AE31002 Aerospace Structural Dynamics

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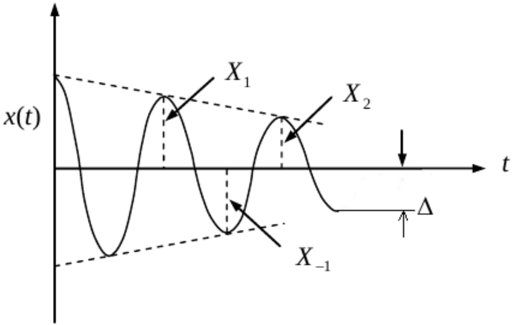
- The sign of the damping force is opposite to that of velocity, the differential equation of motion for each sign is valid only for half cycle intervals.
- The motion will cease, however, when the amplitude is less than Δ, at which position the spring force is insufficient to over come static friction force, which is generally greater than the kinetic friction force.

By work-energy principle



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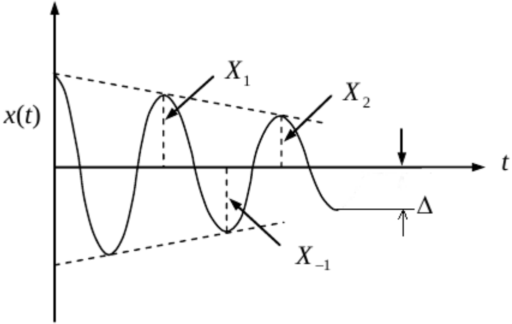
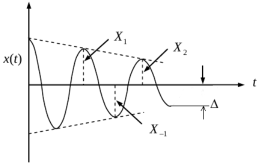


Observation

- The motion will cease, when the amplitude is less than Δ.
- At this stage position the spring force is not sufficient to over come static friction force, which is generally greater than the kinetic friction force.

By work-energy principle



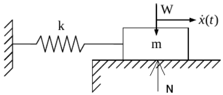
Choose a half cycle with amplitude X_1 to X_{-1} and equate the energy of the system when the kinetic energy is zero because the velocity at both the places are zero and the change in potential energy of the spring must equal the energy released by the damping mechanism.

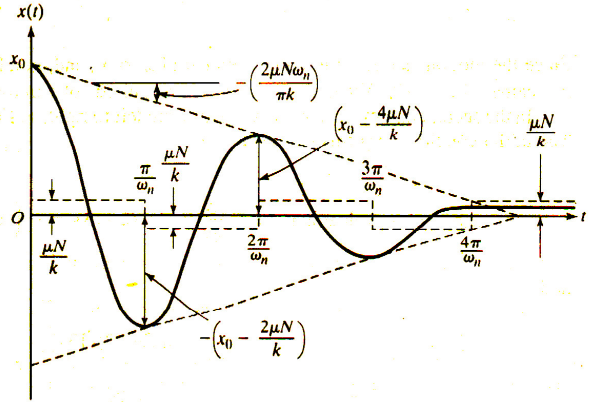
$$\frac{1}{2}k(X_1^2 - X_{-1}^2) - F_d(X_1 + X_{-1}) = 0$$

Change in amplitude is $= X_1 - X_{-1} = \frac{2F_d}{k}$
Decay of amplitude per cycle$= = X_1 - X_2 = \frac{4F_d}{k}$

Trigonometric approach

$$F_d = \mu w = \mu N$$





Trigonometric approach

The equation of motion

$$m\ddot{x} + F_d \operatorname{sgn}(\dot{x}) + kx = 0 \quad ,$$

Where '$sgn(\dot{x})$' denotes sign of $\dot{x}$ and represents a function having the value $+1$ if its argument $\dot{x}$ is positive and the value -1 if its arguments is negative.

Mathematically

$$\operatorname{sgn}(\dot{x}) = \frac{\dot{x}}{|\dot{x}|}$$

So, we can write

$$m\ddot{x} + kx = -F_d \qquad \mathit{for} \;\; \dot{x} > 0$$

$$m\ddot{x} + kx = F_d \qquad \mathit{for} \;\; \dot{x} < 0$$

Trigonometric approach

Considering the 2nd equation first and bringing back the reference of homogeneous solution of the SDOF undamped system
$\Rightarrow \quad x = x_0 \; cos \; \omega_n t + \frac{v_0}{\omega_n} sin \; \omega_n t$, where $\omega_n = \sqrt{k/m}$
The equation becomes

$$m\ddot{x} + kx = F_d = f_d k \quad \text{(Assumed)}$$

$\ddot{x} + \omega_n^2 x = \omega_n^2 f_d$, where, $f_d = F_d/k$ is equivalent to displacement.
Now,
$x - f_d = Acos\omega_n t + Bsin\omega_n t$ is a general solution of the above equation. Solving the above equation with initial conditions as $x(0) = x_0$ and $\dot{x}(0) = 0$ and the solution is

$$x(t) = (x_0 - f_d) \; cos \; \omega_n t + f_d$$

This equation is valid for $0 \leq t \leq t_1$, where t_1 is the time at which the velocity again reduces to zero. Now the velocity is

$$\dot{x}(t) = -\omega_n (x_0 - f_d) \; sin \; \omega_n t$$

Trigonometric approach

The lowest nontrivial solution satisfies the condition $\dot{x}(t_1) = 0$ is $t_1 = \pi/\omega_n$ and the associated displacement is $x(t_1) = -(x_0 - 2f_d)$
If the above amplitude is sufficient enough to overcome the static friction, the mass acquires a positive velocity and , so that the motion must satisfy the other equation of motion in Coulomb damping, i.e.,

$$\ddot{x} + \omega^2 x = -\omega^2 f_d$$

With the initial conditions $x(t_1) = -(x_0 - 2f_d)$ and $\dot{x}(t_1) = 0$

$$x(t) = (x_0 - 3f_d)\ cos\ \omega_n t - f_d$$

The equation is valid for the time period , $t_1 \leq t \leq t_2$, where $t_2 = 2\pi/\omega_n$
At the end of complete cycle, decay of amplitude is $4f_d$, since, $x(t_2) = x_0 - 4f_d$

The amplitude drop in each half cycle is $2f_d$ or $\frac{2\mu N}{k}$,
The number of half cycle elapsed before the motion ceases is
$x_0 - r\frac{2\mu N}{k} \leq \frac{\mu N}{k}$

$$r \geq \left\{ \frac{x_0 - \frac{\mu N}{k}}{\frac{2\mu N}{k}} \right\}$$

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- The motion is periodic in case of Coulomb damping, while it can be non-periodic in a viscously damped system (over-damped case)
- The system comes to rest after some time with Coulomb damping, whereas the motion theoretically continues for ever (perhaps with an infinitely small amplitude) with viscous damping.

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- The system comes to rest after some time with Coulomb damping, whereas the motion theoretically continues for ever (perhaps with an infinitely small amplitude) with viscous damping.
- The amplitude reduces linearly with Coulomb damping, whereas it reduces exponentially in case of viscous damping.

For torsional damping

Let us assume the magnitude of the frictional torque is T.
Equation of motion for the first half cycle is

$$I_0\ddot{\theta} + k_\theta \theta = -T$$

and for the other half is

$$I_0\ddot{\theta} + k_\theta \theta = T$$

The frequency of the system remains the same, i.e.,

$$\omega_n = \sqrt{\frac{k_\theta}{I_0}}$$

Amplitude of motion at the end of r-th half cycle (θ_r), is given by

$$\theta_r = \theta_0 - r\frac{2T}{k_\theta}$$

For torsional damping

The motion ceases when

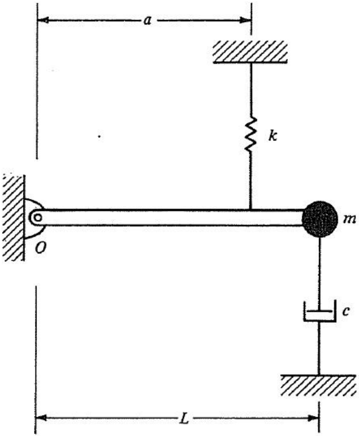
$$r \geq \left\{ \frac{\theta_0 - \frac{T}{k_\theta}}{\frac{2T}{k_\theta}} \right\}$$

A metal block placed on a rough surface , is attached to a spring and is given an initial displacement of 10cm from its equilibrium position. After five cycles of oscillation in 2sec, the final position of the metal block is found to be 1cm, from its equilibrium position. Find the coefficient of friction berween the surface and the metal block. (Assume Coulomb damping)

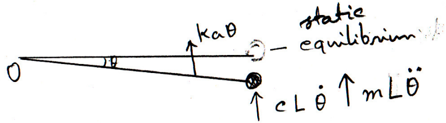
Since five cycle of oscillation were observed to take place in 2sec. the period of oscillation is $2/5 = 0.4$sec, and the frequency of oscillation $\omega_n = \sqrt{\frac{k}{m}} = \frac{2\pi}{T} = \frac{2\pi}{0.4} = 15.708$rad/sec.
Reduction of amplitude of oscillation in 5 cycles is

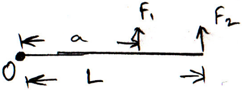
$$5\left(\frac{4\mu mg}{k}\right) = 0.10 - 0.01 = 0.09m$$

$$\mu = \frac{0.09k}{20mg} = \frac{0.09\omega_n^2}{20g} = 0.1132$$



Calculate the frequency of the damped oscillation of the system shown for the values k = 4000 lb/in (7.0051 x 10^5 N/m), c = 20 lb s/in (3502.54 N s/m), m 10 lb s^2/in (1751.27 kg), a = 50 in (1.27m), and L = 100 in (2.54 m). Determine the value of critical damping.





$$mL\ddot{\theta} + cL\dot{\theta} + F_2 = 0,$$

$$mL\ddot{\theta} + cL\dot{\theta} + k\frac{a^2}{L}\theta = 0,$$

$$\ddot{\theta} + \frac{c}{m}\dot{\theta} + \frac{k}{m}\frac{a^2}{L^2}\theta = 0,$$

$$F_1 a = F_2 L \Rightarrow$$

$$F_2 = F_1\frac{a}{L} = k\frac{a^2}{L}\theta$$

$$\ddot{\theta} + 2\xi\omega_n\dot{\theta} + \omega_n^2\theta = 0, \quad where \quad \frac{c}{m} = 2\xi\omega_n \quad and \quad \omega_n^2 = \frac{k}{m}\frac{a^2}{L^2}$$

Now,

$$\omega_d = \omega_n\sqrt{(1-\xi^2)} = \sqrt{\left(\frac{k}{m}\frac{a^2}{L^2} - \frac{c^2}{4m^2}\right)} = 9.95 rad/s$$

to find c_c, find ω_n, then ξ from $c/2m = \xi\omega_n$ and get c_c