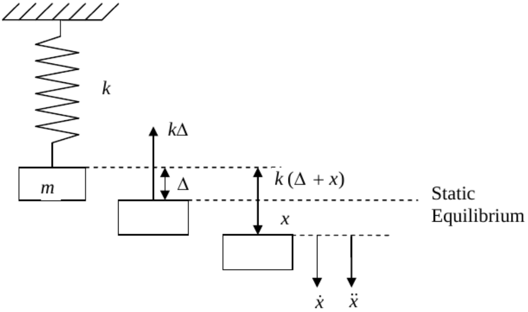
AE31002 Structural Dynamics
Natural Frequency of SDOF System

Anup Ghosh

Simple Harmonic Motion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k (Δ + x)
x
Static
Equilibrium
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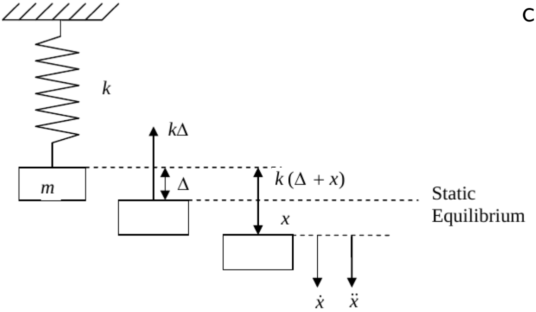

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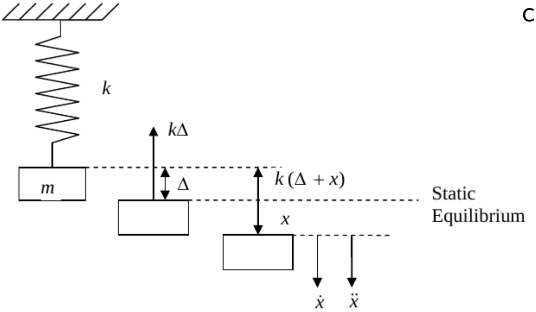

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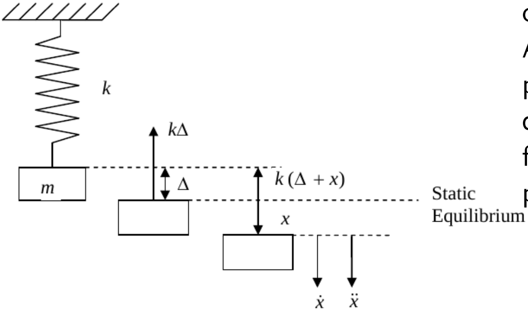

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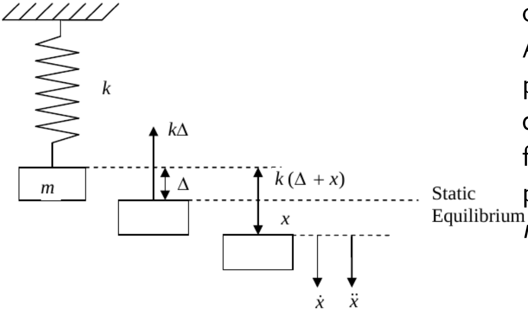

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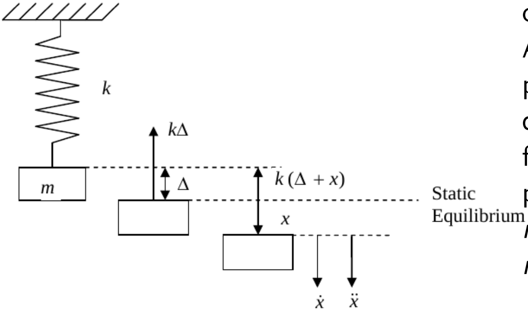

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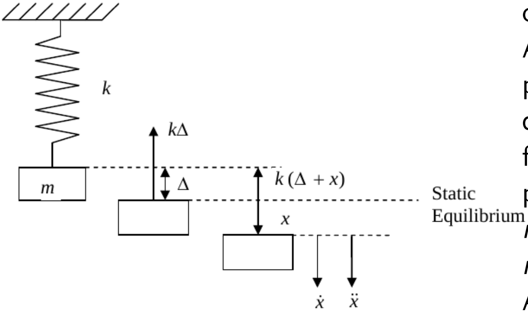

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A 2nd order differential equation.

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$\Rightarrow \quad x = x_0\ cos\ \omega_n t + \frac{v_0}{\omega_n} sin\ \omega_n t$

Also may be written as $x = C\ sin(\omega_n t + \alpha)$

where, $tan\ \alpha = \frac{x_0}{v_0/\omega_n}$

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Substituting the above values,

$x = (A_1 + A_2)\ cos\ \omega_n t + i(A_1 - A_2)\ sin\ \omega_n t$

if we consider, $(A_1 + A_2) = cos\ \phi, \quad i(A_1 - A_2) = sin\ \phi$

$x = A\ cos(\omega_n t - \phi)$

SDOF – Complex number solution

To find out the values of the constants we need to solve a initial value problem,

For $t = 0, \ x = x_0, \ and \ \dot{x} = v_0$

$x_0 = A \ cos \ \phi$ and $v_0 = A\omega_n \ sin \ \phi$

$A = \sqrt{x_0^2 + (\frac{v_0}{\omega_n})^2}$ and $tan \ \phi = \frac{v_0/\omega_n}{x_0}$

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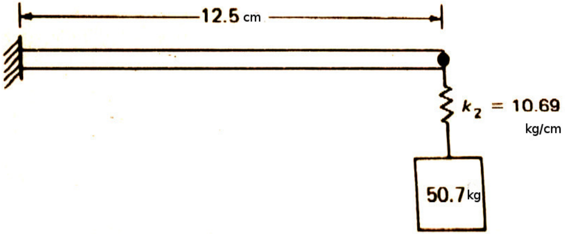

Determine the natural frequency of the system shown in the figure. The system consists of a weight of 50.7kg to a horizontal cantilever beam through a coil spring k_2. The cantilever beam has a thickness $t = \frac{1}{4}$cm, a width $b = 1$cm and modulus of elasticity $E = 30x10^6\ kg/cm^2$, and a length $L = 12.5\ cm$. The coil spring has a stiffness, $k_2 = 10.69\ kg/cm$.

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SDOF – Examples

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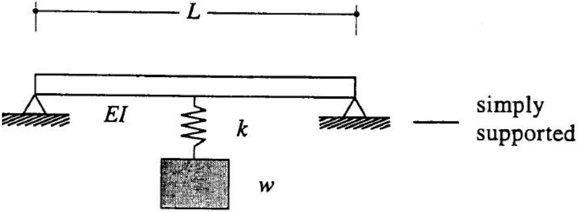
Equivalent stiffness of the system

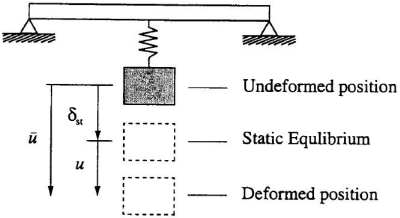
$$k_{eq} = \{\frac{1}{k_1} + \frac{1}{k_2}\}^-1$$

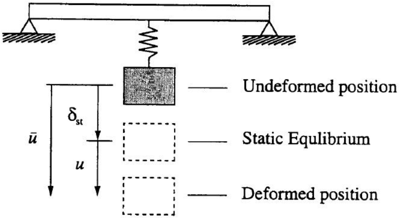
$$k_{eq} = \{\frac{1}{60} + \frac{1}{10.69}\}^{-1} = 9.07 kg/cm^2$$

Natural frequency of the system is
$\omega = \sqrt{k_e/m} = \sqrt{9.07\ x\ 0.981/50.7}$

Determine the natural frequency of a weight w suspended from a spring at the mid point of a simply supported beam shown below. The length of the beam is L, and its flexural rigidity is EI. The spring stiffness is k. Assume the beam to be mass-less.




Undeformed position
δ_{st}
$\bar{u}$
Static Equlibrium
u
Deformed position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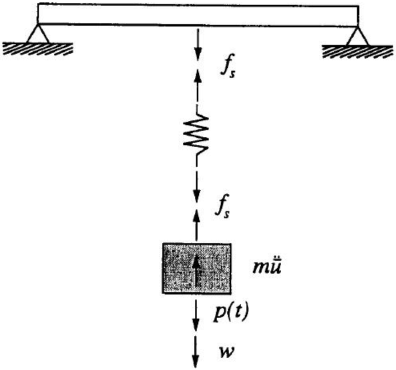
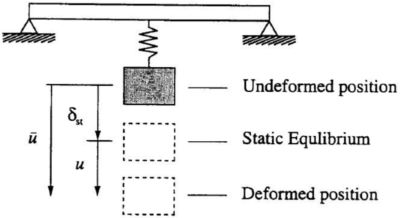
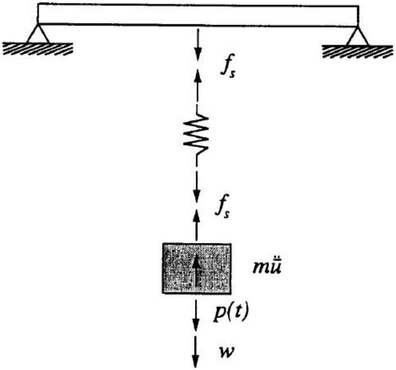
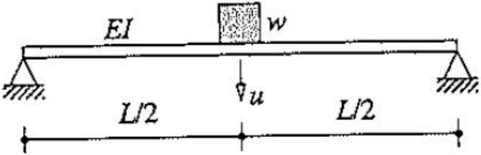

fs
fs
mü
p(t)
w

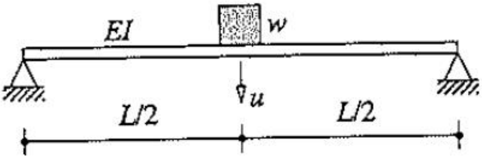




The Static equilibrium position may be considered as the origin. So the equation of motion becomes m



$\Delta = PL^3/48EI$



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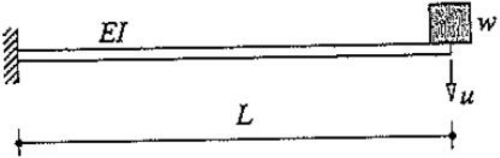

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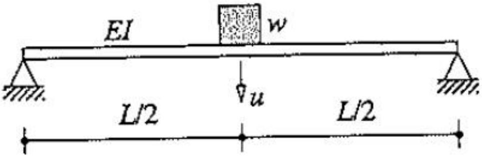

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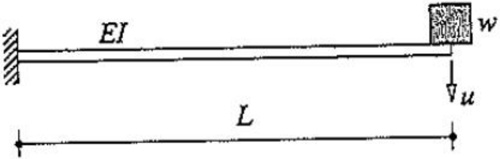

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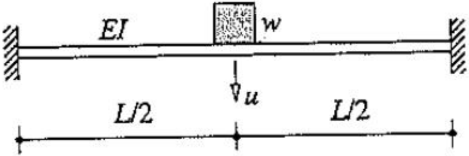

$$\Delta = PL^3/192EI$$