (2.3.5)$$

The subscript cold stands for the cold flow: the flow leaving the compressor. Similarly, hot stands for the flow leaving the turbine. We also define C_{min} as the lowest of the coefficients C_{cold} and C_{hot} . Now, the heat exchanger effectiveness E is given by

$$E = \frac{C_{cold} \frac{T_{0_{out,cold}} - T_{0_{in,cold}}}{C_{min}}}{\frac{T_{0_{in,hot}} - T_{0_{out,cold}}}{C_{min}}} = \frac{C_{hot} \frac{T_{0_{in,hot}} - T_{0_{out,hot}}}{C_{min}}}{\frac{T_{0_{out,hot}} - T_{0_{in,cold}}}{C_{min}}}. \quad (2.3.6)$$

The term in the top of the fraction indicates the amount of heat exchanged. The term in the bottom is a measure of the amount heat that can be exchanged.

We can try to simplify the above relation. If the mass flow and the specific heat are constant ($\dot{m}_{cold} = \dot{m}_{hot}$ and $c_{p_{cold}} = c_{p_{hot}}$), then the effectiveness is given by

$$E = \frac{T_{03.5} - T_{03}}{T_{05} - T_{03}}. \quad (2.3.7)$$

3. Gas turbine types

Gas turbines can be used for many purposes. They can be used to deliver power, heat or thrust. Therefore, different gas turbine types exist. In this chapter, we will examine a few. First, we examine shaft power gas turbines. Then, we examine jet engine gas turbines.

3.1 Shaft power gas turbines

A **shaft power gas turbine** is a gas turbine whose goal is mainly to deliver shaft power. They are often also referred to as **turboshaft engines**. These gas turbines are often used in industrial applications. Gas turbines used for electricity production are also of this type.

3.1.1 The shaft power

For shaft power gas turbines, the shaft power is very important. It mainly depends on the temperature drop in the **power turbine** (PT). This drop is given by

$$T_{0g} - T_{05} = T_{0g} \eta_{is,PT} \left(1 - \left(\frac{p_{05}}{p_{0g}} \right)^{\frac{k_{gas}-1}{k_{gas}}} \right) = T_{0g} \left(1 - \left(\frac{p_{05}}{p_{0g}} \right)^{\eta_{\infty,PT} \frac{k_{gas}-1}{k_{gas}}} \right). \quad (3.1.1)$$

The efficiencies $\eta_{is,PT}$ and $\eta_{\infty,PT}$ are the isentropic and polytropic efficiencies, respectively, of the power turbine. To apply the above equation, we do need to know the properties of point 5. These properties can be derived from the exhaust properties, according to

$$\Delta p_{exh} = p_{05} - p_{exh} \quad \text{and} \quad T_{05} = T_{0,exh} = T_{exh} + \frac{c_{exh}^2}{2c_{p,gas}}, \quad (3.1.2)$$

where c_{exh} is the velocity of the exhaust gas. (By the way, the exhaust point is often also denoted by point 9.) Once this data is known, the actual shaft power can be derived. For this, we use the equation

$$P_{shaft} = \dot{m} c_{p,gas} (T_{0g} - T_{05}) \eta_{m,PT}, \quad (3.1.3)$$

with $\eta_{m,PT}$ being the mechanical efficiency of the gas turbine.

3.1.2 Other performance parameters

There are also some other parameters that are important for shaft power gas turbines. Of course, the **thermal efficiency** $\eta_{thermal}$ is important. (The thermal efficiency is not the thermodynamic efficiency η_{th} , introduced in chapter 1. In fact, it is lower. This is because the thermal efficiency also takes into account various types of losses.) This efficiency is given by

$$\eta_{thermal} = \frac{P_{shaft}}{\dot{m}_{fuel} H_{fuel}}, \quad (3.1.4)$$

where H_{fuel} is the **heating value** of the fuel. Other important parameters are the **specific fuel consumption** sfc and the **heat rate**. The sfc is given by

$$sfc = \frac{\dot{m}_{fuel}}{P_{shaft}} = \frac{1}{H_{fuel} \eta_{thermal}}. \quad (3.1.5)$$

The heat rate is given by

$$\text{heat rate} = \frac{\dot{m}_{fuel} H_{fuel}}{P_{shaft}} = \frac{1}{\eta_{thermal}}. \quad (3.1.6)$$

Finally, there is the **equivalence ratio** λ , also known as the **percentage excess air**. It is defined as

$$\lambda = \frac{\dot{m}_{air} - \dot{m}_{st}}{\dot{m}_{st}}, \quad (3.1.7)$$

where \dot{m}_{st} is the air mass flow required for a complete combustion of the fuel. If there is just enough air to burn the fuel, then we have a **stoichiometric combustion**. In this case, $\lambda = 1$. However, usually λ is bigger than 1.

3.1.3 Exhaust gas of the shaft power turbine

Shaft power gas turbines usually produce quite some heat. This heat can be used. When applying **cogeneration**, we use the heat of the exhaust gas itself. (For example, to produce hot water or steam.) In a **combined cycle** the heat is used to create additional power. This can be done by expanding the heat in a steam turbine.

The process of creating steam deserves some attention. In this process, heat is exchanged from the exhaust gas to the water/vapor. This is caused by the temperature difference between the exhaust gas and the water/vapor. An important parameter is the temperature difference ΔT_{pinch} at the so-called **pinch point**. This is the point where the water just starts to boil. The entire process, including the pinch point, is also visualized in figure 3.1.

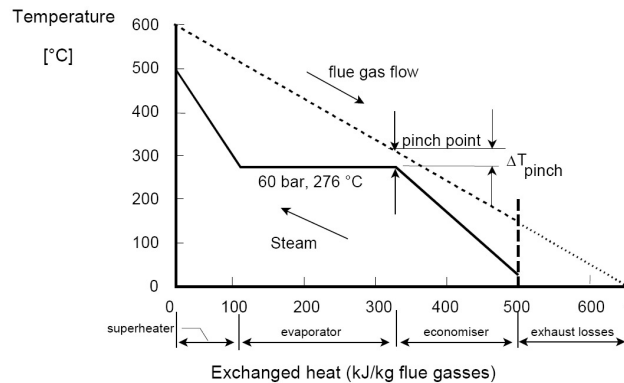


Figure 3.1: Exchanged heat versus temperature diagram for creating steam, using exhaust gas.

We want to minimize the exhaust losses. One way to do that, is to increase the initial exhaust gas temperature. This makes the heat exchange a lot easier. Another option is to split up the process into multiple steps, where each step has a different pressure. In this case, there will also be multiple pinch points.

3.2 Jet engine gas turbines

A **jet engine gas turbine** is a turbine whose goal is mainly to deliver **thrust**. This can be done in two ways. We can let the gas turbine accelerate air. We can also let the gas turbine shaft power a propeller. Often, both methods are used. Jet engine gas turbines are mainly used on aircraft.

3.2.1 Finding the thrust

The goal of a jet engine is to produce thrust. The **net thrust** F_N can generally be found using

$$F_N = \dot{m} (c_j - c_0). \quad (3.2.1)$$

c_0 indicates the air velocity before the air entered the gas turbine. c_j indicates the air velocity after it entered the turbine. $(c_j - c_0)$ indicates the acceleration of the flow.

We can use the above equation to find the thrust of an actual aircraft. But to do that, we have to define some control points. Point 0 is infinitely far upstream, point 1 is at the inlet entry, point 8 is at the exhaust exit and point ∞ is infinitely far downstream. To calculate the thrust, we have to use points 0 and ∞ . The velocity c_0 at point zero is simply equal to the airspeed of the aircraft. However, c_∞ is very hard to determine. Luckily, we can use the momentum relation, which states that

$$\dot{m}(c_\infty - c_8) = A(p_8 - p_\infty) = A(p_8 - p_0), \quad (3.2.2)$$

where we have used that $p_0 = p_\infty = p_{atm}$. If we use this relation, then the net thrust becomes

$$F_n = \dot{m}(c_\infty - c_0) = \dot{m}(c_8 - c_0) + A(p_8 - p_0). \quad (3.2.3)$$

The parameter A is the area of the gas turbine at the exhaust. Once the thrust has been found, we can derive the **effective jet velocity** c_{eff} . It is the velocity that is (theoretically) obtained at point ∞ , such that the thrust F_n is achieved. It satisfies

$$F_n = \dot{m}(c_{eff} - c_0). \quad (3.2.4)$$

3.2.2 Jet engine power

From the net thrust, we can find the **thrust power**. It is defined as

$$P_{thrust} = F_n c_0 = \dot{m} c_0 (c_{eff} - c_0). \quad (3.2.5)$$

This is the ‘useful’ power of the aircraft. We can also look at the change in kinetic energy of the air passing through our jet engine. If we do that, then we find the **propulsion power** P_{prop} . This can be seen as the ‘input’ power. It is defined as

$$P_{prop} = \frac{1}{2} \dot{m} (c_{eff}^2 - c_0^2). \quad (3.2.6)$$

The difference between the propulsion power and the thrust power is known as the **loss power**. It is defined as

$$P_{loss} = P_{prop} - P_{thrust} = \frac{1}{2} \dot{m} (c_{eff} - c_0)^2. \quad (3.2.7)$$

3.2.3 Efficiencies

From the propulsion power and the thrust power, we can derive the **propulsive efficiency** η_{prop} , also known as the **Froude efficiency**. It is given by

$$\eta_{prop} = \frac{P_{thrust}}{P_{prop}} = \frac{2}{1 + \frac{c_{eff}}{c_0}}. \quad (3.2.8)$$

To increase the propulsive efficiency, we should keep c_{eff} as low as possible. So it’s better to accelerate a lot of air by a small velocity increment, than a bit of air by a big velocity increment. Now let’s examine the **thermal efficiency** $\eta_{thermal}$ of the jet engine. It is given by

$$\eta_{thermal} = \frac{P_{prop}}{\dot{m}_{fuel} H_{fuel}}. \quad (3.2.9)$$

It is a measure of how well the available thermal energy has been used to accelerate air. We can also look at the **jet generation efficiency** η_{jet} . It is defined as

$$\eta_{jet} = \frac{P_{prop}}{P_{gg}}. \quad (3.2.10)$$

This is a measure of how well the energy from the power turbine has been used to accelerate air. And now, we can finally define the **total efficiency**. It is given by

$$\eta_{total} = \frac{P_{thrust}}{\dot{m}_{fuel} H_{fuel}} = \eta_{prop} \eta_{thermal}. \quad (3.2.11)$$

3.2.4 Other important parameters

Next to the efficiencies, there are also quite some other parameters that say something about jet engines. First, there is the **specific thrust** F_s . It is defined as the ratio between the net thrust and the air intake. So,

$$F_s = \frac{F_N}{\dot{m}}. \quad (3.2.12)$$

Second, we have the **thrust specific fuel consumption** $TSFC$, which is the ratio between the fuel flow and the thrust. We thus have

$$TSFC = \frac{\dot{m}_{fuel}}{F_N} = \frac{c_0}{\eta_{total} H_{fuel}}. \quad (3.2.13)$$

3.2.5 Improving the jet engine

We have previously noted that it's best to give a small velocity increment to a large amount of air. However, jet engines themselves usually give quite a big acceleration to the air. For this reason, most commercial aircraft engines have big propellers, called **turbofans**. These fans are driven by the shaft of the jet engine. And they give, as is required, a small velocity increment to a large amount of air. This air then **bypasses** (flows around) the jet engine itself.

The relation for the thrust of a turbofan engine is similar to that of a normal jet engine. The only difference, is that we need to add things up. So,

$$F_N = \dot{m}_{jet} (c_8 - c_0) + A_{jet} (p_8 - p_0) + \dot{m}_{fan} (c_8 - c_0) + A_{fan} (p_8 - p_0). \quad (3.2.14)$$

The rest of the equations also change in a similar way.

4. Combustion

The combustion chamber is the part where energy is inserted into the gas turbine. In this chapter, we're going to examine it in detail.

4.1 The combustion process

First, we will look at the combustion process. What reaction is taking place? And what parameters influence this reaction?

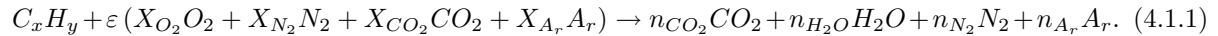
4.1.1 The combustion process

The combustor provides the energy input for the gas turbine cycle. It receives air, inserts fuel, mixes the two components and then it lets the mixture combust. This process is known as **internal combustion**. In a gas turbine, it is generally done at constant pressure. (Although small pressure losses are generally present.)

An important parameter during combustion is the T_{04} temperature. It effects the power output and the thermal efficiency. T_{04} is generally limited by material properties. The materials must be able to withstand large temperatures and temperature gradients. If not, the gas turbine might fail.

4.1.2 The reaction

Gas turbines mainly use kerosene-type fuels. These fuels consist of **hydrocarbons** with the chemical composition C_xH_y . They are mixed with air and then combusted. The **ideal reaction** is then given by



In this equation, the X 's indicate the composition of air. We thus have

$$X_{O_2} = 0.2095, \quad X_{N_2} = 0.7808, \quad X_{CO_2} = 0.0003 \quad \text{and} \quad X_{A_r} = 0.0094. \quad (4.1.2)$$

The term ε is the number of moles of air necessary for every mole of fuel. It is given by

$$\varepsilon = \frac{x + \frac{1}{4}y}{X_{O_2}}. \quad (4.1.3)$$

Finally, the n 's denote the amount of **reaction products**. We find that we have

$$n_{CO_2} = x + \varepsilon X_{CO_2}, \quad n_{H_2O} = \frac{1}{2}y, \quad n_{N_2} = \varepsilon X_{N_2} \quad \text{and} \quad n_{A_r} = \varepsilon X_{A_r}. \quad (4.1.4)$$

In reality, we don't have this ideal reaction. In the real world, not all fuel gets combusted. Not all carbon atoms form carbon dioxide CO_2 . (We will also have carbon monoxide CO .) And there will be various other reaction products as well. We won't examine all those reaction products though.

4.1.3 Reaction parameters

When combusting the mixture, the **fuel-to-air ratio** $FAR = \dot{m}_{fuel}/\dot{m}_{air}$ is an important parameter. We know that, to combust 1 mole of fuel, we need ε moles of air. In stoichiometric conditions, there is precisely enough air to combust all fuel. The stoichiometric fuel-to-air ratio is therefore given by

$$FAR_{st} = \left(\frac{\dot{m}_{fuel}}{\dot{m}_{air}} \right)_{st} = \frac{1}{\varepsilon} \frac{M_{C_xH_y}}{M_{air}} = \frac{X_{O_2}}{x + \frac{1}{4}y} \frac{M_{C_xH_y}}{M_{air}}. \quad (4.1.5)$$

The symbols $M_{C_xH_y}$ and M_{air} denote the molar mass of the fuel and the air, respectively. Similar to the FAR , we also have the **air-to-fuel ratio** AFR . This is simply given by $AFR = 1/FAR$. Since the value of the FAR is usually quite small, the AFR might be easier to work with.

Of course, in the real world, the reaction isn't stoichiometric. In fact, we define the **fuel-to-air equivalence ratio** φ as

$$\varphi = \frac{FAR}{FAR_{st}} = \frac{AFR_{st}}{AFR} = \frac{\dot{m}_{fuel}}{\dot{m}_{air}} \frac{1}{FAR_{st}}. \quad (4.1.6)$$

If $\varphi = 1$, then our combustion is performed at stoichiometric conditions. If we have more fuel, then $\varphi > 1$ and we have a **fuel-rich mixture**. If we have less fuel, then $\varphi < 1$ and we have a **fuel-lean mixture**. From φ , we can also find the **percentage stoichiometric air**, defined as

$$\text{percentage stoichiometric air} = \frac{100\%}{\varphi}. \quad (4.1.7)$$

We can also use φ to find the percentage excess air λ . We do this according to

$$\lambda = \frac{1 - \varphi}{\varphi}. \quad (4.1.8)$$

If λ is positive, then there is plenty of air: we have a fuel-lean mixture.

4.1.4 The heat formed during combustion

We will now examine the heat produced during the combustion. To do this, we will use a concept called **absolute enthalpy**. The absolute enthalpy doesn't only take into account the temperature of a substance. Also the enthalpy of formation plays a role.

Now let's examine a combustion chamber operating at a constant temperature. So-called **reactants**, with absolute enthalpy h_{react} , enter the chamber. Reaction **products**, with enthalpy h_{prod} , leave the chamber. The heat Δh_c leaving the chamber (called the **enthalpy of combustion**) is now given by

$$\Delta h_c = h_{react} - h_{prod}. \quad (4.1.9)$$

The unit of Δh_c is Joule per mole of mixture. Usually, we want to know the lower heating value LHV of the fuel. This is the heat produced per kilogram of fuel. To find it, we can use

$$LHV = \frac{\Delta h_c}{M_{mix}} \frac{\dot{m}_{mix}}{\dot{m}_{fuel}}, \quad (4.1.10)$$

where M_{mix} is the molar mass of the mixture. We can apply these ideas to the combustion process. If we then also make a few simplifying assumptions, then we wind up with

$$\dot{m}_{fuel} \eta_c LHV = \dot{m}_{air} c_{p,gas} (T_{04} - T_{03}). \quad (4.1.11)$$

The left part of the equation indicates the effective heat input. The right part indicates the result of this heat input. The parameter η_c is known as the **combustion efficiency**. We can also rewrite the above equation, by making use of the fuel-to-air equivalence ratio. We then get

$$T_{04} = T_{03} + \frac{\varphi FAR_{st}}{c_{p,gas}} \eta_c LHV. \quad (4.1.12)$$

Due to all the approximations, this equation is not very accurate for higher temperatures and equivalence ratios.

Finally, there is the concept of the adiabatic flame temperature. To introduce it, we suppose that we burn the mixture in a closed adiabatic chamber. In this case, no heat leaves the chamber. The absolute enthalpy thus stays the same ($\Delta h_c = 0$). However, the temperature T does rise. In fact, it rises from the initial reactant temperature to the **adiabatic flame temperature** T_{ad} . So, we find that T_{ad} is defined such that

$$h_{prod}(T_{ad}) = h_{react}(T_{react}). \quad (4.1.13)$$

4.2 The layout of the combustion chamber

Designing a combustion chamber is not an easy thing. There are several complicated parts in it. So let's examine how a combustion chamber is build up.

4.2.1 The general layout

There are several types of combustion chambers. But virtually all combustion chambers have a diffuser, a casing, a liner, a fuel injector and a cooling arrangement. The entire general layout is visualized in figure 4.1.

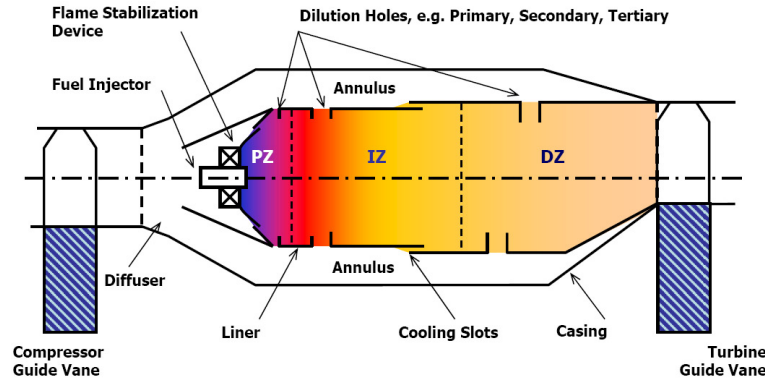


Figure 4.1: The layout of the combustion chamber.

4.2.2 The diffuser

The gas entering the combustion chamber usually has quite a high velocity. This velocity will be responsible for a pressure drop. (This pressure drop is called the **cold loss**.) Also, the flame in the combustion chamber can not survive if the air has a high velocity. Therefore, the airflow needs to be slowed down. And this is exactly the task of the **diffuser**.

Diffusers can be distinguished by how quickly they decrease the velocity. In the **aerodynamic diffuser**, the flow is slowed down gradually, while in the **dump diffuser**, it is slowed down quickly. The dump diffuser has more losses, but is also smaller. It is therefore mainly used in aircraft jet engines.

4.2.3 The casing and the liner

After the airflow has passed the diffuser, it is split up by the **liner**. One part of the airflow goes through the region between the liner and the casing. This region is called the **annulus**. Another part of the airflow enters the mixing chamber, where fuel is injected.

There are several reasons for splitting up the flow. First, the air-to-fuel should have the right value. If it is too high, the mixture will not ignite. Also, the velocity of the flow leaving the diffuser is still too high. The part of the flow that will be ignited has to be slowed down even further.

The liner is divided into three sections. There is a **primary zone (PZ)**, a **secondary/intermediate zone (SZ/IZ)** and a **tertiary/dilution zone (TZ/DZ)**. The main function of the PZ is to provide enough time for the fuel to mix and combust. The goal of the SZ is to provide enough time to achieve full combustion. This significantly reduces bad reaction products like **carbon monoxide CO** and **unburned**

hydrocarbons (UHC). Finally, the goal of the DZ is to reduce the temperature of the outlet stream, such that it is acceptable for the turbine.

4.2.4 The fuel injector

The **fuel injector** injects the fuel into the flow. It is important that the fuel is vaporized before it enters the flame zone. Otherwise, it might not combust properly.

To promote vaporization, the fuel should be **atomized**. This means that the fuel is converted into small drops. This increases vaporization rates. To accomplish this, an **atomizer** is used. To atomize fuel, it has to be given a high relative velocity, with respect to the airflow. So-called **pressure-assist atomizers** give the fuel a high velocity. On the other hand, **air blast atomizers** inject slow-moving fuel into a high-velocity air stream.

4.2.5 Flame stabilization

After the fuel has been injected into the flow, the flow will enter the flame region. It does this with quite a high velocity. To make sure the flame isn't blown away, **flow reversal** can be applied in the PZ. This causes the flow to reverse direction. The best way to reverse the flow, is to **swirl** it. This is done using **swirlers**. The two most important types of swirlers are **axial swirlers** and **radial swirlers**.

The most important advantage of flow reversal, is that the flow speed varies a lot. So there will be a point at which the airflow velocity matches the **flame speed**. (The flame speed is the speed, relative to the airflow, at which the flame can move.) This is the point where the flame **anchors**.

4.2.6 Cooling

The liner is exposed to high temperatures, during combustion. It therefore needs to be cooled, using the airflow in the annulus. There are several ways to do this.

In **film cooling**, stacks of holes are put in the liner. These holes inject air along the inner surface of the liner, providing a protective cooling film. A downside is that the liner is not cooled evenly. The best way to cool the liner evenly, is by using **transpiration cooling**. Now, the liner wall is constructed from a porous material that allows air to pass through it.

Next to these cooling methods, we can also put **metallic tiles** onto the inner surface of the liner. This provides some protection. Finally, the airflow passing through the annulus will also automatically provide some **convection cooling**.

4.2.7 Combustor types

We can distinguish combustors by their shape. **Can-type combustors** (also known as **tubular combustors**) consist of several cylindrical tubes, placed around the turbine shaft. Each tube has both a liner and a casing. Although these types of turbines are easy to develop, their weight is relatively high.

Another type of combustor is the **annular combustor**. The annular combustor has a single ring-shaped liner mounted inside a single ring-shaped casing. Due to the low weight, this type is mostly used in aircraft jet engines. A downside is that it's sensitive to buckling loads.

Finally, we can combine the can-type and the annular combustor. We then get a **can-annular combustor**. We now have several cylindrical liners (like in the tubular combustor) placed in a single ring-shaped casing (like in an annular combustor).

4.3 Unwanted phenomena

Gas turbines don't just deliver power, without any side-effects. We will now look at the bad sides of the combustion chamber.

4.3.1 Combustion stability

An important property of a combustion chamber is **combustion stability**. To have combustion stability, the flame must remain stable at varying fuel mixtures, inlet temperatures, turbulence levels, flow speeds and so on. If it doesn't, things can go wrong.

To simplify the idea of combustion stability, we usually only consider the mixture. The combustion stability now depends on the range of the *FAR* at which the flame remains stable. If the flame dies due to too much fuel, we have **rich extinction**. Similarly, if there is too little fuel, we have **weak extinction**.

The two limits mainly depend on the mass flow of air \dot{m}_{air} passing through the combustion chamber. Flames have trouble surviving at high flow velocities. And a high flow velocity is, of course, linked to a high mass flow. So too high flow velocities/mass flows aren't good. On the other hand, if the flow velocity is too low, the flame will move upstream. This is called **flashback**. It's not very good either. So, we don't want a low flow velocity either.

4.3.2 Losses

Several losses occur in the combustion chamber. First, there is a heat loss. This has several causes. Heat is being spent on heating up/vaporizing the fuel, and on heating the combustor itself. Another cause is **incomplete combustion**. If part of the fuel does not combust, then heat is wasted. To take this into account, we define the **factor of complete combustion** $\eta_{complete}$ as

$$\eta_{complete} = \frac{\dot{m}_f^{reacted}}{\dot{m}_f^{injected}} = \frac{\dot{m}_f^{evaporated}}{\dot{m}_f^{injected}} \frac{\dot{m}_f^{mixed}}{\dot{m}_f^{evaporated}} \frac{\dot{m}_f^{reacted}}{\dot{m}_f^{mixed}}. \quad (4.3.1)$$

Luckily, we have $\eta_{complete} \approx 100\%$ at normal operating conditions.

Next to heat losses, there are also pressure losses. The so-called **cold losses** are caused by skin friction and turbulence effects. These are, in turn, caused by the diffuser, the swirler and the liner. So, we have

$$(\Delta p_{03-04})_{cold} = (\Delta p)_{diffuser} + (\Delta p)_{swirler} + (\Delta p)_{liner}. \quad (4.3.2)$$

The **hot losses** are caused by changes in momentum of the flow. They are caused by temperature changes. An estimation of the hot pressure losses is given by

$$(\Delta p_{03-04})_{hot} = \frac{1}{2} \rho_3 V_3 \left(\frac{T_4}{T_3} - 1 \right), \quad \text{where} \quad V_3 = \frac{\dot{m}_3}{\rho_3} A_{casing}. \quad (4.3.3)$$

4.3.3 Pollutants

Hydrocarbon-fueled gas turbines usually have several unwanted combustion products. The most important combustion products are H_2O , CO_2 , O_2 and N_2 . These are the so-called **products of complete combustion**, and make up 99% of the combustion products.

The remainder of the combustion products can be split up into two groups. The group of **gaseous pollutants** consists of **nitrogen oxides** NO_x , **carbon monoxide** CO and a variety of **unburned**

hydrocarbons UHCs. The amount of gaseous pollutants is usually given by the **emission index** (EI). This is defined as

$$EI = \frac{\text{mass of the produced pollutant in } g}{\text{mass of fuel used in } kg}. \quad (4.3.4)$$

The second group of remaining combustion products is called **smoke**. It mainly consists of **soot particles**, which are particles with a high amount of carbon in them.

5. Compressor and turbines

In this chapter, we will look at the compressor and the turbine. They are both **turbomachinery**: machines that transfer energy from a rotor to a fluid, or the other way around. The working principle of the compressor and the turbine is therefore quite similar.

5.1 Axial compressors

We will mainly look at axial compressors. This is because they are the most used type of compressors. Also, axial compressors work very similar to axial turbines.

5.1.1 Euler's equation for turbomachinery

Let's examine a rotor, rotating at a constant angular velocity ω . The initial radius of the rotor is r_1 , while the final radius is r_2 . A gas passes through the rotor with a constant velocity c . The rotor causes a moment M on the gas. The power needed by the rotor is thus $P = M\omega$.

It would be nice if we can find an expression for this moment M . For that, we first look at the force F acting on the gas. It is given by

$$F = \frac{d(mc)}{dt} = \dot{m}c, \quad (5.1.1)$$

where we have used the assumption that c stays constant. Only the tangential component F_u contributes to the moment. Every bit of gas contributes to this tangential force. It does this according to

$$dF_u = \dot{m}dc_u, \quad (5.1.2)$$

where c_u is the tangential velocity of the air. Let's integrate over the entire rotor. We then find that

$$M = \int_1^2 dM = \int_1^2 r dF_u = \dot{m} \int_1^2 r dc_u = \dot{m} (c_{u,2}r_2 - c_{u,1}r_1). \quad (5.1.3)$$

The power is now given by

$$P = M\omega = \dot{m} (c_{u,2}r_2 - c_{u,1}r_1)\omega = \dot{m} (c_{u,2}u_2 - c_{u,1}u_1). \quad (5.1.4)$$

In this equation, u denotes the speed of the rotor at a certain radius r . We have also used the fact that $\omega = u_1/r_1 = u_2/r_2$. The above equation is known as **Euler's equation for turbomachinery**.

5.1.2 The axial compressor power

Let's examine an axial compressor. This compressor has **rotors** and **stators**. The rotors are moving. They increase the kinetic energy of the gas. The stators are not moving. They are used to turn the kinetic energy into an increase in pressure.

Let's examine the velocities of the gas, as it passes through a rotor and a stator. The situation is shown in figure 5.1. At the point we're examining, the rotor is moving with a velocity u . The velocity of the gas, relative to the rotor, is denoted by v . The angle between the flow velocity c and the shaft axis is denoted by α . The angle between the rotor blade angle and the shaft axis is denoted by β .

The component of the velocity c in axial direction is denoted by c_a . It is assumed to be constant along the compressor. There is a relation between c_a and u . It is given by

$$u = c_a (\tan \alpha_1 + \tan \beta_1) = c_a (\tan \alpha_2 + \tan \beta_2). \quad (5.1.5)$$

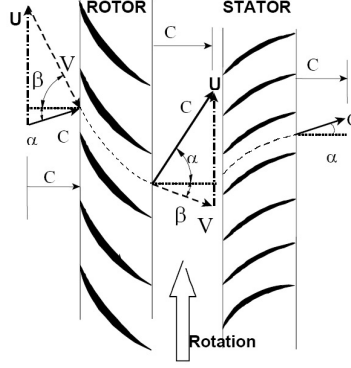


Figure 5.1: The velocities of the gas in the axial compressor.

It follows that

$$(\tan \alpha_1 + \tan \beta_1) = (\tan \alpha_2 + \tan \beta_2). \quad (5.1.6)$$

Let's try to use Euler's equation for turbomachinery. We know that $u_1 = u_2 = u$. We can also use the relations

$$c_{u,1} = c_a \tan \alpha_1 \quad \text{and} \quad c_{u,2} = c_a \tan \alpha_2. \quad (5.1.7)$$

Finally, we divide Euler's equation by the mass flow. This gives us the **specific power** W_s . We thus find that

$$W_s = u (c_{u,2} - c_{u,1}). \quad (5.1.8)$$

In reality, this isn't the compressor power. This is mainly because the axial flow velocity c_a varies for different radial distances r . To take this effect into account, we use the **work-done factor** λ . The power of the compressor is now given by

$$P = \lambda \dot{m} W_s. \quad (5.1.9)$$

Generally, λ becomes small for compressors with multiple stages.

5.1.3 Axial compressor properties

An important effect of the compressor is a rise in the total (stagnation) temperature. In fact, this rise is given by

$$\Delta T_{0s} = \frac{W_s}{c_p} = \frac{\lambda u c_a}{c_p} (\tan \alpha_2 - \tan \alpha_1). \quad (5.1.10)$$

If we take into account the isentropic efficiency η_{is} of the compression stage, then we can also find the pressure ratio. It is given by

$$\varepsilon = \left(1 + \eta_{is} \frac{\Delta T_{0s}}{T_{01}} \right)^{\frac{\gamma}{\gamma-1}} = \left(1 + \frac{\eta_{is} \lambda u c_a (\tan \alpha_2 - \tan \alpha_1)}{c_p T_{01}} \right)^{\frac{\gamma}{\gamma-1}}. \quad (5.1.11)$$

From this, we can also derive the degree of reaction. The **degree of reaction** Λ states which part of the total temperature rise is caused by the rotor. We thus have

$$\Lambda = \frac{\Delta T_{0s,rotor}}{\Delta T_{0s,rotor} + \Delta T_{0s,stator}}. \quad (5.1.12)$$

It can be shown that, for an axial compressor, we have

$$\Lambda = \frac{c_a}{2u} (\tan \beta_2 + \tan \beta_1) = 1 - \frac{c_a}{2u} (\tan \alpha_1 + \tan \alpha_2). \quad (5.1.13)$$

5.1.4 Three-dimensional axial compressors

Previously, we have considered axial compressors as two-dimensional. In reality, they are three-dimensional. The most important effect is that the flow is pushed to the outside of the compressor. This causes a pressure gradient, according to

$$\frac{dp}{dr} = \rho \frac{c_u^2}{r}. \quad (5.1.14)$$

We can combine this equation with the thermodynamic relation

$$dh = T ds + \frac{1}{\rho} dp = T ds + \frac{c_u^2}{r} dr. \quad (5.1.15)$$

We should also examine the definition of the total enthalpy. It is given by

$$h_0 = h + \frac{1}{2} c^2 = h + \frac{1}{2} (c_u^2 + c_a^2). \quad (5.1.16)$$

Let's take the derivative of the last two equations, with respect to r . The entropy usually doesn't vary much in radial direction, so we may neglect ds/dr . Combining all data will then give

$$\frac{dh_0}{dr} = \frac{c_u^2}{r} + c_u \frac{dc_u}{dr} + c_a \frac{dc_a}{dr}. \quad (5.1.17)$$

This equation is called the **vortex energy equation**. Usually, we also assume that the total enthalpy h_0 and the axial velocity c_a don't vary in radial direction. (So $dc_a/dr = dh_0/dr = 0$.) In this case, we can solve for c_u . It is given by

$$c_u r = \text{constant}. \quad (5.1.18)$$

This condition is known as the **free vortex condition**. It's often used, when designing compressors. The necessary twist of the compressor blades can be derived from it.

5.2 Compressor and turbine behaviour

Compressors and turbines often show strange behaviour. Let's take a look at the kind of phenomena that may occur, and how we can analyze them.

5.2.1 The characteristic

The compressor has several important parameters. There are the mass flow \dot{m} , the initial and final temperatures T_{02} and T_{03} , the initial and final pressures p_{02} and p_{03} , the shaft speed ω (also denoted as N or Ω), the rotor diameter D , and so on.

Let's suppose we'll be working with different kinds of compressors. In this case, it would be nice if we could compare these parameters in some way. To do that, dimensionless parameters are used. By using dimensional analysis, we can find that there are four dimensionless parameter groups. They are

$$\frac{\dot{m} \sqrt{RT_{02}}}{p_{02} D^2}, \quad \frac{p_{03}}{p_{02}}, \quad \frac{\omega D}{\sqrt{RT_{02}}} \quad \text{and} \quad \eta. \quad (5.2.1)$$

These parameter groups are known as the **mass flow parameter group**, the **pressure ratio**, the **shaft speed parameter group** and the **efficiency**. The efficiency can be either polytropic or isentropic. (These two efficiencies depend on each other anyway.)

The relation between the four dimensionless parameters can be captured in a graph, known as a **characteristic**. An example of a characteristic is shown in figure 5.2.

When applying dimensional analysis to a turbine, the same results will be found. However, this time the initial and final pressures are p_{04} and p_{05} . The initial and final temperatures are T_{04} and T_{05} .

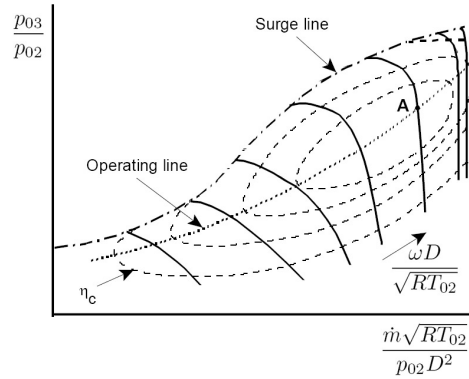


Figure 5.2: The characteristic graph of a compressor.

5.2.2 Stall

Let's examine the air entering the rotor. Previously, we have assumed that this air has exactly the right **angle of incidence** i to follow the curvature of the rotor blade. In reality, this is of course not the case. In fact, if the angle of incidence is too far off, then the flow can't follow the curvature of the rotor blades. This is called **stall**.

Stall usually starts at one rotor blade. However, this stall alters the flow properties of the air around it. Because of this, stall spreads around the rotor. And it does this opposite to the direction of rotation of the rotor. This phenomenon is called **rotating stall**.

Often, only the tips of the rotor blades are subject to stall. This is because the velocity is highest there. This is called **part span stall**. If, however, the stall spreads to the root of the blade, then we have **full span stall**.

For high compressor speeds ω , stall usually occurs at the last stages. On the other hand, for low compressor speeds, stall occurs at the first stages. Generally, the possibility of stalling increases if we get further to the left of the characteristic. (See also figure 5.2.)

5.2.3 Surge

Let's suppose we control the mass flow \dot{m} in a compressor, running at a constant speed ω . The mass flow \dot{m} effects the pressure ratio p_{03}/p_{02} . There can either be a positive or a negative relation between these two.

Let's examine the case where there is a negative relation between these two parameters. Now let's suppose we increase the mass flow \dot{m} . The pressure at the start of the compressor will thus decrease. However, the pressure upstream in the compressor hasn't noticed the change yet. There is thus a higher pressure upstream than downstream. This can cause **flow reversal** in the compressor.

Flow reversal itself is already bad. However, it doesn't stop here. The flow reversal causes the pressure upstream in the compressor to drop. This causes the compressor to start running again. The pressure upstream again increases. Also, the mass flow increases. But this again causes the pressure downstream to increase. Flow reversal thus again occurs. A rather unwanted cycle has thus been initiated.

This cyclic phenomenon is called **surge**. It causes the whole compressor to start vibrating at a high frequency. Surge is different from stall, in that it effects the entire compressor. However, the occurrence of stall can often lead to surge.

There are several ways to prevent surge. We can **blow-off** bleed air. This happens halfway through the compressor. This provides an escape for the air. Another option is to use **variable stator vanes**

(VSVs). By adjusting the stator vanes, we try to make sure that we always have the correct angle of incidence i . Finally, the compressor can also be split up into parts. Every part will then have a different speed ω .

Contrary to compressors, turbines aren't subject to surge. Flow simply never tends to move upstream in a turbine.

5.2.4 Choked flow

Let's examine the pressure ratio p_{04}/p_{05} in a turbine. Increasing this pressure ratio usually leads to an increase in mass flow \dot{m} . However, after a certain point, the mass flow will not increase further. This is called **choked flow**. It occurs, when the flow reaches supersonic velocities.

Choked flow can also occur at the compressor. If we look at the right side of figure 5.2, we see vertical lines. So, when we change the pressure ratio p_{03}/p_{02} at constant compressor speed ω , then the mass flow remains constant.

6. Loads and materials

The parts in a gas turbine are subject to several kinds of loads. To cope with these loads, the right materials need to be used. In this chapter, we'll take a look at the loads and materials used.

6.1 Loads and failures

We will take a look at the load types that are present in a gas turbine. We also examine how these load types can cause failure.

6.1.1 Load types

A lot of types of loads occur in the gas turbine. Let's take a look at a few

- **Centrifugal loads** are caused by high rotational speeds. The centrifugal force is proportional to $m\omega^2 r$.
- **Thermal loads** are caused by temperature changes. These loads are only present if parts can't expand freely. This can be caused by two reasons. Either the two connected parts are heated differently, or the parts have a different **coefficient of thermal expansion** (CTE).
- **Vibrational loads** occur when an object is excited by a frequency close to its **natural frequency**. We generally want to prevent vibrational loads. Therefore, components need to be designed such that their natural frequency isn't near to a frequency occurring in the gas turbine.
- **Pressure loads** occur due to pressure differences. They mainly occur in the compressor and the turbine. This causes bending of the blades. To prevent this, the blades are usually made such that the pressure loads are cancelled out by the centrifugal loads.

6.1.2 Types of failure

There are many ways in which a part of a gas turbine can fail. Again, we examine a couple of them.

- **Static failure** (or **overload**) is quite simple. The present load is bigger than the material strength. To make sure that static failure does not occur, the right material needs to be selected. Special attention has to be paid to the **yield strength** and the **ultimate strength**. These parameters often depend on the temperature T . Also, a (usually big) safety margin has to be taken into account.
- **Fatigue** is a failure type caused by a cyclic load. We can make a distinction between **High Cycle Fatigue** (HCF) and **Low Cycle Fatigue** (LCF). In HCF, a lot of cycles (usually over 10^6) are needed to cause fatigue failure. Only elastic stresses are present. HCF failure is often caused by vibrations. LCF is fatigue due to inelastic stresses.

An important parameter for fatigue is the number of cycles until failure. This can be derived from a **Wöhler curve**. In this curve, the number of cycles until failure is plotted for various stress levels. The Wöhler curve generally depends on the temperature.

Fatigue failure is caused by growing cracks. Cracks can be initiated by **foreign object damage** (FOD), like dirt or birds hitting the gas turbine. After the crack has been initiated, it will grow. The **crack growth** da/dN is expressed in the increase in crack distance da per cycle dN . Crack growth occurs in three stages. First, there is the **slow crack growth region**, in which the crack

growth is low. Second, there is the **power law region**. In this region, the crack growth increases. It does this according to the **Paris law**

$$\frac{da}{dN} = C (\Delta K)^n, \quad (6.1.1)$$

where C and n are constants. ΔK is the **load factor**. The third and last step is **final failure**, in which the growth rate diverges.

- **Creep** is inelastic deformation, caused by low forces at high temperatures. The magnitude of the **creep rate** $\dot{\epsilon}_{cr}$ highly depends on the temperature T . It depends on the stress level σ as well. The **creep strain** ϵ_{cr} shows a similar behaviour as the crack growth. Again, there are three stages. This time, the second stage is characterised by a power law like

$$\dot{\epsilon}_{cr} = AT^n \sigma^m. \quad (6.1.2)$$

- In **corrosion**, a corrosive medium (like oxygen, nitrogen or sulfur) reacts with the metal surface. Due to this, an embrittled layer is formed, from which large flakes can break away. Corrosion generally doesn't cause failure itself, but it encourages the other failure types.

Different parts are subject to different types of failure. Fatigue mainly occurs in rotating/moving parts, like the compressor and the turbine. Damage caused by **thermal degradation** is mostly present in parts with high temperatures, like the combustion chamber and the turbine.

6.2 Building the gas turbine

Let's take a look at how we build a gas turbine. What materials do we select? What coatings do we apply? And how do we make sure that the gas turbine stays in working order?

6.2.1 Materials

First, we look at the compressor blades. They are subjected to high mechanical loads, due to the high rotational speed. We thus want to have materials with a high **specific strength** σ_{UTS}/ρ and a high **specific stiffness** E/ρ . Stainless steel, titanium and nickel alloys are suitable for this. Of these three, titanium gives the largest weight savings. Drawbacks are the high cost of titanium and its flammability.

Now we examine the turbine blades. The first stage turbine rotor blades are the most severely loaded components in the gas turbine. We thus want to have a high quality material. It should have a high specific strength, a high thermal mechanical fatigue resistance, a high creep resistance, sufficient ductility (to sustain FOD) and a reasonable oxidation and corrosion resistance. All these properties need to be maintained at high temperatures. Although **blade-cooling systems** limit the temperature as much as possible, the turbine blades will still get quite hot.

6.2.2 Manufacturing

Manufacturing gas turbine parts can be quite difficult. This especially holds for the turbine blades. Due to the high-strength materials, forging is not possible. **Casting** techniques are therefore used. To be precise, the method of **investment casting** is used.

Coatings are also often applied. This can be done for different reasons. First of all, coatings protect against oxidation/corrosion. A second reason for applying coatings, is for thermal protection. These so-called **thermal barrier coatings** (TBCs) have a low thermal conductivity. They therefore thermally insulate the part they're covering.

When applying coatings, either **diffusion coating** or **overlay coating** is used. In diffusion coating, a chemical reaction is used to put the coating onto the metal part. In overlay coating, the coating is simply sprayed/deposited onto the part.

6.2.3 Design philosophies

Let's ask ourselves an important question: When do we replace parts? The answer to this question depends on the **design philosophy** we apply.

In the **safe-life** philosophy, no inspections are applied. The part is simply replaced after it has reached its **life limit**. The life limit is determined using Wöhler curves. A downside of this method is that parts are often replaced, that do not need replacing yet. Also, components are often found damaged before the life limit is reached.

In the **damage tolerance** philosophy, the possibility of cracks/flaws is taken into account. Therefore, regular inspections are performed. These inspections are performed such that cracks are detected before they can cause serious harm. However, at the end of the life limit, the parts are replaced, no matter if damage has occurred or not.

In the **retirement for cause** (RFC) philosophy, life extension beyond the life limit is applied. To do this, **risk assessments** are used. First, the allowable risk is set. This allowable risk then determines the increase of the life limit. In the RFC philosophy, it is exactly known how much risks are taken. This is an advantage.