AE31002 Aerospace Structural Dynamics
Vibration Instruments

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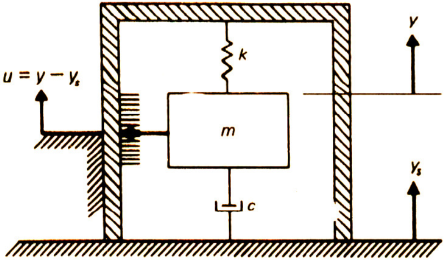
With low natural frequencies

Let us think again about the transmissibility of relative displacement of the oscillator due to support motion, i.e.,

$$\frac{u(t)}{x_0} = \frac{r^2 \; sin \; (\bar{\omega}t - \theta)}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$

With respect to the amplitude and the figure below, it becomes



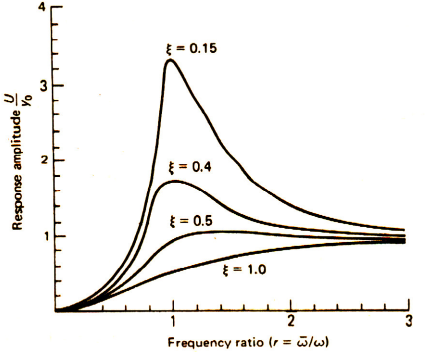
$$\frac{U}{y_0} = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$

$$where, \quad y_s(t) = y_0 \; sin \; \bar{\omega}t$$

The instrument logs the relative displacement of the oscillator w.r.t. the cashing, it is known as **seismograph**.

With low natural frequencies – Seismograph

The frequency response w.r.t. relative displacement of the system is



Observation

– Response is essentially constant w.r.t the frame for frequency ratio r $>$ 1 and ξ=0.5.

With low natural frequencies – Seismograph

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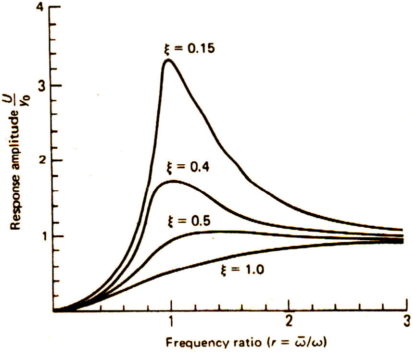

Observation

- Response is essentially constant w.r.t the frame for frequency ratio r $>$ 1 and ξ=0.5.
- The response of a properly damped instrument of this type is essentially proportional to the base-displacement amplitude for high frequencies of motion of base.

With low natural frequencies – Seismograph

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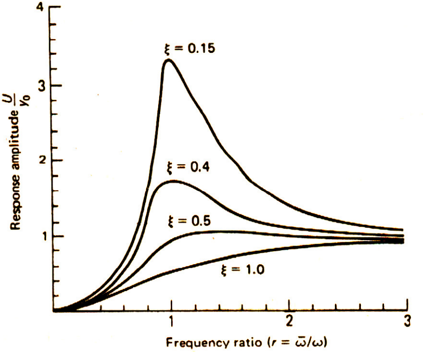

Observation

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- The response of a properly damped instrument of this type is essentially proportional to the base-displacement amplitude for high frequencies of motion of base.

The range of applicability of the instrument is increased by reducing the natural frequency, i.e., by reducing the spring stiffness or increasing the mass. ($\omega = \sqrt{k/m}$)

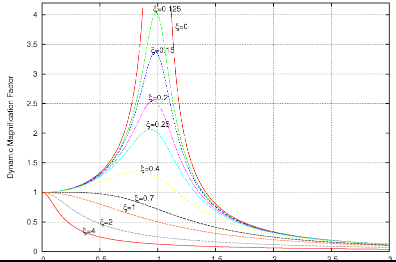
With high natural frequencies – Accelerometer

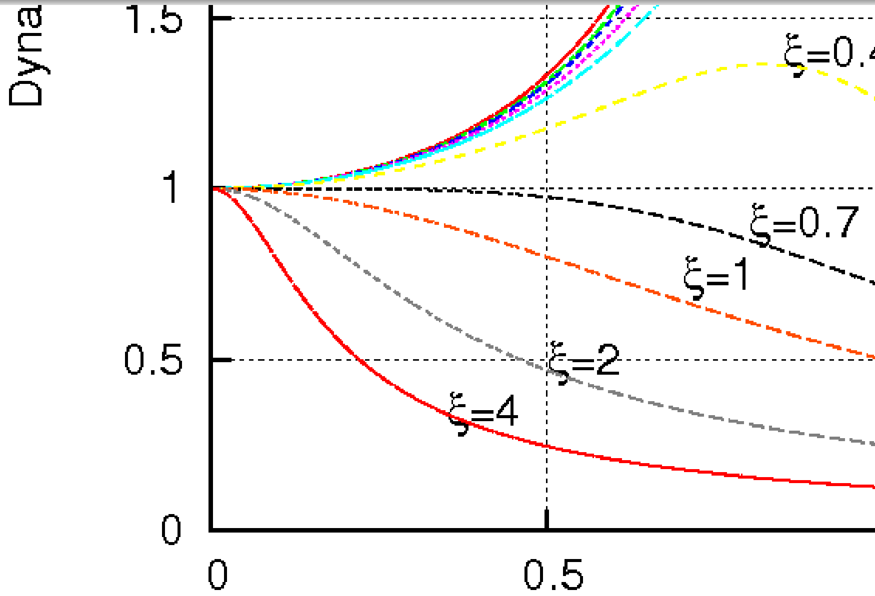
Let us consider a similar instrument under a harmonic acceleration of $\ddot{x}_s = \ddot{x}_0 \ sin \ \bar{\omega} t$ (considering the basic support motion derivation)

$$m\ddot{u} + c\dot{u} + ku = -m\ddot{x}_0 \ sin \ \bar{\omega} t$$

The steady state response of this system expressed as the dynamic magnification factor is them

$$D = \frac{U}{m\ddot{x}_0/k} = \frac{1}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$




Dyna
1.5
1
0.5
0
0
0.5
ξ=0.4
ξ=0.7
ξ=1
ξ=2
ξ=4

With high natural frequencies – Accelerometer

Observations

- For a damping ratio ξ=0.7, the value of the response is nearly constant in the frequency range $0 < r < 0.6$ (approx).

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- Response indicated by this instrument will be directly proportional to the base-acceleration amplitude for frequencies up to about 6/10 of natural frequency.
- Its range of application will increase by increasing the natural frequency, i.e., by increasing the stiffness of the spring or by decreasing the mass of the oscillator.