AE31002 Aerospace Structural Dynamics
Energy Dissipated by Damping

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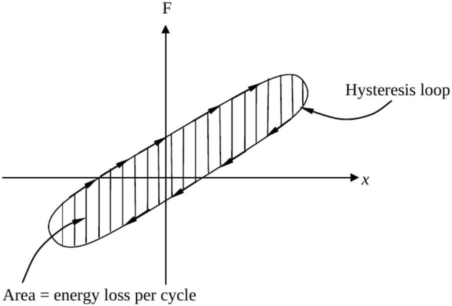
In all cases, however, the force-displacement curve will enclose an area, referred to as the **hysteresis loop**, that is **proportional to the energy loss per cycle**.

In this section ω is the forcing frequency



The energy lost per cycle due to a damping force F_d is

$$W_d = \oint F_d dx$$

$$= \oint c\dot{x}(\dot{x})dt = \oint c\dot{x}^2 dt$$

where $F_d = c\dot{x}$. For the simplest case of damping let the steady state response be

$$\begin{aligned} x &= X\sin(\omega t - \phi) \\ \dot{x} &= \omega X\cos(\omega t - \phi) \\ \ddot{x} &= -\omega^2 X\sin(\omega t - \phi) \end{aligned}$$

The energy dissipated per cycle W_d

$$\begin{aligned} &= c\omega^2 X^2 \int_0^{\frac{2\pi}{\omega}} \cos^2(\omega t - \phi)dt \\ &= c\omega^2 X^2\left(\frac{\pi}{\omega}\right) \\ &= \pi c\omega X^2 \end{aligned}$$

In a similar line work done by spring and inertia forces can be written as

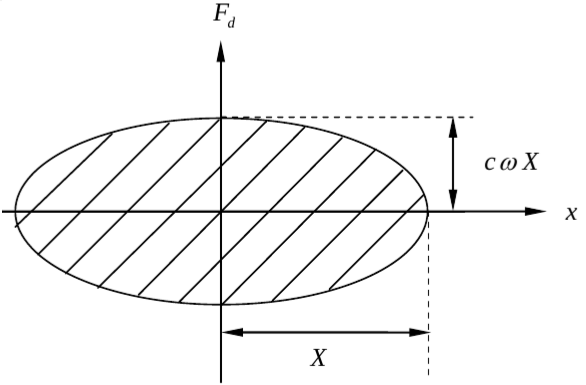
$$
\begin{aligned}
W_s &= \oint F_s\, dx & F_s &= kx \\
&= \oint k\, x\, \dot{x}\, dt \\
&= \oint k\, X^2 \omega \sin(\omega t - \phi) \cos(\omega t - \phi) dt \\
&= 0 \\
W_I &= \oint F_I\, dx & F_I &= m \ddot{x} \\
&= \oint m\, \ddot{x}\, \dot{x}\, dt = \oint m \{-\omega^2 X \sin(\omega t - \phi)\}\{\omega X \cos(\omega t - \phi)\} \\
&= -\omega^3 m X^2 \oint \sin(\omega t - \phi) \cos(\omega t - \phi)\, dt = 0
\end{aligned}
$$

Let,

$$\begin{aligned}\dot{x} &= \omega X \cos(\omega t - \phi) \\ &= \pm \omega X \sqrt{1 - \sin^2(\omega t - \phi)} \\ &= \pm \omega \sqrt{X^2 - x^2(t)}\end{aligned}$$

the damping force become $F_d = c\dot{x} = \pm c\omega\sqrt{X^2 - x^2}$

or $\left(\dfrac{F_d}{c\omega X}\right)^2 + \left(\dfrac{x}{X}\right)^2 = 1$ an equation of ellipse.



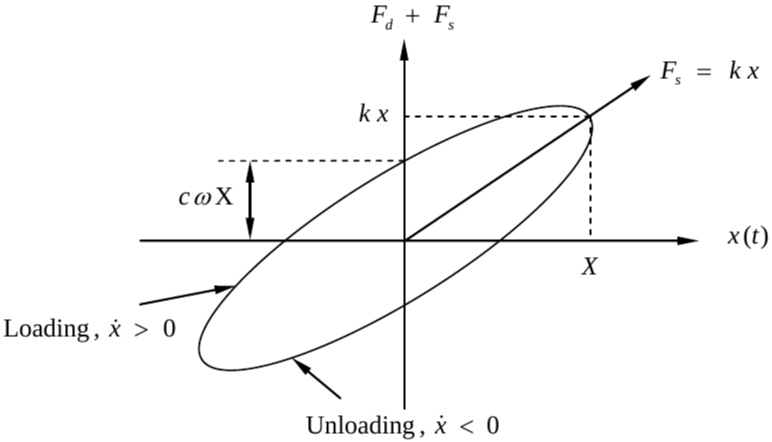
$$\text{Area} = \pi(c\,\omega X)(X) = \pi c\,\omega X^2$$

$$\left(\frac{F_d}{c\,\omega X}\right)^2 + \left(\frac{x}{X}\right)^2 = 1$$

It is of interest to examine the total (elastic & damping) resisting force that is measured in an experiment.

$$\begin{aligned} F_d + F_s &= c\dot{x} + kx \\ &= kx \pm c\omega\sqrt{X^2 - x^2} \end{aligned}$$

A plot of $F_s + F_d$ against x is an ellipse. The energy dissipated by damping is still the area enclosed by the ellipse because the area enclosed by the single valued elastic force, $F_s = kx$, is zero.



Damping properties of materials are listed in many different ways depending on the technical areas to which they are applied. Of these we list two relative energy units that have wide usage.

First of these is **specific damping capacity**, defined as the energy loss per cycle W_d divided by the pick potential energy U.

$$\text{Specific damping capacity} = \frac{E_d}{E_{so}}$$

$$\text{Where} \quad E_{so} = \tfrac{1}{2} k X^2$$

$$\therefore \quad \text{Specific damping capacity} = \frac{\pi c \omega X^2}{\frac{1}{2} k X^2} = \frac{2\pi c \omega}{k}$$

The second quantity is the **loss coefficient** or **specific damping factor**, is define as the ratio of damping energy loss per radian, $W_d/2\pi$, divided by the peak potential or strain energy U.

$$\eta = \frac{w_d}{2\pi U} = \frac{c\,\omega}{k}$$

Structural Damping or Solid Damping

When materials are cyclically stressed, energy is dissipated internally within the material itself. **Experiments** by several investigators indicate that for **most structural metals**, such as steel, aluminium, the energy dissipated per cycle is **independent of the frequency over a wide frequency range and proportional to the square of the amplitude of vibration.** Internal damping fitting this classification is called **solid damping or structural damping**.

Energy dissipated by structural damping may be written as $W_d = \alpha X^2$ Where α is constant with units of force/displacement. Using the concept of equivalent viscous damping gives

$$\pi C_{eq} \omega X^2 = \alpha X^2$$

$$\text{or,} \quad C_{eq} = \frac{\alpha}{\pi\omega}$$

and

$$m\ddot{x} + \left(\frac{\alpha}{\pi\omega}\right)\dot{x} + kx = F(t)$$

Complex stiffness: In the calculation of the **flutter** speed of air plane wings and tail surfaces, the concept of complex stiffness is used. It is arrived at by assuming the oscillations to be harmonic, which enables to be written as

$$m\ddot{x} + \left(k + i\frac{\alpha}{\pi}\right)x = F_0 e^{i\omega t}$$

Difining $\gamma = \frac{\alpha}{k\pi}$, the equation becomes $m\ddot{x} + k(1 + i\gamma)x = F_0 e^{i\omega t}$. The quantity $k(1 + i\gamma)$ is called the complex stiffness and γ is the **structural damping factor**.

With the solution $x = Xe^{i\omega t}$, the steady state amplitude becomes

$$X = \frac{F_0}{(k - m\omega^2) + i\gamma k}$$

with an amplitude of resonance $|X| = \frac{F_0}{\gamma k}$ Comparing this with the resonance amplitude of a system with viscous damping. $|X| = \frac{F_0}{2\xi k}$ or $\gamma = 2\xi$. **We conclude that with equal amplitudes at resonance, the structural damping factor is equal to twice the viscous damping factor.**